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Research Article

A new bi-level data envelopment analysis model to evaluate the Human Development Index

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Abstract

In the 1990s, the united nations development programme (UNDP) introduced the human development index (HDI) to determine the development degrees of countries. One deficiency in the HDI calculation is the use of equal weights for its sub-indicators. Many scholars have tried to solve this problem using a data envelopment analysis (DEA) method, particularly the one enhanced by weight restrictions. Indeed no specific method

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has been yet suggested to determine the parameters of the weight restrictions. In this paper, we use four DEA/benefit of the doubt (BoD) models enriched by the assurance regions type I (AR-I) constraints to assess human development; we aim to objectively determine the AR-I bounds. Therefore, we consider a basis as the accepted human development values and propose a bi-level optimization problem to extract the AR-I bounds in such a way that the efficiency scores are almost the same as the basic values. On the other hand, the HDI is a globally accepted index that shows small changes year by year. So, if the UNDP decides to apply a BoD model for calculating the HDI instead of the traditional method, then it is better than the scores obtained by the BoD model, showing small changes in comparison with the HDI, at least in the first few years. Therefore, the HDI values are considered as the basis. Moreover, the objectively achieved AR-I bounds provide us with an insight into the way the sub-indicators affect the development scores. The bounds can be modified by the experts opinions, in the future.

AMS subject classifications (2020): 91B06; 90C29

Keywords: Human development index (HDI); Data envelopment analysis (DEA), Benefit of the doubt (BoD); Assurance regions type I (AR-I); Bi-level optimization problem (BOP).

1 Introduction

In the 1990s, the united nations development programme (UNDP) introduced the human development index (HDI) [87] to show that people's capabilities should be measured not only by economic advances but also by human well-being. The HDI is an incorporation of three important dimensions of human development: a long and healthy life, education, and standards of living. The sub-indicators used for the assessment of these dimensions, respectively, are, expected years of life at birth, mean of years of schooling for adults and expected years of schooling for children, and gross national income per capita.

Since the introduction of the HDI, it has been criticized from different aspects [47, 46, 42]. One main criticism is the arbitrariness that is caused by subjective choices in the HDI construction. To solve this problem, many authors proposed the use of the data envelopment analysis (DEA) technique [58, 27, 21, 7, 61, 60]. The method used in most papers is to construct an index by applying the benefit of the doubt (BoD) approach, which is a DEA model considering only desirable attributes.

DEA is a nonparametric method for evaluating the performance of decision-making units (DMUs), where each DMU consumes multiple inputs to produce multiple outputs. Since the appearance of the first DEA model, it has been extended and developed vastly. One of the most important developments is the introduction of weight restrictions, which limit the total flexibility in choosing the weights by the DMUs. These weight restrictions also increase the discrimination power of the DEA models. So, a DEA model with weight restrictions seems to be a suitable model for evaluating the HDI (see Section 2.2). However, the parameters associated with the weight restrictions are arbitrarily determined. Blancard and Hoarau [9] suggested that the arbitrariness of choosing these parameters should be eliminated. So, in this paper, we focus on a procedure that objectively determines the parameters of the weight restriction constraints.

Here, we use four BoD approaches (like Mariano, Ferraz, and Oliveira Gobbo [60]), namely, the traditional, the multiplicative, the slacks-based measure (SBM), and the range-adjusted measure (RAM) approaches. We enrich these BoD models with the constraints of the assurance regions type I (AR-I) [83]. Our proposed procedure is to consider a set of values as the basis and then objectively determine the bounds of the AR-I constraints in such a way that the enriched BoD model produces the basis. This procedure is somewhat inspired by the work of Edirisinghe and Zhang [31], where input/output selection is endogenously performed by an iterative procedure that seeks the maximum correlation between the DEA-based performance scores and the stock returns. Also, it is somehow similar to the approach used in Alrezaee, Hajinezhad, and Paradi [2], in which the returns to scale are extracted from the data by using a method based on data mining. The key question is how to select the basis. To answer this question, we have reviewed those research papers that used DEA to solve the problem of equal weights for evaluating the HDI (see Section 2.2). In some papers, the scores obtained by the DEA models were compared with the HDI [58, 27, 28, 56]; in cases where there were large differences, the reasons were explained. Now, two ambiguities arise: 1) why did the researchers refuse to justify small differences? 2) why does it seem that a small difference between the two rankings is desirable?

On the other hand, if the UNDP decides to shift the HDI evaluation from geometric average to a BoD model, then the HDI values and rankings may change dramatically, which is not usual; by the current evaluation method, the ranking will change a little between two consecutive years, because of the nature of the HDI sub-indicators; except for gross national income per capita, the changes in other sub-indicators are very slow from year to year. So, it would be wise to manipulate the BoD approach in such a way that its results make the lowest changes in the human development values or the rankings in comparison with the last year. Since the UNDP's HDI has such a property, we consider it as the basis and try to extract the required information for setting up the BoD model to obtain efficiency scores that are closest to the basis. By using the proposed method, the effects of the sub-indicators on the calculation of human development values are determined and can be manipulated by the experts' opinions.

To objectively determine the bounds of the AR-I constraints, we propose a bi-level optimization problem (BOP) in which the upper-level is the minimization of the differences between the bases, namely, the UNDP's HDI values and the enriched BoD scores; the lower-level is a linear multi-objective optimization problem derived from the integration of the enriched BoD model for all units. The proposed model is NP-hard. To search for a globally optimal solution for the proposed BOP, we apply a nested approach; the upper-level is optimized by a genetic algorithm (GA) that is enhanced by two local search methods. Also, the lower-level linear multi-objective optimization problem is transformed into N linear optimization problems, which are solved via an exact method. However, the final solution of the BOP is the one which produces the scores that are closest to the UNDP's HDI values, solutions with the objective function (of the upper-level) less than a determined value are saved in *BestSet* during the process of solving the BOP. If the final solution is not desirable, then a solution from *BestSet* can be applied. Of course, the bounds can be manipulated according to the experts' opinions, and they can be gradually modified each year.

