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Convergence of approximate solution of delay Volterra integral equations

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Abstract

In this paper, sinc-collocation method is discussed to solve Volterra functional integral equations with delay function $\theta(t)$. Also the existence and uniqueness of numerical solutions for these equations are provided. This method improves conventional results and achieves exponential convergence. Numerical results are included to confirm the efficiency and accuracy of the method.

Keywords: Volterra functional integral equations; delay function; sinc-collocation.

1 Introduction

Delay integral equations arise widely in scientific fields such as physics, biology, ecology, control theory, etc. Due to the practical application of these equations, they must be solved successfully with efficient numerical approaches. In recent years, there have been extensive studies in convergence properties and stability analyses of these numerical methods, see, for example, [10]. The numerical solutions of integral equations with delays have also been discussed by several authors such as Brunner [1], Li and Kuang [5], Linz and Wang [6].

Sinc methods for approximating the solutions of Volterra integral equations have received considerable attention mainly due to their high accuracy. These approximations converge rapidly to the exact solutions as the number of sinc points increases. Systematic introduction of these methods can be found in [9]. In [11] sinc-collocation method is emplyed to solve Volterra

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integral equations with smooth kernels. The analytical and numerical techniques used in these works can be extended to delay integral equations.

The main objective of the current study is to implement the sinc-collocation method for Volterra functional integral equation of the form

$$y(t) = g(t) + (\mathcal{V}y)(t) + (\mathcal{V}_\theta y)(t), \qquad t \in I := [0, T]. \tag{1}$$

The Volterra delay integral operators $\mathcal{V}$ and $\mathcal{V}_\theta$ (from $C(I) \to C(I)$) describing these equations are defined by

$$(\mathcal{V}y)(t) := \int_0^t K_1(t, s)y(s)ds$$

and

$$(\mathcal{V}_\theta y)(t) := \int_0^{\theta(t)} K_2(t, s)y(s)ds,$$

respectively, and the delay function θ is subject to the following conditions:
(D1) $\theta(0) = 0$, and θ is strictly increasing on the interval I;
(D2) $\theta(t) \leq \overline{q}t$, $t \in I$, for some $\overline{q} \in (0, 1)$;
(D3) $\theta \in C^d(I)$ for some $d \geq 0$.

We will refer to a θ that satisfies (D1) as a vanishing delay function (or, in short, a *vanishing delay*). The linear case, $\theta(t) = qt = t - (1 - q)t =: t - \tau(t)$ $(0 < q < 1)$ (proportional delay) is also known as the pantograph delay function [4]. In this paper we consider vanishing delay but our methods can be use with nonvanishing delay too.

The layout of this paper is as follows. In Section 2, the solvability of equation (1) is stated. Section 3 outlines some of the main properties of sinc function that is necessary for the formulation of the delay integral equation. Sinc-collocation method is considered in Section 4. In section 5, we analyze the existence and uniqueness of numerical solutions. In Section 6, the order of scheme convergence using the new approach is described. Finally, Section 7 contains the numerical experiments.

2 Existence and uniqueness of solutions

In the present section, we state the solvability of integral equations with vanishing delay. The following theorem generalizes Volterras 1897 classical result on the existence and uniqueness of solutions for the equation (1) with $\theta(t) = qt$ $(0 < q < 1)$.

Theorem 1. *[3] Assume that the given functions g, K_1 and K_2 in (1) satisfy 1) $g \in C(I)$, $K_1 \in C(D)$, and $K_2 \in C(D_\theta)$, where*

$$D := \{(t,s) : 0 \le s \le t \le T\}, \quad D_\theta := \{(t,s) : 0 \le s \le \theta(t), t \in I\};$$

2) $\theta(t)$ is subject to the assumptions (D1)-(D3).
Then for each $g \in C(I)$ there exists a unique function $y \in C(I)$ which solves the equation (1) on I.

