



## Payment scheduling under project crashing based on project progress

M. Mortazavi Nejad, H.R. Tareghian and Z. Sari\*

### Abstract

In this paper, we address a new problem in the context of project payment scheduling when project activities are allowed to be crashed with the purpose of maximizing the contractors *net present value* (NPV). We assume that the contractor is paid at some pre-specified points of time according to the volume of work performed. Upon completion of activities, the cost of their execution is paid. Two different approaches are used to determine the volume of work performed at so called review points. In the first approach, only completed activities are considered. In the second approach, any portions of the activities that are executed are considered. To increase the volume of work performed at the review points, the contractor may decide to crash some activities and as such may possibly increase his NPV. As activity crashing costs the contractor money, a compromise needs to be made. Two mathematical models are developed to study each approach and hence help the contractor to make the best decision. These models offer a means of investigating whether it is advisable to crash some activities and are therefore of practical importance. It is shown that the contractor may increase his NPV, even when he pays for the activity crashing costs. The performance of the mathematical models is illustrated using a numerical example.

**Keywords:** Payment scheduling; Progress payment; Project crashing; Contractor's net present value.

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\*Corresponding author

Received 29 April 2015; revised 8 November 2016; accepted 3 Desember 2016

M. Mortazavi Nejad

Department of Applied Mathematics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad, Mashhad, Iran. e-mail: mamortazavi2012@gmail.com

H.R. Tareghian

Department of Applied Mathematics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad, Mashhad, Iran. e-mail: taregian@ferdowsi.um.ac.ir

Z. Sari

Department of Applied Mathematics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad, Mashhad, Iran. e-mail: zeynabsari89@gmail.com

## 1 Introduction and Background

The *Critical path method* (CPM) is essentially a time-oriented technique based on network structure which is mainly used to schedule project activities under resource availability or scarce resources. Time cost trade-off problem (TCTP) in which the compression of the project schedule is sought in order to achieve an improved outcome in terms of project duration, cost, and projected revenues is one of the earliest applications of the CPM in which cost factors are considered. The objective of the TCTP is to compress the project to the optimum duration which minimizes the total project costs. Other project cost monitoring are mainly focused on reporting the total amount of project costs and no consideration is given to the time value of money [13]. Considering that large scale projects usually have long life cycles, *net present value* (NPV) is suggested as a criterion for the financial control of projects. Russell introduced the concept of cash flow in a project [15]. He assumed that the project is represented in *activity-on-arrow* (AOA) format and that the payments, i.e. outward cash flows and the receipts, i.e. inward cash flows occur at some or all of the project network's nodes. Considering the inward and outward cash flows the problem is to schedule the network's events in a manner that the present value of cash flows is maximized. In order to maximize the NPV of the projects cash flows, Russell developed a mathematical model with a nonlinear objective function and a set of linear constraints [15]. The model's objective function was then linearized by the Taylor's series expansion around a feasible solution. The dual of the linearized model which possess the characteristics of the minimum cost network flow can be rapidly solved in small networks and can be efficiently solved in larger networks. The final solution is then obtained iteratively. In addition, Russell came to some interesting conclusions: *the critical path may not be very cost significant but that there exists a cost-critical tree of activities to each of which a marginal cost of lengthening the duration can be ascribed.*

In 1972, Grinold considered a deadline for the project and transformed the non-linear model of Russell into an equivalent linear model. The linear model that has the structure of a weighted distribution problem was then solved using an efficient procedure [7]. In 1982, Talbot developed a mixed integer programming model for the project cash flow problem under the limitation of resources [18]. In 1990, Elmaghraby et al. developed a procedure to solve the cash flow problem with fixed amounts of inward and outward flows [6]. They showed that it is beneficial to advance the events that correspond to positive flows and delay the events that relate to negative flows as much as possible (see e.g. the review papers [11] and [19]).