It is worthwhile to note that shifting from a traditional performance assessment system that uses fixed equal weights to a more complicated one, namely, a DEA model, could be difficult from the approval aspect. For example, suppose that a DEA model is used instead of the arithmetic average to evaluate the performance of bank branches. The DEA model may produce efficiency scores that are substantially different from the results of the traditional model. Moreover, it may show big changes in comparison with the last assessment. As a result, it is difficult for the managers to accept the new rankings. For this reason, it is advisable to apply the proposed BOP to extract the AR-I bounds in such a way that the DEA results do not differ dramatically from the results of the traditional method. However, the bounds could be manipulated by the experts' opinions and also be changed in the future. Moreover, considering a set of rankings as the base and applying the proposed BOP for extracting the parameters of weight restrictions are helpful in cases where the experts cannot define the parameters explicitly but are able to determine a ranking of the units.

The contributions of this research can be described as follows:

1) We propose a multi-objective BOP to objectively determine the AR-I bounds in the enriched BoD models, and we apply their results for evaluating human development values.

2) The proposed BOP represents a road to shift slowly from the traditional evaluation of the HDI (or any index) to a BoD approach (or any DEA model) with the least changes in the efficiency values or rankings.

3) This structure, which is used for initializing the parameters of a BoD model, can be applied to objectively determine all parameters that appeared in the DEA models.

This paper is organized as follows. Section 2 reviews the literature related to DEA, the HDI, and BOP. In Section 3, the BoD approaches for evaluating the human development are reviewed. In Section 4, the BOP is proposed for tuning the AR-I bounds. We propose an approach to solve the BOP model in Section 5. In Section 6, the numerical results of the proposed method are presented. The last section is devoted to a brief conclusion.

2 Literature review

This section reviews the literature related to DEA method.

2.1 Data envelopment analysis

The origins of DEA date back to Farrell's seminal paper [34]. Charnes, Cooper, and Rhodes [17] developed Farrell's idea and presented a model that is able to measure the efficiencies of units having multiple inputs and multiple outputs. This first model of DEA is called CCR. It has inspired several extensions, most notably the BCC model of Banker, Charnes, and Cooper [5] and the additive model of Charnes et al. [16].

The most well-known advantage of the DEA models is the flexibility in choosing weights. As a result of this, a DMU that is determined to be inefficient cannot argue that its inefficiency is due to the weights selected for its inputs and outputs. However, this full flexibility has been criticized from different aspects (see [3, 65]). Although the DMUs may have their own circumstances and objectives in general, they are assumed to be homogeneous. So, it is not sensible that the weights significantly differ from one DMU to another [71]. To resolve this problem, weight restriction is introduced to ensure that all inputs and outputs are considered and that higher weights are assigned to the more important ones.

Now, let us mention the most prominent weight restriction constraints in the DEA literature. The most straightforward weight restriction is to set simple numerical limits on each of the weights [30, 69]. This may lead to infeasible linear programming.

Thompson et al. [83] proposed the concept of assurance regions type I (AR-I), which incorporates into the model the relative importance of the inputs/outputs. Moreover, they introduced assurance regions type II (AR-II), which is the relative value of inputs to outputs.

It should be noted that applying the AR-II constraints may cause infeasibility. The Cone-Ratio (CR) technique, developed by Charnes et al. [20] in 1989, is the most well-known weight restriction that acts on the data and transforms them [82]. AR-I and CR are related in the sense that an AR-I constraint can be represented as a CR one. However, CR is more general than AR-I.

Wong and Beasley [91] presented a weight restriction based on virtual inputs/outputs which limited the importance of input i to DMUj .

The DEA literature is vast, leading to numerous conducted reviews. Liu, Lu, and Lu [52] delved into the evolving landscape of DEA research, employing a network clustering method to identify four distinct research fronts, shedding light on key areas such as bootstrapping, undesirable factors, cross-efficiency, and network DEA. Building on this, Zhou et al. [96] offered a comprehensive overview of DEA's origins, popular models, applications, and its evolving trends through a bibliometric analysis. Complementing these works, Camanho and D'Inverno [13] provided a historical overview of DEA, discussing its main models, recent developments, and successful applications, emphasizing its relevance in organizational management and public policy. Furthermore, Krmac and Mansouri Kaleibar [49] conducted a systematic review of DEA's applications in port efficiency evaluation, revealing its potential as a valuable tool for assessing future port performance, while also identifying research gaps. Emrouznejad et al. [33] contributed to the discourse by outlining recent theoretical developments and novel applications of DEA, showcasing its versatility across diverse fields. Liu et al. [53] reviewed the areas in which DEA has been applied. They concluded that the top five industries addressed by researchers were banking, healthcare, agriculture, transportation, and education. The applications of DEA are surveyed on the field of banking [64], healthcare [44], agriculture [50], transportation [59], and education [81].

2.2 The human development index and DEA

DEA was originally proposed in the microeconomic environment to measure the performance of schools, hospitals, and so on. It is also well-suited for the evaluation of macroeconomic performance [55]. When DEA is extended to analyze social performance, the concepts of "input" and "output" are not considered in the same sense as their traditional sense of DEA; instead, they are considered desirable and undesirable attributes, respectively, [67].

