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New classes of infinite groups

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Abstract

In this article, we consider some new classes of groups, namely, M_p-groups, T_0-groups, ϕ-groups, ϕ_0-groups, groups with finitely embedded involution, which were appeared at the end of twenties century. These classes of infinite groups with finiteness conditions were introduced by V.P. Shunkov. We give some review of new results on these classes of groups.

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1 Introduction

At the beginning of twentieth century the classes of Frobenius groups and Chernikov groups were introduced.

A group of the form $G = F \leftthreetimes H$ is called a *Frobenius group*, if the following conditions are satisfied:

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1) $H^g \cap H = 1$, $g \in G \setminus H$;

2) $G \setminus F = \bigcup_{g \in G} H^g \setminus \{1\}$.

Any finite extension of a direct product of finite number of quasi cyclic groups is called a *Chernikov group*.

In this article we consider some new classes of groups: M_p-groups, T_0-groups, Φ-groups, Φ_0-groups, groups with finitely embedded involution, which were introduced at the end of twentieth century. These classes of groups are closely connected with Frobenius and Chernikov groups. One of these classes, namely, T_0-groups firstly was discussed in Iran, while V.P. Shunkov took part as an invited speaker at the 22nd Annual Iranian Mathematics Conference held in Mashhad in 1991 and gave couple of talks on the concept of "T_0-groups" [20, 21].

Let G be a group, then an element $a \in G$ is called an *involution* or *almost regular* if it is of order two or its centralizer $C_G(a)$ is finite, respectively. The element $b \in G$ is called strictly real with respect to involution a, say, if it is transferred to its inverse by conjugating with a.

2 M_p-groups

Definition 2.1 *A group G is called an M_p-group, if for its infinite normal complete Abelian p-subgroup B with minimality condition and for any element a of order p the following conditions are satisfied:*

a) locally finite p-subgroups of $C_G(a)B/B$ are finite;

b) if some complete Abelian p-subgroup C of the group G contained in the set $\bigcup_{g \in G} \langle a, a^g \rangle$, then $C \leq B$.

The subgroups B and $\langle a \rangle$ of the group G are called the kernel and the hand of M_p-group, respectively.

We remind that the concept of M_p-groups was introduced by V.P. Shunkov at the end of 1983.

If a is the element of order p in an M_p-group G then the following properties hold:

1) all locally finite p-subgroups of $C_G(a)$ are finite;

2) all locally finite p'-subgroups of $C_G(a)$ are finite;

3) all locally finite subgroups of $C_G(a)$ are finite.

In the corresponding properties of the M_p-group G, the hand $\langle a\rangle$ is called p-finite hand, p'-finite hand and finite hand, respectively.

The following are the examples of M_p-groups (with finite hands):

Any Chernikov group with infinite p-subgroup and with almost regular element of order p is an M_p-group [15].

Any golomorfic extension of an infinite Chernikov p-group using the group of external automorphisms is also an M_p-group [15].

The kernel of an M_p-group may be different from the maximal complete Abelian p-subgroup of the group. Indeed, let $G = H \times T$, where $H = P \leftthreetimes (c)$, let P be an infinite complete Abelian p-group, $|c| = p$, $C_H(c)$ be finite, T is a free product of quasicyclic p-group S and cyclic group (b) of order p. Clearly G is an M_p-group with kernel P and with finite hand (bc), besides P is included in the maximal complete Abelian p-subgroup $P \times S \neq P$ [15].

Next theorem gives us the sign of non simplicity of an infinite group.

Theorem 2.2 *(Shunkov V.P. [15]). Let G be a group without involutions and B be its infinite complete Abelian p-subgroup, which satisfies the following conditions:*

1) $H = N_G(B)$ is an M_p-group with kernel B and p-finite hand $\langle a\rangle$;

2) for an arbitrary element $g \in G \setminus H^{\#}$, the subgroup $\langle a, a^g\rangle$ is finite;

3) $|C_G(a) : H \cap C_G(a)| < \infty$ and $H \cap C_G(a)$ contains all p'-elements of finite order from $C_G(a)$;

4) if Q is a finite $\langle a\rangle$-invariant q-subgroup from H with $Q \cap C_G(a) \neq 1$ and $q \neq p$, then $N_G(Q) \leq H$. Then $B \triangleleft G$.