3 Review of the sinc approximation

In this section, we will review sinc function properties, sinc quadrature rule, and the sinc method. These are discussed thoroughly in [9]. For any $h > 0$, the sinc basis functions are given by

$$S(j,h)(z) = \mathrm{sinc}(\frac{z - jh}{h}), \qquad j = 0, \pm 1, \pm 2, \ldots,$$

where

$$\mathrm{sinc}(z) = \begin{cases} \frac{\sin(\pi z)}{\pi z}, \; z \ne 0; \\ 1, \; z = 0. \end{cases}$$

The sinc function form for the interpolating point $z_k = kh$ is given by

$$S(j,h)(kh) = \begin{cases} 1, \; k = j; \\ 0, \; k \ne j. \end{cases}$$

They are based in the infinite strip D_d in the complex plane

$$D_d = \{w = u + iv : |v| < d\}.$$

To construct approximation on the interval $[0,T]$, we consider the conformal map

$$\phi(z) = \ln(\frac{z}{T - z}).$$

The map ϕ carries the eye-shaped region

$$D = \big\{z \in \mathcal{C} : |\arg(\frac{z}{T - z})| < d\big\}.$$

The function

$$z = \phi^{-1}(w) = \frac{Te^w}{1 + e^w}$$

is an inverse mapping of $w = \phi(z)$. We define the range of ϕ^{-1} on the real line as

$$\Gamma = \{\psi(u) = \phi^{-1}(u) \in D : -\infty < u < \infty\}.$$

The sinc grid points $z_k \in (0, T)$ in D will be denoted by x_k because they are real. For the evenly spaced nodes $\{kh\}_{k=-\infty}^{\infty}$ on the real line, the image which corresponds to these nodes is denoted by

$$x_k = \phi^{-1}(kh) = \frac{Te^{kh}}{1+e^{kh}}, \qquad k = \pm 1, \pm 2, \ldots .$$

Definition 1. Let D be a simply connected domain which satisfies $(a, b) \subset D$ and α and c_1 be a positive constant. Then $\mathcal{L}_\alpha(D)$ denotes the family of all functions $u \in \mathrm{Hol}(D)$ which satisfy

$$|u(z)| \leqslant c_1 |Q(z)|^\alpha \tag{2}$$

for all z in D where $Q(z) = (z-a)(b-z)$.

The next theorem shows the exponential convergence of the sinc approximation.

Theorem 2. *Let $u \in \mathcal{L}_\alpha(D)$, let N be a positive integer, and let h be selected by the formula $h = \sqrt{\frac{\pi d}{\alpha N}}$, then there exists positive constant c_2, independent of N, such that*

$$\sup_{t\in(a,b)} |u(t) - \sum_{j=-N}^{N} u(t_j) S(j,h)(\phi(t))| \leqslant c_2 \sqrt{N} e^{-\sqrt{\pi d \alpha N}}.$$

The error analysis of the sinc indefinite integration has been given in [7].

Theorem 3. *Let $uQ \in \mathcal{L}_\alpha(D)$ for d with $0 < d < \pi$. Let $h = \sqrt{\frac{\pi d}{\alpha N}}$. Then there exists a constant c_2, which is independent of N, such that*

$$\sup_{t\in(a,b)} \left| \int_a^t u(s)ds - h \sum_{j=-N}^{N} \frac{u(t_j)}{\phi'(t_j)} J(j,h)(\phi_{SE}(t)) \right| \leqslant c_3 e^{-\sqrt{(\pi d \alpha N)}}, \tag{3}$$

where

$$J(j,h)(x) = \frac{1}{2} + \int_0^{\frac{x}{h}-j} \frac{\sin(\pi t)}{\pi t} dt.$$

4 Sinc-collocation method

In this section, we apply sinc-collocation method to solve equation (1) which we state again for the convenience of the reader:

$$y(t) = g(t) + \int_0^t K_1(t,s)y(s)ds + \int_0^{\theta(t)} K_2(t,s)y(s)ds$$

if $t = 0$ we have $y(0) = g(0)$. For ease of calculation, we employ the transformation

$$u(t) = y(t) - \frac{T-t}{T} g(0),$$

in this case $u(0) = 0$. Then the above problem becomes

$$u(t) = f(t) + \int_0^t K_1(t,s)u(s)ds + \int_0^{\theta(t)} K_2(t,s)u(s)ds \tag{4}$$

where

$$f(t) := g(t) - \frac{1}{T}(T-t)g(0) + \frac{1}{T}g(0)\left\{\int_0^t K_1(t,s)(T-s)ds + \int_0^{\theta(t)} K_2(t,s)(T-s)ds\right\}.$$

Now, let $u(x)$ be the exact solution of (4) that is approximated by the following expansion

$$u_n(t) = \sum_{j=-N}^{N} u(t_j)S(j,h)o\phi(t) + u(t_{N+1})w(t), \tag{5}$$

we choose $w(t)$ so that above formula interpolate function u at the points t_j, so

$$w(t) = \frac{1}{T}\left(t - \sum_{j=-N}^{N} t_j S(j,h)(\phi(t))\right)$$

where the points t_j are defined by

$$t_j = \begin{cases} \phi^{-1}(jh),\ j = -N, \ldots, N; \\ T,\ j = N+1. \end{cases}$$