The first applications of DEA to the evaluation of macroeconomic performance can be found in [63, 39]. In this area, researchers have used DEA to construct a composite index and to measure the efficiencies of countries or regions in generating welfare [61]. The most important advantage of using DEA is that the arbitrariness in weighting the sub-indicators is eliminated, and they are determined endogenously [45]. In fact, the DEA model is used to retrieve some information on appropriate weighting hidden in the performance of a country; the relatively strong (poor) performance of the country in one indicator shows that its policies consider that dimension as more (less) important than the others [21].

For constructing an index, the DEA approach with only the desirable attributes is alternatively called the BoD (benefit of the doubt). This was originally proposed by Melyn and Moesen in 1991 [63]. In the BoD approach, all the indicators or sub-indicators are considered as outputs, along with a dummy input equal to 1 for all the units. Unlike the original HDI calculated by the equal weights, the HDI obtained using the BoD approach applies the most advantageous weights for each country or region [10]. A good review of the BoD approach and the different weight restrictions, which can be used in it, is provided by Cherchye et al. [21].

The first application of the BoD approach for recalculating the HDI dates back to the work of Mahlberg and Obersteiner [58]. They stated two reasons for using the DEA method.

(a) The HDI should be evaluated against the best performance of the other countries.

(b) The weights of the sub-indicators should be determined directly from the data and independently for each country.

Initially, they used an output-oriented CCR model. Then, to improve the discrimination of the efficient countries, a BoD approach with AR-I constraints was applied; they examined three different arbitrary intervals for the AR-I constraints. The ranking of countries resulting from this method and the ranking obtained by the UNDP had a correlation of 0.83. Although, in some cases, there were significant differences. They believed that the main reason for such differences was the weights expertly assigned to the sub-indicators of the HDI.

Despotis [27] presented a method for evaluating the HDI based on a modified DEA model. In this method, a BoD approach is used, and then, by using a goal linear programming model, common weights for all the countries are obtained. Thus, a new estimate of the HDI is achieved, which improves comparability. According to Despotis [28], when the HDI is used to evaluate the human development of countries, the sub-indicators of life expectancy, education, and standards of living are considered as means for achieving social welfare. Although this approach is accepted, human development could also be considered differently; standards of living improve the sub-indicators of life expectancy and education. In such cases, the competency of countries in converting income into a higher quality of life can be modeled and evaluated. Thus, to evaluate the human development of countries, Despotis used a DEA model with variable returns to scale in which the sub-indicator of standards of living was the input and the sub-indicators of life expectancy and education were the outputs.

Lee, Lin, and Fang [51] presented a fuzzy multi-objective DEA model to evaluate the performance of countries based on the human development perspective. In their approach, like Despotis' model [28], the efficiencies of all countries were maximized using a common set of weights.

In 2010, the UNDP changed the method of measuring the HDI [88]. One of these changes was the use of geometric averages instead of arithmetic averages, which eliminated compensation among the HDI sub-indicators. However, the problem of equal weights has remained unresolved. To solve this problem, Zhou et al. [95, 93] used two BoD approaches in the multiplicative form [18, 19] to calculate the best and the worst efficiency scores. Then, the convex combination of these two efficiency scores was regarded as the efficiency of the country under study.

In some DEA results, the optimal weights of some sub-indicators may be equal to 0. To resolve this problem, Zhou et al. applied the weight restriction proposed by Wong and Beasley [91] and set the parameters arbitrarily. It should be noted that by using the logarithmic transformation, the multiplicative model is changed into the additive model. Also, Blancard and Hoarau [8] used a similar BoD approach with the same weight restriction to evaluate the HDI. One of the drawbacks of their approach is that the weight parameter in the convex combination of the best and worst efficiency scores significantly affects the value and rank of the HDI [40]. In addition, the multiplicative models in [95, 93, 8] are not scale-invariant, which is problematic for linearizing the model because the values of sub-indicators lie between 0 and 1 (see Section 3).

Hatefi and Torabi [40, 41] used a set of common weights to recalculate the HDI of Asian and Pacific countries. To do so, they used a one-stage optimization model to calculate the common weights in such a way that the maximum deviation from the full efficiency score was minimized. The drawback of this model is that the weights of the sub-indicators are substantially influenced by the countries that have poor performance [84]. By using the dual of the proposed model, Hatefi and Torabi [41] performed an analysis to improve the performance of countries with low HDI values.

Tofallis [85] applied a multiplicative DEA model. The model involves an extra factor that absorbs the effect of multiplying any sub-indicator by a constant. The method proposed in [85] is somewhat similar to Despotis' model [27, 28]. Firstly, the DEA model is applied to all countries. Subsequently, a set of common weights is estimated by using the least squares regression.

Puyenbroeck and Rogge [89] compared groups instead of individuals to cope with the problems caused by intermediate steps in the construction of the HDI. They used a BoD approach with weight restrictions to analyze the HDI levels in seven regions. The method they utilized to limit the weights was based on the work of Wong and Beasley [91], with the difference that only the unit under study was considered.

Mariano, Sobreiro, and Rebelatto [61] provided a valuable review of the research works that used DEA to recalculate the HDI. They highlighted twenty gaps in this field. To address the lack of systematic studies considering the advantages and challenges of applying different DEA approaches to the evaluation of the HDI, Mariano, Ferraz, and Oliveira Gobbo [60] compared multiple BoD approaches, including the traditional, the multiplicative, the SBM, the RAM, weight restrictions, common weights, and tiebreaker methods. For the weight restriction, they used the method proposed by Puyenbroeck and Rogge [89]. The analysis was done for raw and normalized data by using social network analysis (SNA) and the information derived from the model itself. Finally, they made the following useful suggestions:

- The normalized data are preferable because of avoiding the outliers.
- The nonradial models are more appropriate because they do not generate false efficiencies.
- To improve the results, it is recommended that weight restrictions be considered in the DEA model.