Theorem 2.3 *(Shunkov V.P. [15]). Let G be a group without involutions, a an element of prime order p of G with centralizer $C_G(a)$, which is a finite p-subgroup, satisfies the condition that all subgroups $\langle a, a^g\rangle$, $g \in G$ are finite. Then G has* *complete Abelian normal p-subgroup B such that in G/B Sylow p-subgroups, containing element aB are finite and conjugate, and the number of such subgroups is finite.*

Condition of finiteness of subgroups $\langle a, a^g \rangle$ in the above theorem is necessary. Infact, it is enough to consider example of free product of infinite Chernikov p-group with almost regular element of order p ($p \neq 2$) and some non-trivial periodic group without involutions. Periodic product of such groups give us an example of periodic group without involutions, in which all the conditions of the theorem are valid, except the mentioned condition, but the statement of the theorem for such a group is incorrect.

The hand $\langle a \rangle$ is called *reduced* in M_p-group G with kernel B, if $B \cap C_G(a)$ is a reduced Abelian group. Remind that a *reduced Abelian group* is an Abelian group which dose not have any complete Abelian subgroups.

Theorem 2.4 *(Shunkov V.P. [16]). Let G be a group without involutions, B be its complete Abelian p-subgroup, a be an element of order p of G, satisfies the following conditions:*

1) $H = N_G(B)$ is an M_p-group with p-kernel B and reduced hand $\langle a \rangle$;

2) for every element $g \in G \setminus H^{\#}$, the subgroup $\langle a, a^g \rangle$ is finite;

3) $|C_G(a) : H \cap C_G(a)| < \infty$ and H contains all p'-elements of finite orders from $C_G(a)$;

4) if Q is a finite $\langle a \rangle$-invariant q-subgroup of H with condition $Q \cap C_G(a) \neq \langle 1 \rangle$ and $q \neq p$, then $N_G(Q) \leq H$. Then B is normal in G and G is an M_p-group with kernel B and reduced hand $\langle a \rangle$.

In M_p-group G the hand $\langle a \rangle$ is called *regular* if $C_G(a)$ is finite and every locally finite $\langle a \rangle$-invariant primary subgroup of G is finite.

In all well-known examples of M_p-groups with regular hand, which are not periodic almost nilpotent groups, satisfy the condition that any regular hand generates an infinite subgroup with some conjugates with it.

In this connection the next question will be occurred: Is M_p-group with regular hand exist which does not satisfy this condition and is not a periodic almost nilpotent group?

The answer to this question is negative in the class of groups without involutions (see the following theorem), that is in the class of groups without involutions periodic almost nilpotent M_p-groups with regular hand and only such groups do not satisfy this condition. But for M_p-groups with regular hand with involutions ($p \neq 2$), this question is still open even in the class of locally finite groups. The solution of this question is connected with the characterization of well-known simple groups in the class of periodic groups.

Now, we give some examples of M_p-groups with regular hands, see [17]:

— Infinite dihedral group is an M_2-group with regular hand;

— Novikov-Adian group [1] is an M_p-group with regular hand for any prime number p from the set of prime divisors of orders of elements of the group;

— Free product of non-trivial finite groups is an M_p-group with regular hand;

— Periodic product of groups without involutions [2] is an M_p-group with regular hand.

Theorem 2.5 *(Shunkov V.P. [17]). A group G is an M_p-group without involutions with regular hand $\langle a \rangle$ if and only if it is a periodic almost nilpotent group, when it satisfies the condition that: the subgroups $\langle a, a^g \rangle$, $g \in G$, are finite.*

Theorem 2.6 *(Shunkov V.P. [17]). A group G is an M_2-group with regular hand $\langle a \rangle$ if and only if is periodic almost Abelian group with finite Sylow subgroups, whenever all the subgroups $\langle a, a^g \rangle$, $g \in G$ are finite.*

For more information of the properties of the class of M_p-groups one may refer to V.P. Shunkov's monograph [18].

In [9], there was the sign of non-simplicity of an infinite group. As a corollary from that result the characterization of M_p-groups with involutions was studied.

M_p-groups with hands of orders not equal to two, were studied in the class of groups without involutions in [15] by V.P. Shunkov. M_p-groups with hands of orders 2 were studied by V.O. Gomer [6]. In the next theorem, the characterization of non-simplicity of M_p-group with hand of order not equal to three is discussed.

If G is an M_p-group with finite hand $\langle a \rangle$, then $C_G(a)$ can have infinite p-subgroups. For example, it is enough to take direct product of the above mentioned groups and Novikov-Adian free periodic group [1].