In the majority of the articles, the flows are assumed to be of constant and known values. The contractor knows the costs of performing the project activities and therefore can negotiate the amount and time of the client's

payments in order to maximize his returns. As payments are usually done according to the volume of the work performed (*progress payment*), scheduling of activities find significant importance. The problem of determining the amount as well as the timing of payments to maximize NPV is called the *payment scheduling problem* (PSP). It was first studied from the contractor's view point by Dayanand et al. [2]. Kazaz et al. considered the problem for projects with unlimited resources and developed a mixed integer programming model to maximize NPV. They showed that Bender's decomposition can reduce the running time of the model significantly [13]. Sepil et al. studied the performance of a number of heuristics to solve the PSP under resource limitation [16].

In continuation of their previous research, Dayanand et al. developed a 0-1 programming model together with a number of heuristics. Dayanand et al. altered Russell, Grinold and Talbot's models by adding some new constraints and claimed that their models can be used for both the contractor's and client's view points subject to some modifications [3]. Ulusoy et al. considered the PSP from the simultaneous view points of the contractor and the client. They developed a two loop genetic algorithm to study the problem. The outer loop represents the client and the inner loop represents the contractor. In the outer loop and under the assumptions of fixed schedules as well as the timing of payments, the client changes the amount of payments. Fixing the amount of payment, the inner loop is executed such that by rescheduling the activities, the contractor's NPV is maximized. The two loops negotiate their solutions until they come to a "fair" solution that is accepted by both parties [20]. Four types of payment scheduling models: *lump sum payment at the terminal event-LSP*, *payments at event occurrences-PEO*, *the equal time intervals-ETI* and *progress payment-PP* were distinguished by Ulusoy et al. They used the two loops genetic algorithm developed in [20] to study the PSP under resource limitations and in respect of each of the payment scheduling models [21]. Dayanand et al. developed a two stage search heuristic to solve PSP. They utilized a simulated annealing model in the first stage to find a set of payments. To improve the solution, they tuned the solution in the second stage by rescheduling activities [4]. Dayanand et al. modeled the PSP from the client's view point and developed a number of mixed integer programming models to study the problem [5]. Vanhoucke et al. considered the PSP under unlimited resources and developed a branch and bound procedure to study and analyze it [22].

Szmereskovsky developed a branch-and-bound procedure for a novel PSP model where the client selects the payment schedule and the contractor protects his interests by selecting the activity schedule. However, the contractor rejects the payment schedule if his NPV does not exceed a given threshold [17]. The multi-mode resource constrained project scheduling problem with discounted cash flow under the four payment scheduling models pro-

posed by Ulusoy et al., was studied by Mika et al. who considered only positive cash flows and employed a simulated annealing as well as a genetic algorithm to solve the problem [14].

Discrete time cost trade-off problem (DTCTP) involves the selection of a set of execution modes in order to achieve a certain objective. Although DTCTP has been combined with the maximum NPV project scheduling problem (see e.g. [23]), its influence on project payment scheduling has not been studied extensively [9]. The resulting problem is called the *multi-mode project payment scheduling problem* (MPSP) where the objective is to assign activity modes and progress payments so as to maximize the NPV under the constraint of project deadline. He et al. analyzed the effect of the bonus-penalty structure on payment scheduling and found that this structure can enhance the flexibility of payment scheduling greatly [8]. In 2009, He et al. developed two heuristics, i.e. simulated annealing and tabu search to study the MPSP and compared their performance on a data set constructed randomly [9].

Kavlak et al. investigated client-contractor bargaining in project scheduling with limited resources using *activity-on-node* (AON) networks. They considered two different payment models. In the first model, contractor receives payments at predetermined regular time intervals. In the second model, he receives only one payment at activity completions. They developed a simulated annealing and a genetic algorithm as solution procedures. The client and the contractors desire to seek a compromise are reflected in an objective function [12]. He et al. defined a new problem called the multi-mode capital-constrained project payment scheduling problem (MCCPSP) as the combination of the capital-constrained project scheduling problem and the MPSP. They studied the MCCPSP where the objective is to assign activity modes and payments concurrently so as to maximize the NPV of the contractor under the constraint of capital availability [10].