A group of researchers has investigated the efficiency of countries or regions in transforming the minimum possible units of resources into the maximum possible levels of the sub-indicator of quality of life [4, 66]; it can be seen as the sustainable HDI [9, 15].

2.3 Bi-level optimization problems

Multi-level and bi-level optimization problems have emerged as important research areas in mathematical programming [80]. These optimization problems are widely used in the modeling of real-world problems in which there are many decision-makers at different levels. A BOP is a special case of multilevel optimization problems (MOPs) with two levels, where the upper-level is optimized subject to the best solution of the lower-level. The first formulation of BOPs goes back to the work of Bracken and McGill [12]. Later, Candler and Norton [14] proposed the term “bi-level programming” in a technical report. Since then, many researchers have focused on this area of optimization.

MOPs and BOPs have been applied to various fields, including supply chain management [68], energy-efficient scheduling [92], and robot motion planning [97]. Recently, BOPs have been applied to solve parameter tuning problems, where the procedure of algorithm configuration is designed by using a BOP [78, 62].

BOPs are strongly NP-hard [76]. Many studies have been dedicated to solving BOPs using classical methods such as the Karush–Kuhn–Tucker (KKT) approach [6, 74], descent methods [90], and the penalty function methods [57]. However, in the case of complex BOPs, evolutionary techniques are more suitable. For a review of solution methodologies, we refer the reader to [76, 22]. One of the most popular evolutionary algorithms for solving BOPs is the nested methods, where the lower-level optimization problem is solved completely corresponding to any given upper-level decision [77]. The upper-level optimization is handled using an evolutionary algorithm, and the lower-level is handled by using a classical method or an evolutionary one. As generalizations of BOPs, multi-objective BOPs have been considered by a number of researchers [75, 70, 26].

3 Some BoD approaches used to evaluate the HDI

Here, the different BoD approaches addressed in this work are represented (for more details; see Mariano, Ferraz, and Oliveira Gobbo [60]). We utilize the multiplier form of the BoD models in the subsequent sections. Additionally, for a clearer comprehension of the models, we present the envelopment forms. Suppose that E_j, H_j, and G_j are the sub-indicators of education, health, and standards of living, and represent the outputs for country j ($j = 1, 2, \ldots, N$), respectively, and assume that W_E, W_H, and W_G are the weights of the sub-indicators of education, health, and standards of living, respectively. Also, E_o, H_o, and G_o are the respective sub-indicators related to the DMU under evaluation. Moreover, v is a scalar value being free in sign. A dummy input equal to 1 is considered for all units, and N is the number of countries under evaluation. For the envelopment form, suppose that S_E, S_H, and S_G are nonnegative slacks of the education, health, and standards of living, respectively, and that λ_j is the importance value of benchmark j for the

Table 1: Traditional BoD

Traditional BoD in multiplier form	Traditional BoD in envelopment form
min $\frac{1}{E_o} = v$ Subject to: $W_E \cdot E_o + W_H \cdot H_o + W_G \cdot G_o = 1,$ $-v + W_E \cdot E_j + W_H \cdot H_j + W_G \cdot G_j \leq 0,$ for all j, $W_E, W_H, W_G \geq 0.$	max $\frac{1}{E_o} = \theta$ Subject to: $\sum \lambda_j E_j \geq E_o,$ $\sum \lambda_j H_j \geq H_o,$ $\sum \lambda_j G_j \geq G_o,$ $\sum \lambda_j = 1,$ $\lambda_j \geq 0.$

country under analysis. The traditional BoD approach [63] is the output-oriented BCC model. The traditional BoD approach 57 is the output-oriented BCC model. The multiplier and envelopment forms are as follows:

The multiplier and envelopment forms of the multiplicative BoD approach, which are similar to the model presented by Tofallis [85], are as follows:

Table 2: Multiplicative BoD

Multiplicative BoD in multiplier form	Multiplicative BoD in envelopment form
Max $Ef_o = e^v \times E_o^{W_E} \times H_o^{W_H} \times G_o^{W_G}$ Subject to: $W_E + W_H + W_G = 1,$ $e^v \times E_j^{W_E} \times H_j^{W_H} \times G_j^{W_G} \leq 1,$ for all j , $W_E, W_H, W_G \geq 0.$	Min $Ef_o = \theta$ Subject to: $\frac{\theta \times \prod_j \lambda_j}{S_E} = E_o,$ $\frac{\theta \times \prod_j \lambda_j}{S_H} = H_o,$ $\frac{\theta \times \prod_j \lambda_j}{S_G} = G_o,$ $\sum \lambda_j = 1,$ $\lambda_j, S_E, S_H, S_G \geq 0.$

Applying the natural logarithm to the objective function and the constraints, the multiplicative BoD in Table 2 are linearized as follows:

Table 3: Linearized Multiplicative BoD

Linearized Multiplicative BoD in multiplier form	Linearized Multiplicative BoD in envelopment form
Max $\ln(E_f) = v + W_E \ln(E_o)$ $+ W_H \ln(H_o) + W_G \ln(G_o)$ Subject to: $W_E + W_H + W_G = 1,$ $v + W_E \ln(E_j) + W_H \ln(H_j) + W_G \ln(G_j) \leq 0,$ for all j, $W_E, W_H, W_G \geq 0.$	Min $\ln(E_f) = \ln(\theta)$ Subject to: $\ln(\theta) + \sum_j \ln(E_j)\lambda_j - \ln(S_E) = \ln(E_o),$ $\ln(\theta) + \sum_j \ln(H_j)\lambda_j - \ln(S_H) = \ln(H_o),$ $\ln(\theta) + \sum_j \ln(G_j)\lambda_j - \ln(S_G) = \ln(G_o),$ $\sum_j \lambda_j = 1,$ $\lambda_j, \ln(S_E), \ln(S_H), \ln(S_G) \geq 0.$

The multiplier and envelopment forms of the SBM-BoD model, which are based on the output-oriented SBM model (Tone [86]), are as follows. Note that, as the slack related to the input is zero, the input term has been removed, and the inverse of the objective function is maximized.