Theorem 2.7 *(Kozulin S.N., Senashov V.I., Shunkov V.P. [9]) Let G be a group, B be its infinite complete Abelian p-subgroup ($p \neq 3$), satisfying the following conditions:*

1) $H = N_G(B)$ is an M_p-group with a kernel B and p-finite hand $\langle a \rangle$;

2) for every $g \in G \setminus H^{\#}$, subgroups of the form $\langle a, a^g \rangle$ are finite and solvable;

3) $|C_G(a) : H \cap C_G(a)| < \infty$ and $H \cap C_G(a)$ contains all p'-elements of finite order from $C_G(a)$;

4) if Q is a finite $\langle a \rangle$-invariant q-subgroup from H with the condition that $Q \cap C_G(a) \neq 1$ and $q \neq p$, then $N_G(Q) \leq H$;

5) in G all finite $\langle a \rangle$-invariant p'-subgroups are solvable subgroups.

Then $B \lhd G$.

We shall remind, that the hand of M_p-group is called p-finite, if in $C_G(a)$, locally finite p-subgroups are finite.

This theorem generalizes V.O. Gomer's Theorem [6], for $p = 2$.

3 T_0-groups

At the beginning of last century the concept of a T_0-group appeared in the articles of V.P. Shunkov. This class of groups is defined by finiteness conditions. We recall the definition of the class of T_0-groups.

Definition 3.1 *Let G be a group with involutions, i be some of its involution. We call G to be a T_0-group, if it satisfies the following conditions:*

1) all subgroups of the form $< i, i^g >$, $g \in G$, are finite;

2) Sylow 2-subgroups of G are cyclic or generalized quaternions groups ;

3) the centralizer $C_G(i)$ is infinite and has a finite periodic part;

4) the normalizer of any non-trivial $< i >$-invariant finite subgroup of G is either contained in $C_G(i)$, or has a periodic part being a Frobenius group with Abelian kernel and with finite complement of even order;

5) $C_G(i) \neq G$ and for any element c of $G \setminus C_G(i)$, strictly real relating to i (i. e. such that $c^i = c^{-1}$), there exists an element s_c in $C_G(i)$, such that the subgroup $< c, c^{s_c} >$ is infinite.

Now we give the construction of Shunkov's example of T_0-group from [22] based on well-known example of S.P. Novikov and S.I. Adjan [1].

Example of T_0-group (a). Let $A = A(m, n)$ be a torsion-free group $A(m, n)$, which is a central extension of cyclic group with the group $B(m, n)$, for $m > 1$, $n > 664$ an odd number [1]. The group $A(m, n)$ has non-trivial center $Z(A) =< d >$ and $A/ < d >$ is isomorphic with $B(m, n)$ [1]. Let's consider a group $B = A \wr < x >$, where x is an involution.

Now take an element $u = d \cdot d^{-x}$ from $A \times A^x$. It is obvious, that $u \in Z(A \times A^x)$ and $u^x = u^{-1}$. As it is shown in [22], the group $G = B/ < u >$ and its involution $i = x \cdot < u >$ satisfy conditions (1)–(5) from the definition of T_0-group and $G = V \leftthreetimes < i >$, $C_G(i)$ is an infinite group with periodic part $< i >$, the all subgroups $< i, i^g >$ in G are finite and every maximal finite subgroup G with involution i is a dihedral group of order $2n$ and hence G is a T_0-group.

(b). Let $V = O(p)$ (see the definition of groups of the type $O(p)$, $C(\infty)$ in [11]). The group V has non-trivial center $Z(V) = (t)$ and $V/Z(V) = V/(t) \simeq C(\infty)$ [8].

Consider the group $T = V \wr (k) = (V \times V) \leftthreetimes (k)$, where k is an involution. Take the element $b = (t, t^{-1})$ of the group $V \times V$. Obviously, $b \in Z(V \times V)$ and $b^k = b^{-1}$. Assume $M = T/(b)$ be the factor group of T and take an involution $j = k(b)$ in M. From abstract properties of the groups $V = O(p)$, $C(\infty)$ [11], it is easy to show that the group M and its involution j satisfy the conditions 1) —

5) of the definition of T_0-group. Hence, $M = T/(b)$ is a T_0-group (with respect to the involution $j = k(b)$). Also note that in M any maximal periodic subgroup containing the involution j is the dihedral group of order $2p$.

Now, we deduce some results on T_0-groups. The detail of such kind of results can be found in [24].