In this paper, we consider the PSP with unlimited resources, and address the question of activity compression from the contractor's view point. Our assumptions are as follows: (i) a deadline is specified for the project; (ii) the negative cash flows occur at the completion of activities; (iii) the payments are done at review points; (iv) the amount of each payment is based on the volume of work completed; (v) any extra costs for compression of activities are paid by the contractor. Two approaches are considered for the determination of the volume of finished work: (i) only finished activities are considered; (ii) any part of work completed by the review point is accounted for. Considering that by increasing the volume of work at each review point, the contractor can increase the amount of payments due to him and as such increase his NPV, an interesting question arises about the suitability of compressing some activities so that more activities can be performed within the current review points. From now on, we call this problem *crash max NPV<sup>PP</sup>*

problem and develop some mathematical models to study it.

The rest of the paper is organized as follows. In Section 2, we present two models for the problem to account for the two approaches proposed to determine the volume of finished work. In Section 3, we tailor the models of Section 2 for the problem. In Section 4, we discuss the results and determine the most significant parameters. Some conclusions are drawn in Section 5.

## 2 *max NPV<sup>PP</sup>* Problem

To study *max NPV<sup>PP</sup>* Problem, we assume that the project with activities is represented in AON format. Finish to start precedence with zero lag governs the relations between activities. The following notations are used in our analysis:

$d_i$	: Duration of activity $i$ ( $1 \leq i \leq n$ )
$c_i$	: Cost of performing activity $i$ ( $1 \leq i \leq n$ )
$s_i$	: Starting time for activity $i$ ( $1 \leq i \leq n$ )
$f_i$	: Completion time for activity $i$ ( $1 \leq i \leq n$ )
$Ef_i$	: Earliest finish time for activity $i$ ( $1 \leq i \leq n$ )
$Lf_i$	: Latest finish time for activity $i$ ( $1 \leq i \leq n$ )
$\alpha$	: Discount rate
D	: Project deadline
$c_t$	: Discount factor for time $t$ , $c_t = e^{-\alpha t}$ , $t = 0, 1, \dots, D$
T	: A constant period of time, $mT \geq D$ , $(m-1)T < D$
$P_t$	: Payment amount at review point $t$ , $t = T, 2T, \dots, mT$
$\omega_{it}$	: Percentage of activity $i$ completed in the interval $(t-T, t]$ , $t = T, 2T, \dots, mT$
$W_{it}$	: Percentage of activity $i$ completed in the interval $(0, t]$ , $t = T, 2T, \dots, mT$

The negative and positive cash flows with respect to activity  $i$  is  $c_i$  and  $(1 + \gamma_i) c_i$  respectively. Let  $\{T, 2T, \dots, mT\}$  denotes the set of review points (if  $mT > D$  then  $mT = D$ ). The amount of payment is based on two different approaches. We use example of Fig.1 to illustrate the differences between the two approaches. There are three activities in Fig.1 whose durations, costs and marginal profits are 20, 600 and 20 respectively. we assume that it is required to determine the amount of payments at  $T = 30$  and  $2T = 60$ . We have:

**Approach 1** -In Approach 1 we only consider the cost of those activities that are finished in the time interval  $(t-T, t]$ . With reference to Fig.1,  $P_T = 0$ , because no activity is completed in the interval  $(0, T]$ . However, in the interval  $(T, 2T]$  all the three activities are completed, therefore

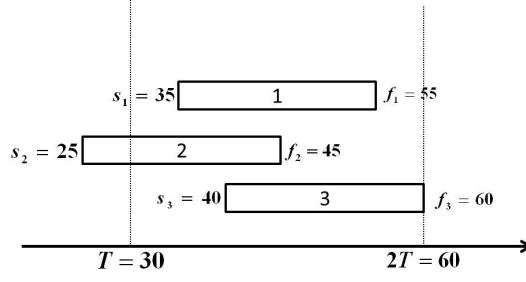


Figure 1: Schedule of three activities.

$$P_{2T} = (1 + \gamma_1) c_1 + (1 + \gamma_2) c_2 + (1 + \gamma_3) c_3 = 2160.$$