Table 4: SBM-BoD

SBM-BoD in multiplier form	SBM-BoD in envelopment form
$Min\frac{1}{Ef_o} = v - W_E E_o - W_H H_o - W_G G_o$	$Max\frac{1}{Ef_o} = 1 + \frac{1}{3}(\frac{S_E}{E_o} + \frac{S_H}{H_o} + \frac{S_G}{G_o}$
Subject to:	Subject to:
$v - W_E E_j - W_H H_j - W_G G_j \geq 1,$ for all j,	$\sum_j \lambda_j E_j - S_E = E_o,$
$W_E \geq \frac{1}{3E_o},$	$\sum_j \lambda_j H_j - S_H = H_o,$
$W_H \geq \frac{1}{3H_o},$	$\sum_j \lambda_j G_j - S_G = G_o,$
$W_G \geq \frac{1}{3G_o}.$	$\sum_j \lambda_j = 1,$
	$\lambda_j, S_E, S_H, S_G \geq 0.$

Finally, the RAM-BoD model, derived from the output-oriented RAM model (Aida et al. [1]), is scale and translation invariant. It is represented in the multiplier and envelopment forms as follows:

Table 5: RAM-BoD

RAM-BoD in multiplier form	RAM-BoD in envelopment form
$MaxEf_o = v + W_E E_o + W_H H_o + W_G G_o$	$MinEf_o = 1 - \frac{1}{3}(\frac{S_E}{R_E} + \frac{S_H}{R_H} + \frac{S_G}{R_G})$
Subject to:	Subject to:
$v + W_E \cdot E_j + W_H \cdot H_j + W_G \cdot G_j \leq 1,$ for all j,	$\sum_j \lambda_j E_j - S_E = E_o,$
$W_E \geq \frac{1}{3R_E},$	$\sum_j \lambda_j H_j - S_H = H_o,$
$W_H \geq \frac{1}{3R_H},$	$\sum_j \lambda_j G_j - S_G = G_o,$
$W_G \geq \frac{1}{3R_G}.$	$\sum_j \lambda_j = 1,$
	$\lambda_j, S_E, S_H, S_G \geq 0.$

Here, R_E, R_H, and R_G are the ranges of education, health, and standards of living in the N countries, respectively. It is worth noting that $\frac{1}{3}(\frac{S_E}{R_E} + \frac{S_H}{R_H} + \frac{S_G}{R_G})$ represents the range-adjusted inefficiency of the country under evaluation.

We enrich the aforementioned BoD models by using the AR-I restrictions

$$L_1 \leq \frac{W_E}{W_H} \leq U_1, \tag{1}$$

$$L_2 \leq \frac{W_E}{W_G} \leq U_2, \tag{2}$$

where L_1, U_1, L_2, and U_2 are constant and nonnegative values. From here on, we refer to RAM-BoD mode (Table 5) with the AR-I constraints (1)–(2) as the enriched RAM-BoD model or enriched model (Table 5). The parameters of the AR-I constraints could be determined by experts' opinions. However, to avoid problems associated with the application of the BoD model instead of the traditional assessment system, we objectively determine the unknown AR-I parameters in such a way that the ranking of the UNDP is reproduced by the enriched BoD models. To this end, we propose a BOP. This provides a useful tool to better understand the relative effects of sub-indicators in calculating the HDI values and manipulating them later. The BOP is explained in the next section.

4 The proposed BOP

In this section, the HDI values presented by the UNDP are considered an acceptable base. It should be noted that the HDI classifications are based on fixed cutoff values. We set up the BoD model (each of the models in Tables 1, 3, 4, and 5) with the AR-I parameters (constraints (1)–(2)) in such a way that it results in the base values. To do so, we determine the unknown bounds of the AR-I constraints by using a BOP, where the upper-level is to minimize the total difference between the enriched BoD scores and the HDI values; the lower-level is a multi-objective optimization problem for evaluating the BoD efficiency scores corresponding to each country subject to that BoD constraints and also the AR-I constraints with variable bounds. In other words, the upper-level optimizes the values of unknown AR-I parameters in the lower-level based on the upper-level objective function, while the lower-level maximizes the efficiency scores under the parameters assigned by the upper-level, subject to the BoD and AR-I constraints.

We want to reproduce the HDI values by the enriched BoD model (each of the models in Tables 1, 3, 4, and 5); so, in the best case, we obtain $Ef_t = HDI_t$ for $t = 1, \ldots, N$. Thus, the upper-level objective function is defined as the minimization of the sum of absolute differences between the efficiency scores (Ef_t) and the HDI values (HDI_t) for countries $t = 1, \ldots, N$:

$$\min \ z = \sum_{t=1}^{N} |EF_t - HDI_t|. \tag{3}$$

Here, the decision variables of the upper-level are constrained as follows:

$$0 \leq L_1^v \leq U_1^v, \tag{4}$$
$$0 \leq L_2^v \leq U_2^v. \tag{5}$$

It should be noted that if the ranking is considered as the base, then the upper-level objective function is changed to the absolute difference between the two rankings.