Theorem 3.2 *(Shunkov V.P. [23, 26]). Let G be a group and a be an element of prime order p, satisfying the following conditions:*

(1) subgroups of the form $< a, a^g >$, $g \in G$, are finite and almost all are solvable;

(2) in the centralizer $C_G(a)$ the set of elements of finite order is finite;

(3) the normalizer of any non-trivial $< a >$-invariant finite subgroup of G has periodic part;

(4) for $p \neq 2$ and $q \in \pi(G)$, $q \neq p$, any $< a >$-invariant elementary Abelian q-subgroup of G is finite.

Then either G has almost nilpotent periodic part, or G is a T_0-group and $p = 2$.

Corollary 3.3 *Let G be a group and a an element of prime order $p \neq 2$, satisfying conditions 1) – 4) of previous Theorem. Then G has almost nilpotent periodic part.*

The following statement is equivalent to previous theorem and gives an abstract characterization of T_0-groups in the class of all groups.

Corollary 3.4 *Let G be a group and a an element of prime order p. The group G is a T_0-group and $p = 2$ if and only if for the pair (G, a) the conditions (1) – (4) of previous theorem are satisfied and the subgroup $< a^g | g \in G >$ is not periodic almost nilpotent.*

The particular case when $p = 2$ requires special consideration, since in this case condition (4) of previous theorem is superfluous, i.e. the following statements are true.

Corollary 3.5 *Let G be a group with involutions and i be one of its involutions, satisfying the following conditions:*

(1) subgroups of the form $< i, i^g >$, $g \in G$, are finite;

(2) the set of elements of finite order of $C_G(i)$ is finite;

(3) the normalizers of non-trivial $< i >$-invariant finite subgroups of G have periodic parts;

Then either G has almost nilpotent periodic part, or G is a T_0-group.

The conditions (1) – (3) of the above corollary are independent, i.e. each of them does not follow from the other two.

Theorem 3.6 *(Shunkov V.P. [23]). Let G be a group and a be an element of prime order p, satisfying the following conditions:*

1) subgroups of the form $< a, a^g >$, $g \in G$, are finite and almost all are solvable;

2) the centralizer $C_G(a)$ is finite;

3) $p \neq 2$ and for $q \in \pi(G)$, $q \neq p$, any $< a >$-invariant elementary Abelian q-subgroup of G is finite.

Then G is a periodic almost nilpotent group.

Theorem 3.7 *(Shunkov V.P. [23]). A non-trivial finitely generated group G is finite if and only if there exists an element $a \in G$ of prime order p satisfying the following conditions:*

(1) the subgroups of the form $< a, a^g >$, $g \in G$ are finite and almost all are solvable;

(2) the centralizer $C_G(a)$ is finite;

(3) when $p \neq 2$ and $q \in \pi(G)$, $q \neq p$, any $\langle a \rangle$-invariant elementary Abelian q-subgroup is finite.

Theory of T_0-groups is created by V.P.Shunkov [24].

4 Φ-groups

We devote this section for investigating the properties of a class of Φ-groups.

This class group is rather broad: among them are groups of Burnside type [1], Ol'shanskii monsters [11]. It is very closely connected with the groups of Burnside type of odd period $n \geq 665$.

Definition 4.1 *Let G be a group, and i be an involution of G, satisfying the following conditions:*

(1) all subgroups of the form $< i, i^g >, g \in G$, are finite;

(2) $C_G(i)$ is infinite and has a layer-finite periodic part;

(3) $C_G(i) \neq G$ and $C_G(i)$ is not contained in any other subgroups of G with a periodic part;

(4) if K is a finite subgroup of G, which is not contained in $C_G(i)$ and $V = K \cap C_G(i) \neq 1$, then K is a Frobenius group with complement V.

A group G with some involution i satisfying these conditions (1–(4) is called a Φ-group.

This class of groups has been introduced by V.P.Shunkov.

The example from previous section is an example of Φ-group

Theorem 4.2 *(Senashov V.I. [7]). An Φ-group G satisfies the properties:*

(1) all involutions are conjugate;

(2) Sylow 2-subgroups are locally cyclic or finite generalized quaternions groups;

(3) there are infinitely many elements of finite orders in G, which are strictly real with respect to the involution i and for every such element c of this set there exists an element s_c from the centralizer of i such that $< c, c^{s_c} >$ is an infinite group.

V.P. Shunkov posed the problem of studying groups with some additional limitations provided that for a given finite subgroup B, the following condition is valid: normalizer of any non-trivial B-invariant finite subgroup has a layer-finite periodic part.