**Approach 2** -In this approach the cost of performing any section of the activities in the time interval  $(t-T, t]$  is considered. Up to the point  $T$ , only 25% of activity 2 is completed. So  $\omega_{2,T} = 5/20$  and  $P_T = (1 + \gamma_2) \omega_{2,T} c_2 = 180$ . In the interval  $(T, 2T]$  we have:  $\omega_{1,2T} = \omega_{3,2T} = 1$  and  $\omega_{2,2T} = 15/20$ . Therefore,  $P_{2T} = (1 + \gamma_1) \omega_{1,2T} c_1 + (1 + \gamma_2) \omega_{2,2T} c_2 + (1 + \gamma_3) \omega_{3,2T} c_3 = 1980$ . As it can be seen, the total amount of payments in the two review points is 2160. In fact, the total amount of payments can be obtained from  $\sum_{i=1}^n (1 + \gamma_i) c_i$ . However, the discounted amount of payments at rate  $\alpha$  at time  $t$  is given by  $P_t e^{-\alpha t}$ . In the next section we present two models for the *max NPV<sup>PP</sup>* according to the given approaches. Model *max NPV<sup>PP</sup>* with respect to Approach 1 is called *max NPV<sub>1</sub><sup>PP</sup>* and with respect to Approach 2 is called *max NPV<sub>2</sub><sup>PP</sup>*.

## 2.1 Models

We develop our models based on the aforementioned assumptions. Note that the contractor's estimate of the amount due to him is based on the marginal profit, the costs of performing the activities and the approach adopted for determining the volume of the work performed. In order to illustrate the features of the proposed models we use the example project given in Appendix A.

### 2.1.1 Model *max NPV<sub>1</sub><sup>PP</sup>*

For each activity  $i$  and time  $t$ , variable  $\delta_{it}$  indicates whether activity  $i$  has occurred before time period  $t$ , namely  $\delta_{it} = 1$  if  $f_i \leq t$  and 0 otherwise. Thus,  $\delta_{iD} = 1$  for  $i = 1, 2, \dots, n$ .

$$Max \sum_{t=T, 2T, \dots, mT} P_t e^{-\alpha t} - \sum_{i=1}^n c_i e^{-\alpha f_i} \quad (1)$$

s.to.

$$f_1 = 0 \quad (2)$$

$$f_j - d_j \geq f_i \quad (i, j) \in A \quad (3)$$

$$f_i \leq t + (1 - \delta_{it})D \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \quad (4)$$

$$f_i \geq t - \delta_{it}D \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \quad (5)$$

$$\sum_{s=T}^t P_s \leq \sum_{i=1}^n (1 + \gamma_i) \delta_{it} c_i \quad t = T, 2T, \dots, mT \quad (6)$$

$$f_i \geq 0 \quad i = 1, 2, \dots, n \quad (7)$$

$$P_t \geq 0 \quad t = T, 2T, \dots, mT \quad (8)$$

$$\delta_{it} \in \{0, 1\} \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \quad (9)$$

The objective function in (1) maximizes the NPV. Constraint (2) enforces the project to start at time 0. The set of constraints in (3) ensure the precedence relations between the activities while constraints (4) and (5) display the relations between  $f_i$ ,  $\delta_{it}$  and  $t$  with respect to activity  $i$  and determine the values of  $\delta_{it}$ . Constraint (6) ensures that the payment at each review point  $t$  is not more than the volume of work performed. Model  $max NPV_1^{PP}$  as described by (1) to (9) is a mixed integer program with a nonlinear objective function and a set of linear constraints. The objective can be linearized as follows. Let  $x_{it} = 1$  denotes the completion of activity  $i$  at time  $t$ , and  $x_{it} = 0$  denotes otherwise. The completion time for an activity can be expressed as  $f_i = \sum_{t=Ef_i}^{Lf_i} t x_{it}$ . Similarly, the variable  $\delta_{it}$  can be expressed as  $\delta_{it} = \sum_{s=0}^t x_{is}$ .