By replacing approximate solution (5) in $t = t_k$ in the equation (4), it follows that

$$\begin{aligned} &\sum_{j=-N}^{N} u_j S(j,h)(\phi(t_k)) + u_{N+1}w(t_k) \\ &= \sum_{j=-N}^{N} u_j \int_0^{t_k} K_1(t_k,s)S(j,h)(\phi(s))ds + u_{N+1}\int_0^{t_k} K_1(t_k,s)w(s)ds \\ &+ \sum_{j=-N}^{N} u_j \int_0^{\theta(t_k)} K_2(t_k,s)S(j,h)(\phi(s))ds + u_{N+1}\int_0^{\theta(t_k)} K_2(t_k,s)w(s)ds \\ &+ f(t_k). \end{aligned} \tag{6}$$

We are interested in approximating the integral in above equation by the quadrature formula presented in (3). Then by using Theorem 3, we obtain

$$\int_0^{t_k} K_1(t_k,s)S(j,h)o\phi(s)ds \approx h\frac{K_1(t_k,t_j)}{\phi'(t_j)}J(j,h)(\phi(t_k)), \quad k=-N,\dots,N+1.$$

From definition of t_k we can write

$$J(j,h)(\phi(t_k)) = \begin{cases} J(j,h)(kh),\ k=-N,\dots,N; \\ 1,\ k=N+1. \end{cases}$$

The analogue of above equation we have

$$\int_0^{\theta(t_k)} K_2(t,s)S(j,h)o\phi(s)ds \approx h\frac{K_2(t_k,t_j)}{\phi'(t_j)}J(j,h)(\phi_k), \quad k=-N,\dots,N+1$$

in which $\phi_k := \phi(\theta(t_k))$, in next section these formula will be discussed. Finally, let

$$\begin{aligned} K_{1k} &= \int_0^{t_k} K_1(t_k,s)w(s)ds, \\ K_{2k} &= \int_0^{\theta(t_k)} K_2(t_k,s)w(s)ds, \\ b_k &= K_{1k}+K_{2k}, \qquad k=-N,\dots,N+1. \end{aligned} \tag{7}$$

By using relation (3), we can approximate b_k in the following form

$$\begin{aligned} K_{1,k} &:= \int_0^{t_k} K_1(t_k,s)w(s)ds \\ &= \frac{1}{T}\int_0^{t_k} K_1(t,s)\left(s-\sum_{j=-N}^{N} t_j S(j,h)(\phi(s))\right)ds \\ &= \frac{1}{T}\left(\int_0^{t_k} sK_1(t,s)ds - \sum_{j=-N}^{N} t_j \int_0^{t_k} K_1(t,s)S(j,h)(\phi(s))ds\right) \\ &= \frac{1}{T}\left(\int_0^{t_k} sK_1(t,s)ds - h\sum_{j=-N}^{N} t_j K_1(t_k,t_j)\frac{1}{\phi'(t_j)}J(j,h)(\phi(t_k))\right) \end{aligned} \tag{8}$$

Thus equation (6) is written as

$$\begin{aligned} &u_k - h\sum_{j=-N}^{N}\frac{1}{\phi'(t_j)}\left\{K_1(t_k,t_j)J(j,h)(\phi(t_k))+K_2(t_k,t_j)J(j,h)(\phi_k)\right\}u_j \\ &-b_k u_{N+1} = f(t_k). \end{aligned} \tag{9}$$

This linear system of equations is equivalent to (4). By solving this system, the unknown coefficients u_j are determined. We rewirte the linear system (7) in matrix form

$$[\mathcal{I}-\mathcal{A}]\mathbf{U}_N=\mathbf{F} \tag{10}$$

where

$$\mathcal{A}_{k,j}=\frac{h}{\phi'(t_j)}\left\{K_1(t_k,t_j)J(j,h)(\phi(t_k))+K_2(t_k,t_j)J(j,h)(\phi_k)\right\},$$
$$k=-N,\dots N+1,\quad j=-N,\dots N,$$

$$\mathcal{A}_{k,N+1}=b_k,\qquad k=-N,\dots N+1,$$
$$\mathbf{U}_N=[u_{-N},\dots,u_{N+1}]^t,\qquad \mathbf{F}=[f(t_{-N}),\dots,f(t_{N+1})]^t.$$

5 Existence and uniqueness of the sinc-collocation solution

In this section, we study the existence and uniqueness of the solution to (8).