This problem is partly solved for the class of locally soluble groups and for the case $\mid B \mid = 2$ under more general limitations, it is solved with Φ-groups accuracy.

Theorem 4.3 *(Ivko M.N., Senashov V.I. [7]). A periodic locally soluble group is layer-finite if and only if for some of its finite subgroup B, the normalizer of any non-trivial B-invariant finite subgroup is layer-finite.*

Theorem 4.4 *(Senashov V.I. [7]). Let G be a group and a be an involution of G, satisfying the following conditions:*

1. All subgroups of the form $< a, a^g >, g \in G$, are finite;

2. Normalizer of every non-trivial $< a >$-invariant finite subgroup has a layer-finite periodic part.

Then either the set of all elements of finite orders forms a layer-finite group or G is a Φ-group.

In the last two theorems, layer-finite groups are characterized for the class of locally solvable groups and groups with a layer-finite periodic part in more general case with Φ-groups accuracy. It is possible to find more information on layer-finite groups in [14].

The following theorem characterizes finite groups with Φ-groups accuracy.

Theorem 4.5 *(Senashov V.I. [7]). Let G be a group with involutions and i be some involution from G satisfying the following conditions:*

(1) G is generated by the involutions which are conjugate with i;

(2) almost all groups $< i, i^g >$ are finite, for all $g \in G$;

(3) normalizer of every $< i >$-invariant finite subgroup has a layer-finite periodic part.

Then G is either finite or a Φ-group.

5 Φ_0-groups

In this section we investigate the properties of the class of Φ_0-groups, which is a subclass of the class of Φ-groups. Such groups are very close to T_0-groups, but in this section we show their differences.

Definition 5.1 *Let G be a group and i be an involution of G, satisfying the following conditions:*

(1) all subgroups of the form $< i, i^g >, g \in G$, are finite;

(2) $C_G(i)$ is infinite and has a finite periodic part;

(3) $C_G(i) \neq G$ and $C_G(i)$ is not contained in any other subgroup of G with a periodic part;

(4) if K is a finite subgroup of G, which is not inside $C_G(i)$ and $V = K \cap C_G(i) \neq 1$, then K is a Frobenius group with complement V.

The group G with some involution i satisfying the above conditions (1)–(4) is called a Φ_0-group.

This class of groups has been introduced by V.P.Shunkov.

In [22], V.P. Shunkov raised the next question for discussion:

Do the classes of Φ_0-groups T_0-groups coincide or not?

In the same article, V.P. Shunkov specially emphasized that the most difficult problem is to establish the satisfiability for Φ_0-group of conditions (4) and (5) from the definition of T_0-group.

In [13] V.I. Senashov proved that Φ_0-group satisfies all conditions from the definition of T_0-group except the fourth condition. In the same article he constructed an example of Φ_0-group which is not a T_0-group, i. e. it was shown that the fourth condition does not hold in every Φ_0-gruop.

Example of Φ_0-group Let's take isomorphic copies of the T_0-groups $G = V \leftthreetimes (i)$ from [22]:

$$G_1 = V_1 \leftthreetimes (i_1), G_2 = V_2 \leftthreetimes (i_2), ..., G_n = V_n \leftthreetimes (i_n), ...$$

In the cartesian product of the groups G_n, $n = 1, 2, ...$, consider the subgroup $U = W \leftthreetimes (j)$, where W is a direct product of subgroups V_n, $n = 1, 2, ...$, and $j = i_1 \cdot i_2 \cdot ...$ is an involution from the cartesian product of G_n, $n = 1, 2, ...$ One can check that such a group U is a Φ_0-group. It is easy to see that the fourth condition from the definition of T_0-group is incorrect for the group U.

6 Groups with Finitely Embedded Involution

It is necessary to introduce the next concept, which was comprehended by V.P. Shunkov at the end of the 80th.

Let G be a group, i some of its involution and $\mathfrak{L}_i = \{i^g | g \in G\}$ the set of conjugations of i in G. We shall call the involution i *finitely embedded in* G, when for any element g of G the intersection $(\mathfrak{L}_i\mathfrak{L}_i) \cap gC_G(i)$ is finite, where $\mathfrak{L}_i\mathfrak{L}_i = \{i^{g_1}i^{g_2} | g_1, g_2 \in G\}$.

Let us give the simplest examples of the groups with a finitely embedded involution.