Since  $x_{if_i} = 1$  and  $x_{it} = 0$  for  $t \neq f_i$  then  $\sum_{t=Ef_i}^{Lf_i} c_t x_{it} = c_{f_i}$ . Now the second part of (1) becomes

$$\sum_{i=1}^n c_i e^{-\alpha f_i} = \sum_{i=1}^n c_i c_{f_i} = \sum_{i=1}^n c_i \sum_{t=Ef_i}^{Lf_i} c_t x_{it}$$

which if substituted in model  $max NPV_1^{PP}$ , produces a linear programming model (10-16) that we call  $linear max NPV_1^{PP}$ .

$$\text{Max} \quad \sum_{t=T, 2T, \dots, mT} P_t c_t - \sum_{i=1}^n c_i \sum_{t=Ef_i}^{Lf_i} c_t x_{it} \quad (10)$$

s.to.

$$x_{1,0} = 0 \quad (11)$$

$$\sum_{t=Ef_i}^{Lf_i} x_{it} = 1 \quad i = 1, 2, \dots, n \quad (12)$$

$$\sum_{t=Ef_j}^{Lf_j} (t - d_j) x_{jt} \geq \sum_{t=Ef_i}^{Lf_i} t x_{it}, \quad (i, j) \in A \quad (13)$$

$$\sum_{s=T}^t P_s \leq \sum_{i=1}^n (1 + \gamma_i) c_i \sum_{s=0}^t x_{is} \quad t = T, 2T, \dots, mT \quad (14)$$

$$P_t \geq 0 \quad t = T, 2T, \dots, mT \quad (15)$$

$$x_{it} \in \{0, 1\} \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \quad (16)$$

Constraint (12) ensures that there is only one finish time for each activity as no activity preemption is allowed. Set of constraints in (13) ensure the precedence relations between the activities. Using model (10-16), the example project is solved by GAMS and BONMIN as the solver [1]. The optimal schedule with the NPV of 2611.977 is shown in Fig.2. As it can be seen activities have been scheduled so that the maximum amount is paid at review points, and also they are scheduled as late as possible to delay the accrument of costs. For instance, in the time interval  $(0, 30]$ , activity 2 and after that, activities 3, 4, 5 and 6 have been scheduled as late as possible. In addition, if activity 2 was to finish on day 8, the NPV would have decreased to 2523.324. This is because the increase in payments due to activities 7 and 8 being finished on day 30 does not compensate the costs of performing the activities that now had to be paid earlier.

### 2.1.2 Model $max NPV_2^{PP}$

In this model any parts of the activities that are completed are considered for the analysis of payments. To calculate  $W_{it}$ , the fraction of activity  $i$  completed at review point  $t$ , three situations might occur (see Fig.3) which should be considered. The amount of  $W_{it}$  is calculated as follows:

$$W_{it} = \delta_{it} + (1 - \delta_{it}) \max\left\{0, \frac{t - (f_i - d_i)}{d_i}\right\} \quad (17)$$

$$0 \leq W_{it} \leq 1 \quad i = 1, 2, \dots, n; t = T, 2T, \dots, mT \quad (18)$$

Using  $W_{it}$ , we rewrite the constraint about the payments as follows:



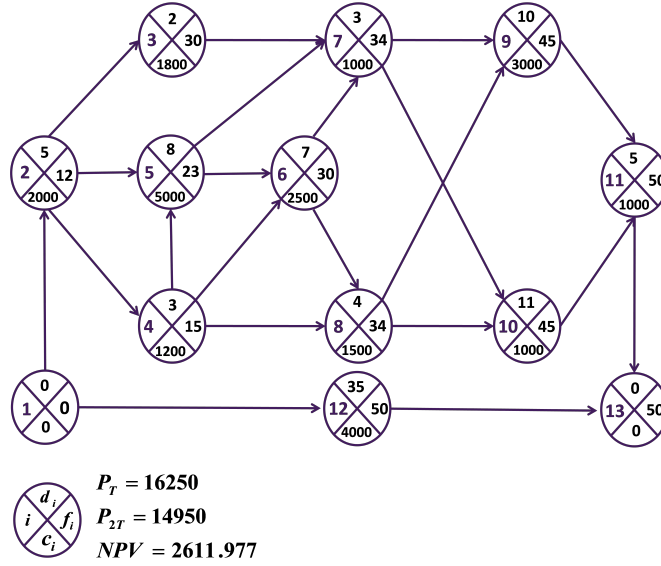


Figure 2: The  $linear\ max\ NPV_1^{PP}$  model - the optimal payment schedule.