The efficiency scores Ef_t, for ($t = 1, 2, \ldots, N$), are the responses of the lower-level multi-objective optimization problem to the upper-level decision on the AR-I variable bounds. The lower-level problem can be one of the BoD approaches in Tables 1, 3, 4, or 5. We consider the RAM-BoD model in Table 5 to explain the BOP. The objectives of the lower-level, similar to the model in Table 5, are as follows:

$$\max \ Ef_t = v_t + W_{E_t} E_t + W_{H_t} H_t + W_{G_t} G_t, \quad t = 1, 2, \ldots, N. \tag{6}$$

In equations (6), the index t is considered for the aforementioned weights, because the weights of all N countries are included in a single model. The variables v_t, W_{E_t}, W_{H_t}, and W_{G_t} are the decision variables of the lower-level optimization problem, where, similar to the model in Table 5, the weights for a country must be assigned in such a way that the following constraints are satisfied:

$$v_t + W_{E_t} E_j + W_{H_t} H_j + W_{G_t} G_j \leq 1, \quad j = 1, 2, \ldots, N, \ t = 1, \ldots, N, \tag{7}$$
$$W_{E_t} \geq \frac{1}{3R_E}, \qquad t = 1, \ldots, N, \tag{8}$$
$$W_{H_t} \geq \frac{1}{3R_H}, \qquad t = 1, \ldots, N, \tag{9}$$
$$W_{G_t} \geq \frac{1}{3R_G}, \qquad t = 1, \ldots, N. \tag{10}$$

Next, we have to consider the AR-I constraints. The relationships between the decision variables of the two levels of the BOP are expressed by the following inequalities:

$$L_1^v \leq \frac{W_{E_t}}{W_{H_t}} \leq U_1^v, \quad t = 1, \ldots, N, \tag{11}$$

$$L_2^v \leq \frac{W_{E_t}}{W_{G_t}} \leq U_2^v, \quad t = 1, \ldots, N. \tag{12}$$

The constraints (11)–(12) are similar to the AR-I constraints (1)–(2) except that the bounds (L_1^v, U_1^v, L_2^v, and U_2^v) are considered as variables. The combination of equations (3)–(12) is our proposed BOP model for evaluating the bounds of the AR-I constraints in the enriched model in Table 5, to achieve the maximum alignment with the UNDP's HDI. The BOP for the enriched BoD models in Tables 1, 3, and 4 is the same.

By solving the proposed BOP, we obtain the bounds L_1^v, U_1^v, L_2^v, and U_2^v, and also the weights W_{G_t}, W_{H_t}, W_{E_t}, and v_t for $t = 1, \ldots, N$. We replace the bounds of the AR-I constraints in the enriched model in Table 5, namely, L_1, U_1, L_2, and U_2 in constraints (1)–(2), with the optimized variable bounds L_1^v, U_1^v, L_2^v, and U_2^v, respectively. Thus, we obtain the enriched RAM-BoD model with objectively determined bounds. These bounds enable us to analyze the effects of sub-indicators on the evaluation of the HDI values.

Let us note that it is necessary to consider both objective functions (equations (3) and (6)) in a bi-level structure. If our BOP is considered with (3) as the only objective function or the objective functions of the two levels are merged in some way, then applying the optimized AR-I bound variables to the enriched model in Table 5 could produce efficiency scores that are different from the ones obtained from the BOP model and are also so far from the UNDP's HDI values.

The objective functions and the constraints of the BOP are linear or at least convex. So, the proposed BOP model is a convex bi-level multi-objective optimization problem. Therefore, it seems effective to change the lower-level optimization problem into the constraints of the upper-level optimization problem by using the KKT optimality conditions. However, since the lower-level is a multi-objective optimization problem, the number of constraints regarding the KKT conditions is so large that it is even too difficult to find a feasible solution. Hence, we use a nested approach in which the upper-level is managed by a GA, and the lower-level optimization problems are solved exactly. The methods we apply to solve the proposed BOP are described in the following section.

5 Solving the proposed BOP

In this section, we propose a nested approach to solve our proposed BOP; the upper-level is handled by a GA, which is described in the following subsection; by replacing the optimized upper-level decision variables (the optimized variable bounds of the AR-I constraints), the lower-level problem changes to a linear multi-objective optimization problem, which in turn can be transformed into N linear single-objective optimization problems. In fact, these linear optimization problems are the enriched model in Table 5 for $o = 1, 2, \ldots, N$, and they are solved exactly.

5.1 The GA heuristic

GAs are heuristic search algorithms that can be applied to an extensive range of optimization problems. For the first time, the basic theoretical concepts of GAs were developed by Holland [43] in 1975. These probabilistic search techniques imitate the natural evolution process to find the best or the nearly best solution. A candidate solution is represented as a chromosome, and the optimization problem is formulated as the fitness function. A GA starts with a random population of chromosomes that evolves using nature-inspired operators: selection, crossover, mutation, and replacement. A more exhaustive overview of GAs can be found in [48].

In our GA, we seek the best solution to the upper-level optimization problem (3)–(5). So, we consider a chromosome as an array of length 4 whose numbers show the lower and upper bounds for equations (11)–(12). The population is of the fixed size $P = 20$. By considering the maximum value 5 for the upper bound, each chromosome is randomly initialized in the interval $[0, 5]$. After producing a chromosome (bounds), the *Evaluate* algorithm is run; the chromosome is checked to be feasible regarding the constraints (4)–(5). If any bound is negative, then its value is fixed to 0, and if the lower bound is greater than its respective upper bound, then it is fixed to be one-half of the value of the upper bound.