Lemma 1. *For $x\in\mathbb{R}$, the function $J(j,h)(x)$ is bounded by*

$$|J(j,h)(x)|\leqslant 1.1.$$

Theorem 4. *Assume that K_1, K_2 and f in the Volterra integral equation (4) are continuous on their respective domains D, D_θ and I. Then there exists an $\overline{h}>0$ so that for any $h\in(0,\overline{h})$ the linear algebraic system (8) has a unique solution $\boldsymbol{U}_N$.*

Proof. We know that

$$\mathcal{A}_{k,j}=\frac{h}{\phi'(t_j)}\left\{K_1(t_k,t_j)J(j,h)(\phi(t_k))+K_2(t_k,t_j)J(j,h)(\phi_k)\right\},$$
$$k=-N,\dots N+1,\quad j=-N,\dots N,$$
$$\mathcal{A}_{k,N+1}=b_k,\qquad k=-N,\dots N+1,$$

so we can write

$$\|\mathcal{A}\|_\infty=\max_{k=-N,\dots N+1}\big\{h\sum_{j=-N}^{N}\frac{1}{\phi'(t_j)}\left|J(j,h)(\phi(t_k))K_1(t_k,t_j)+K_2(t_k,t_j)J(j,h)(\phi_k)\right|+|b_k|\big\}.$$

Using Lemma 1, we have

$$\|\mathcal{A}\|_\infty \leqslant \max_{k=-N,\ldots N+1} \big\{1.1h \sum_{j=-N}^{N} \frac{1}{\phi'(t_j)} |K_1(t_k,t_j)+K_2(t_k,t_j)| + |b_k|\big\}.$$

Theorem 3 and continuty K_1 and K_2 and equations (7) and (8) give

$$|b_k| \leqslant ce^{-\sqrt{\pi d\alpha N}}.$$

Therefore

$$\|\mathcal{A}\|_\infty \leqslant 1.1h \sum_{j=-N}^{N} \frac{1}{\phi'(t_j)} |K_1(t_k,t_j)+K_2(t_k,t_j)| + ce^{-\sqrt{\pi d\alpha N}}.$$

Thus the elements of the matrice $\mathcal{A}$ are all bounded. The Neumann Lemma then shows that the inverse of the matrix $\mathcal{I}-\mathcal{A}$ exists whenever $\|\mathcal{A}\|_\infty < 1$. This clearly holds whenever h is sufficiently small. In other words, there is an $\overline{h} > 0$ so that for any $h < \overline{h}$ matrix $\mathcal{A}$ has a uniformly bounded inverse. The assertion of Theorem 4 now follows. □

6 Convergence analysis

The convergence of the sinc-collocation method which is introduced in the previous sections is discussed in the present section. It is assumed that u is the exact solution of Eq. (4) and $\mathcal{U}_N$ is an approximation of the sinc method. Firstly, we state the following lemma which is used subsequently.

Lemma 2. *([8]) Let $h > 0$. Then it holds that*

$$\sup_{x\in\mathbb{R}} \sum_{j=-N}^{N} |S(j,h)(x)| \leqslant \frac{2}{\pi}(3+\ln N).$$

In the following theorem, we will find an upper bound for the error.

Theorem 5. *Let $\mathcal{U}_N(x)$ is the approximate solution of integral equation (4). Then there exists a constant c_5 independent of N such that*

$$\sup_{x\in(0,T)} |u(x)-\mathcal{U}_N(x)| \leqslant c_5\sqrt{N}\ln N e^{-\sqrt{\pi d\alpha N}}. \tag{11}$$

Proof. For collocation error $e := u - \mathcal{U}_N$

$$\sup_{t\in(0,T)}\left|e(t)-\sum_{j=-N}^{N}S(j,h)(\phi(t))e_j+e_{N+1}w(t)\right|$$
$$\leqslant \sup_{t\in(0,T)}\left|e(t)-\sum_{j=-N}^{N}S(j,h)(\phi(t))e_j\right|+|e_{N+1}|\sup_{t\in(0,T)}|w(t)|$$

$$\leqslant c_1\sqrt{N}e^{-\sqrt{\pi d\alpha N}}+|e_{N+1}|\frac{1}{T}\sup_{t\in(0,T)}\left|t-\sum_{j=-N}^{N}t_jS(j,h)(\phi(t))\right|$$
$$\leqslant c_1\sqrt{N}e^{-\sqrt{\pi d\alpha N}}+|e_{N+1}|\frac{1}{T}c_1'\sqrt{N}e^{-\sqrt{\pi d\alpha N}}$$
$$\leqslant c\sqrt{N}e^{-\sqrt{\pi d\alpha N}},$$

so we can write

$$e(t)=\sum_{j=-N}^{N}S(j,h)(\phi(t))e_j+e_{N+1}w(t)+c\sqrt{N}e^{-\sqrt{\pi d\alpha N}}.$$