1. If in a group G there exists an involution i with finite centralizer $C_G(i)$, then i is a finitely embedded involution in G.

2. If in some group G the involution i is contained in a finite normal subgroup of G, then i is a finitely embedded involution in G.

3. Let G be a Frobenius group with the periodic kernel and infinite complement H, containing an involution i. Then i is a finitely embedded involution in G.

An involution of a group is called *finite* one, if it generates a finite subgroup with all of its conjugations are involution.

Now let us formulate some results, which the following is the main one.

Theorem 6.1 *(Shunkov V.P. [19]). Let G be a group, i be its finite and finitely embedded involution, $\mathfrak{L}_i = \{i^g | g \in G\}$, $B = < \mathfrak{L}_i >$, $R = < \mathfrak{L}_i\mathfrak{L}_i >$, Z be a subgroup generated by all 2-elements from R. Then B, R and Z are normal subgroups in G and one of the following statements is valid:*

(1) B is a finite subgroup;

(2) B is locally finite, $B = R \leftthreetimes (i)$ and Z is a finite extension of a complete Abelian 2-subgroup A_2 with the minimal condition and $ici = c^{-1} (c \in A_2)$.

A number of corollaries follow from the above theorem.

Corollary 6.2 *If a group has a finite involution with a finite centralizer, then it is locally finite.*

Corollary 6.3 *If a periodic group has an involution with a finite centralizer, then it is locally finite.*

Corollary 6.4 *If a finitely embedded involution exists in a group, then its closure is a periodic subgroup.*

Corollary 6.5 *A simple group with involutions is finite if and only if some of its involutions is finite and is finitely embedded.*

As in a periodic group any involution is finite, the next result follows from the last corollary.

Corollary 6.6 *A periodic simple group with involutions is finite if and only if some of its involutions are finitely embedded.*

Corollary 6.7 *Let G be a group, H a subgroup containing a finite involution i, and (G, H) is a Frobenius pair. Then G is a Frobenius group with a periodic Abelian kernel and with a complement $H = C_G(i)$ if and only if i is a finitely embedded involution in G.*

The above corollary is incorrect even for periodic groups, if the involution i is not finitely embedded.

R. Brauer proved the next result in the middle of twentieth century.

Theorem 6.8 *(Brauer R. [3]). There exists only finite number of finite simple groups with a given centralizer of involution.*

Proposition 6.9 *(Shunkov V.P.([25], [27].) For an arbitrary natural number M there is only finite number of finite simple groups G with involution τ satisfies to condition $|C_G(\tau) \cap \tau^G| \leq M$.*

Definition 6.10 *Let G be a group and τ be its involution. The number*

$$t(G, \tau) = \max_{g \in G} |gC_G(\tau) \cap (\tau^G \tau^G)|$$

is called the parameter of embedding of involution τ in the group G.

This concept is the basis of the next generalization for periodic groups of Brauer Theorem which was developed by V.P. Shunkov in 2001 (for the announcement see [25], [27]).

There is only finite number of periodic simple groups with involution and with given finite parameter of embedding of this involution, besides all the others are also finite.

Modulo the classification of finite simple groups, it is enough to verify the hypothesis for infinite families of alternating groups and for groups of Lie type.

Theorem 6.11 *(Golovanova O.V. [5]). Let M be natural number and G a finite simple group with one class of conjugate involutions. Then for any involution τ of G, $|C_G(\tau) \cap \tau^G| > M$, for large enough $|G|$.*

Theorem 6.12 *(Golovanova O.V., Levchuk V.M. [4]). Let M be an arbitrary natural number and $G = S_n,\ A_n$ or $PSL_n(q)$ with even q. Then for any involution τ of G, $|C_G(\tau) \cap \tau^G| > M$, for large enough $|G|$.*

The last two theorems verify hypothesis of V.P. Shunkov for mentioned classes of groups.

For another class of groups with similar assumptions are investigated in the next theorems, which were proved in 2006.

Theorem 6.13 *(Golovanova O.V. [4]).Let M be an arbitrary natural number and G a group of Lie type over the field of even order. If the order of G is large* *enough, then there is an involution τ in the group G with condition $|C_G(\tau)\cap\tau^G| > M$.*

Theorem 6.14 *(Golovanova O.V., Levchuk V.M. [4]).Let M be an arbitrary natural number and $G = PSL_n(q)$ with odd q. If the of G is large enough, then for any diagonalizable involution τ of G the inequality $|C_G(\tau)\cap\tau^G| > M$ is valid.*

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