Once the feasibility of the solution is ensured, its values are replaced in equations (11)–(12). We have already mentioned that solving the lower-level multi-objective optimization problem (6)–(12) with the fixed AR-I bounds is equivalent to solving the enriched model in Table 5 with those fixed AR-I bounds, for $o = 1, 2, \ldots, N$. So, by using the current solution, we set the AR-I bounds in the enriched model in Table 5 and evaluate Ef_o for $o = 1, 2, \ldots, N$. If it is feasible, then the fitness function (equation (3)) is calculated. However, the determined bound variables may cause infeasibility. We use the GAMS (with CPLEX solver) to solve the enriched model in Table 5. In case of infeasibility, we modify the bound variables in a way that provides wider intervals for the corresponding proportion of weights, as follows:

$$\begin{aligned} L_1^v &= \min_t \frac{W_{E_t}}{W_{H_t}}, \quad U_1^v = \max_t \frac{W_{E_t}}{W_{H_t}}, \\ L_2^v &= \min_t \frac{W_{E_t}}{W_{G_t}}, \quad U_2^v = \max_t \frac{W_{E_t}}{W_{G_t}}. \end{aligned}$$

Subsequently, the enriched model in Table 5 with the modified bounds for $o = 1, \ldots, N$ is solved again. If the feasibility is achieved, then the GA proceeds; otherwise, the value of the first cell of the chromosome that is not considered yet is decreased (or increased) by 10% if it shows the lower (or upper) bound. Then, the enriched model in Table 5 with the newly determined bounds is solved. This proportional increase or decrease in the bounds is repeated until the feasibility is achieved.

For any produced solution, the *Evaluate* algorithm is run. Parents are chosen by using a binary tournament; two pools of members are formed, where each one consists of two randomly selected members. The best member of each pool is a parent. The crossover operator randomly takes the bounds of equations (11) and (12) from the two parents to produce a single child. By the probability of 0.3, a child is mutated; a cell in the chromosome is randomly selected, and its value is changed to its respective upper or lower bound. Also, we propose two methods to locally seek solutions. If the mutation is not done, then the child is subject to the local search (LS); one of the methods is chosen with equal probability. The worst member of the population (considering the objective function (3)) is replaced by the new child. This process is repeated until the best fitness function is not improved within M iterations. Here, we set $M = 10$.

During the running of the GA, the best solutions are saved; we consider the solution with the lowest fitness function during the random initialization of the population, then the produced solutions whose fitness functions are lower than that, enter *BestSet*. After the termination of the GA, *BestSet* is sorted and the first, middle, and last members are selected to be improved by using local search methods. Ultimately, the solution with the best fitness function is selected as the final solution. However, the other members of *BestSet* are possible solutions, one of which may be selected by the expert if it seems to be more sensible.

5.2 The LS methods

To improve a solution to the upper-level optimization problem, we use two LS methods. During the first LS method (LS1), each bound is increased (or decreased) by α percent while the fitness function (calculated by the *Evaluate* algorithm) improves. In the second LS method (LS2), each lower bound is increased (or decreased) together with its respective upper bound by α percent while the fitness function (calculated by the *Evaluate* algorithm) improves. In LS2, the distance between the upper bound and its respective lower bound remains fixed. During the running of the GA, we use the LS methods with $\alpha = 10\%$. After the termination of the GA, LS1, LS2, and then LS1 are run with $\alpha = 10\%$. Next, LS1 and then LS2 are run with $\alpha = 1\%$ for the selected solution of *BestSet*.

6 Numerical results and their analysis

To examine the proposed BOP model, we use the UNDP data of 2019. A summary of the HDI and its sub-indicators (from the UNDP data) for 189 countries is presented in Table 6.

Table 6: Summary of the HDI and its sub-indicators for 189 countries (the UNDP data of 2019)

	Education	Life expectancy	Standards of living	HDI
Mean	0.659	0.811	0.714	0.722
Median	0.682	0.832	0.732	0.740

We solve the proposed BOP model by applying the heuristic GA; the optimized variable bounds of the AR-I constraints for the different enriched BoD models are obtained in Table 7.

Table 7: The optimized bounds of the AR-I constraints for the different enriched BoD models

	L_1^v	U_1^v	L_2^v	U_2^v
Traditional BoD in Table 1	2.9737	2.9934	1.0271	1.0285
Multiplicative BoD in Table 3	2.2988	2.3123	0.9091	0.9146
SBM-BoD in Table 4	1.7715	2.6488	0.8487	2.2152
RAM-BoD in Table 5	0.6985	0.7023	0.9911	1.0019

It is worthwhile to note that all three solutions selected from *BestSet* almost converge to the best solution, presented in Table 7, by using the local search methods. For the traditional enriched BoD model, the weight of the education sub-indicator is 2.9 times that of the health sub-indicator. For the enriched BoD models other than the RAM, the weight of the education sub-indicator is at least 1.8 times that of the health sub-indicator. Except for the enriched SBM-BoD model, the bounds for the other enriched BoD models show that the weights of the education and income sub-indicators are almost equal.

We use these AR-I bounds to enrich the BoD models. The correlation between the rankings obtained from the HDI values and the efficiency scores resulting from the BoD models with and without AR-I constraints are represented in Table 8.

Table 8: The correlation between the rankings from the HDI values and the BoD models

		$+AR-I$
Traditional	0.538	0.997
Multiplicative	0.997	0.997
SBM	0.999	0.999
RAM	0.998	0.998

As evident in Table 9, the strong correlations between BoD models and HDI rankings—excluding the traditional BoD model without AR-I constraints—indicate that BoD models are generally suitable for assessing the human development of countries. Nevertheless, Tables 8 and 9 reveal specific differences among these models in detail.

Table 9 presents a summary of the errors: the absolute values of differences between the efficiency scores (E) and the HDI values are ($|HDI - E|$), and the absolute values of differences between the rankings based on the HDI values (R_{HDI}) and the rankings based on the efficiency scores (R_E) are ($|R_{HDI} - R_E|$). Moreover, the errors of the BoD models without the AR-I constraints are represented.

As mentioned before, the classification of countries is based on the fixed cutoff points of the HDI: low (less than 0.550), medium (0.550–0.699), high (0.700–0.799), and very high (0.800 or greater). So, the objective function of the proposed BOP is to minimize the changes in human development values. To further investigate the results, we represent the number of changes in the classification of countries in comparison with the UNDP's HDI in Table 10.