For $t=t_j$ it satisfies the error equation

$$e(t)=(\mathcal{V}e)(t)+(\mathcal{V}_\theta e)(t).$$

The contribution of $\mathcal{V}$ in the above error equation is described by

$$(\mathcal{V}e)(t_j)=\int_0^{t_j}K_1(t_j,s)e(s)ds$$
$$=\int_0^{t_j}K_1(t_j,s)\left\{\sum_{k=-N}^{N}S(j,h)(\phi(s))e_k+e_{N+1}w(s)+c\sqrt{N}e^{-\sqrt{\pi d\alpha N}}\right\}$$

$$=h\sum_{k=-N}^{N}\frac{K_1(t_j,t_k)}{\phi'(t_k)}J(j,h)(\phi(t_k))e_k+K_{1j}e_{N+1}+c_2e^{-\sqrt{\pi d\alpha N}}+t_jc\sqrt{N}e^{-\sqrt{\pi d\alpha N}}$$
$$=h\sum_{k=-N}^{N}\frac{K_1(t_j,t_k)}{\phi'(t_k)}J(j,h)(\phi(t_k))e_k+K_{1j}e_{N+1}+c'\sqrt{N}e^{-\sqrt{\pi d\alpha N}},$$

also for delay operator $\mathcal{V}_\theta$ we have

$$(\mathcal{V}_\theta e)(t_j)=\int_0^{\theta(t_j)}K_2(t_j,s)e(s)ds$$
$$=h\sum_{k=-N}^{N}\frac{K_2(t_j,t_k)}{\phi'(t_k)}J(j,h)(\phi_k)e_k+K_{2j}e_{N+1}+c''\sqrt{N}e^{-\sqrt{\pi d\alpha N}}.$$

Thus, the representation of e_j has the form

$$e_j = h \sum_{k=-N}^{N} \frac{1}{\phi'(t_k)} \left\{ K_1(t_j,t_k)J(j,h)(\phi(t_k)) + K_2(t_j,t_k)J(j,h)(\phi_k) \right\} e_k \\ + b_j e_{N+1} + c\sqrt{N} e^{-\sqrt{\pi d \alpha N}}$$

we may write the collocation equation as

$$[\mathcal{I} - \mathcal{A}]\mathbf{e} = c\sqrt{N}e^{-\sqrt{\pi d\alpha N}}\mathbf{I}.$$

Here, $\mathcal{I}$ and $\mathbf{I}$ denotes the identify matrix and constant vector 1, repectively. It thus follows from Theorem 4 that uniform bound exists for $(\mathcal{I} - \mathcal{A})^{-1}$, so

$$\|\mathbf{e}\| \leqslant c\sqrt{N}e^{-\sqrt{\pi d\alpha N}}. \tag{12}$$

Hence, by Lemma 2 and (12) we can obtain the upper bound (11). □

7 Illustrative examples

In this section, the theoretical results of the previous sections are used for two numerical examples. The numerical experiments are implemented in *Matlab*.

Example 1. The pantograph Volterra integral equation

$$y(t) = g(t) + \int_{\theta(t)}^{t} K(t,s)y(s)ds$$

with $\theta(t) = qt$, $k(t,s) = ts$, and $g(t) = (-t^2 + t + 1)e^t + t(qt-1)e^{qt}$, has the exact solution $y(t) = e^t$. The results are shown in Table 1.