Regarding Tables 7–10, we discuss some points.

- The fitness function of the BOP is the sum of the values of |HDI-E|. So, the best enriched BoD model to reproduce the basis is the SBM, and the worst one is the RAM. Also, the best and worst errors for

Table 9: A summary of the errors of the BoD models without and with the AR-I constraints

BoD model	Weight restriction		Average	Median	Min	Max	Sum
Traditional		$\lvert HDI - E\rvert$	0.171	0.141	0.041	0.606	32.285
		$\lvert R_E - R_{HDI}\rvert$	39.196	29	0	184	7408
	+AR-I	$\lvert HDI - E\rvert$	0.019	0.016	0	0.050	3.512
		$\lvert R_E - R_{HDI}\rvert$	2.900	2	0	17	548
Multiplicative		$\lvert HDI - E\rvert$	0.105	0.093	0.037	0.260	19.824
		$\lvert R_E - R_{HDI}\rvert$	12.847	10	0	45	2428
	+AR-I	$\lvert HDI - E\rvert$	0.019	0.018	0	0.049	3.683
		$\lvert R_E - R_{HDI}\rvert$	2.847	2	0	17	538
SBM		$\lvert HDI - E\rvert$	0.028	0.0131	0	0.062	5.346
		$\lvert R_E - R_{HDI}\rvert$	1.249	1	0	8	236
	+AR-I	$\lvert HDI - E\rvert$	0.012	0.007	0	0.048	2.186
		$\lvert R_E - R_{HDI}\rvert$	1.661	1	0	7	314
RAM		$\lvert HDI - E\rvert$	0.096	0.079	0	0.289	18.134
		$\lvert R_E - R_{HDI}\rvert$	2.730	2	0	10	516
	+AR-I	$\lvert HDI - E\rvert$	0.095	0.079	0	0.289	17.968
		$\lvert R_E - R_{HDI}\rvert$	2.529	2	0	9	478

misclassification are obtained by the enriched SBM and the RAM with or without AR-I constraints, respectively (Table 10).

- It is evident from Tables 9 and 10 that the enriched traditional and multiplicative BoD models show the most improvements in becoming aligned with the UNDP's HDI in comparison with their original models. This is because of the wide feasibility region in the original BoD model that is limited by the AR-I constraints in an optimal way.

- After considering the AR-I constraints, the errors of misclassification (Table 10) obtained for the SBM-BoD model show notable improvement. However, the changes in the RAM-BoD model are not considerable.

- It seems that applying the AR-I constraints to the SBM-BoD model makes the errors of the human development values slightly better; however, it makes the errors of the rankings rather worse. Although, the changes in the new rankings do decrease misclassifications (Table 10).

- Tables 9–10 show as the results of the BoD model without the AR-I constraints get farther away from the UDNP's HDI, the optimized upper and lower bounds of the AR-I constraints get closer to each other (in the traditional BoD), and vice versa (in the SBM-BoD).

Table 10: The number of changes in the human development classification by using the BoD models with or without AR-I constraints

Traditional		Multiplicative		SBM		RAM	
	+AR-I		+AR-I		+AR-I		+AR-I
112	12	102	14	27	6	79	79

7 Discussion

The proposed bi-level optimization framework for the DEA model serves as a versatile structure for initializing parameters within the DEA context. While parameters can often be determined through expert opinions, real-world scenarios may demand a more gradual transition from traditional ranking methods to DEA-based approaches. For instance, organizations such as banks might initially possess baseline efficiencies derived from conventional methods and seek a seamless transition to DEA methods. This gradual transition allows for the calculation of subsequent efficiencies with bearable and acceptable changes, ensuring a smooth adoption process without drastic disruptions to existing operations. One important aspect to consider is the potential challenge of translating certain constraints that can be introduced to DEA models into meaningful relationships among inputs, outputs, or DMUs. In such cases, establishing a baseline efficiency scores, if attainable, proves valuable. The computational time required to solve the bi-level DEA model grows as the number of DMUs, inputs, and outputs increases. Nonetheless, as the lower-level optimization is linear and the upper level is managed by a heuristic, the computational complexity remains linear. Furthermore, the bi-level optimization is solved once, and subsequently, using the obtained parameters, the DEA model is employed to assess DMUs. It is important to note that solving the bi-level DEA model is challenging because it falls under NP-hard problems, leading to difficulties with local optima. The effectiveness of the heuristic method in navigating these challenges hinges on thoughtful de-sign and proper initialization of parameters. Employing a greedy search for parameter initialization is a common strategy that can enhance the model's efficiency and effectiveness.

8 Conclusion

In the 1990s, the UNDP introduced the HDI. This is currently measured as the geometric average of three sub-indicators. One of the drawbacks of this method is the use of equal weights for the three sub-indicators. To resolve this problem, it is suggested that a DEA model be employed, particularly one that is enriched by weight restrictions. In this paper, we considered multiple BoD models enriched by the AR-I constraints. We suggested a novel objective method for determining the parameters associated with the AR-I constraints. The proposed method was based on a BOP, and by considering a basis, it sought the best bounds in such a way that the results of the enriched BoD model were able to reproduce the basis. It should be noted that because of the nature of its sub-indicators, the UNDP's HDI usually shows small changes in comparison with last year's assessment. To maintain this quality for the new method of assessing human development and to make it more acceptable, the UNDP's HDI was fixed as the basis. According to our numerical results, the scores obtained by the enriched SBM-BoD model were similar to those obtained by the UNDP's HDI. Except for the enriched RAM-BoD, which could not be aligned with the UNDP's HDI, the effect of the education sub-indicator on the evaluation of human development was determined to be much more than that of the health sub-indicator.

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