Table 1: Values of $\|E\|_\infty$ for Example 1

$N \setminus q$	0.01	0.09	0.1	0.5	0.99
10	3.7266×10^{-6}	4.5684×10^{-6}	4.1900×10^{-6}	8.8432×10^{-5}	1.7511×10^{-5}
30	5.5006×10^{-10}	3.6148×10^{-9}	5.2450×10^{-9}	4.0946×10^{-7}	7.5530×10^{-8}
50	8.6331×10^{-13}	1.0118×10^{-10}	1.3788×10^{-10}	1.0852×10^{-8}	1.1792×10^{-9}
70	7.9936×10^{-15}	5.4152×10^{-12}	7.3683×10^{-12}	5.8153×10^{-10}	1.0688×10^{-10}
90	3.2196×10^{-15}	4.3609×10^{-13}	5.9374×10^{-13}	4.6795×10^{-11}	8.6006×10^{-12}

Example 2. Consider

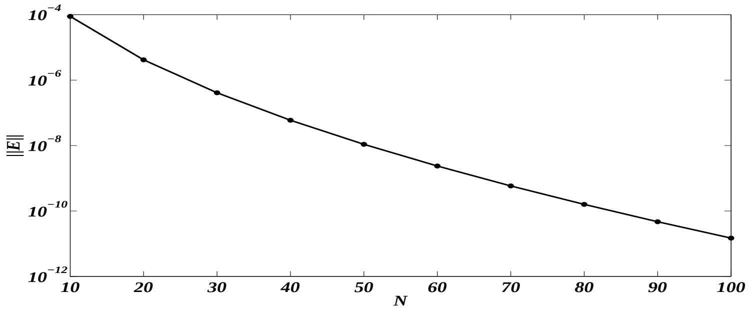

Figure 1: The errors for $q = 0.5$ in Example 1

$$y(t) = g(t) + \int_0^{t^r} K(t,s)y(s)ds$$

with $k(t,s) = s - t$. We choose $g(t)$ so that its exact solution is $y(t) = t - t^2$. Table 2 shows the numerical results.

Table 2: Values of $\|E\|_\infty$ for Example 2

$N \setminus r$	0.01	0.09	0.1	0.5	0.99
10	1.1098×10^{-6}	4.6373×10^{-6}	5.3418×10^{-6}	5.2686×10^{-6}	5.5796×10^{-6}
30	4.0592×10^{-10}	8.1311×10^{-10}	8.1496×10^{-10}	7.9947×10^{-10}	8.1273×10^{-10}
50	8.2652×10^{-13}	1.3631×10^{-12}	1.4027×10^{-12}	1.3289×10^{-12}	1.3898×10^{-12}
70	4.8580×10^{-15}	7.1871×10^{-15}	7.0991×10^{-15}	6.8972×10^{-15}	7.0499×10^{-15}
90	1.4498×10^{-16}	2.2706×10^{-16}	1.9428×10^{-16}	1.6653×10^{-16}	3.3306×10^{-16}

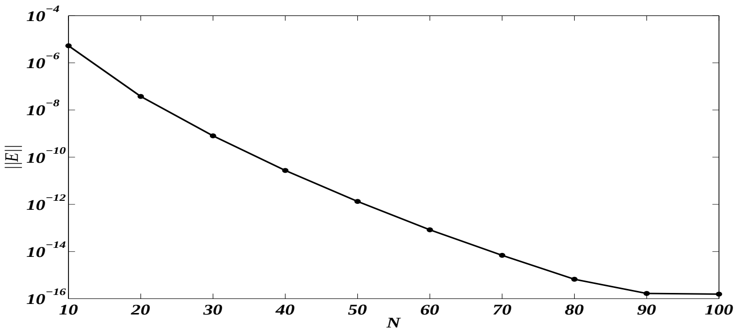

Figure 2: The errors for $r = 0.5$ in Example 2

8 Conclusion

Several methods has been presented for the numerical solution of equation (1) in the special cases for example $\theta(t) = qt$ [2]. We propose a numerical algorithm in order to solve the delay integral equation (Eq. (1)) where θ is general function. Our method has been shown theoretically and numerically to be extremely accurate and achieve exponential convergence with respect to N.

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همگرایی جواب تقریبی معادلات انتگرال ولترای تأخیری

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دریافت مقاله ۲ تیر ۱۳۹۴، دریافت مقاله اصلاح شده ۲۵ آذر ۱۳۹۴، پذیرش مقاله ۱۷ دی ۱۳۹۴

چکیده : در این مقاله، روش هم محلی سینک برای حل معادلات انتگرال تابعی ولترا با تابع تأخیر بحث شده است. هم چنین وجود و یکتایی جواب های تقریبی برای این معادلات اثبات شده است. این روش نتایج متعارف را بهبود می بخشد و همگرایی نمایی را نتیجه می دهد. نتایج عددی برای تأیید دقت و کارایی روش ارائه شده اند.

کلمات کلیدی : معادلات انتگرال تابعی ولترا؛ تابع تأخیر؛ هم‌محلی سینک.