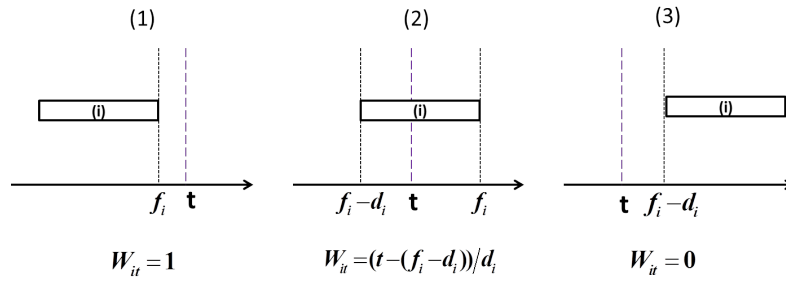


Figure 3: Portions of work completed in the interval  $(0, t]$ .

$$\sum_{s=T}^t P_s \leq \sum_{i=1}^n (1 + \gamma_i) W_{it} c_i \quad t = T, 2T, \dots, mT \quad (19)$$

By adding (17) and (18) to model  $max\ NPV_1^{PP}$  and replacing constraint (6) with (19) we obtain model  $max\ NPV_2^{PP}$ . This is also a mixed integer nonlinear programming model. Using model  $max\ NPV_2^{PP}$  the example project was solved by GAMS with DICOPT as the solver. The acceptable schedule with the NPV of 2980.275 is the same as Fig.2. However the NPV has increased. This is because at review point 30 part of activity 12 which is partially completed has also been accounted for.

### 3 *crash max NPV<sup>PP</sup>* Problem

In most projects, it is possible to compress the duration of all or some of the activities by allocating extra resources. Obviously, allocation of extra resources costs more. In spite of the fact that the contractor pays this extra cost, his motivation for crashing activities in the *crash max NPV<sup>PP</sup>* problem as described previously is the possible benefit that can be gained by the earlier payment of increased volume of work performed. The activity  $i$ 's duration after compression,  $d'_i$ , can be varied between its normal duration,  $d_i$ , and its crashed duration,  $d''_i$ , i.e.  $d''_i \leq d'_i \leq d_i$ . The cost of activity  $i$ , if performed at crashed duration is  $c'_i$ . Therefore the resource utilization rate of activity  $i$  is  $c'_i = (c'_i - c_i)/(d_i - d''_i)$ . The discounted cash flow for activity  $i$  is therefore  $(c_i + c'_i(d_i - d''_i)) e^{-\alpha f_i}$ . Recall that costs are accrued at the completion of activities and payments are done at review points. In the next section we present our models.

#### 3.1 *crash max NPV<sup>PP</sup>* Model based on Approach 1

The following model is based on the *linear max NPV<sub>1</sub><sup>PP</sup>* developed in the previous section.

$$\text{Max} \quad \sum_{t=T, 2T, \dots, mT} P_t c_t - \sum_{i=1}^n (c_i + c'_i(d_i - d'_i)) \sum_{t=0}^D c_t x_{it} \quad (20)$$

s.to.

$$x_{1,0} = 0 \quad (21)$$

$$\sum_{t=0}^D x_{it} = 1 \quad i = 1, 2, \dots, n \quad (22)$$

$$\sum_{t=0}^D (t - d'_j) x_{jt} \geq \sum_{t=0}^D t x_{it} \quad (i, j) \in A \quad (23)$$

$$\sum_{s=T}^t P_s \leq \sum_{i=1}^n (1 + \gamma_i) c_i \sum_{s=0}^t x_{is} \quad t = T, 2T, \dots, mT \quad (24)$$

$$d'_i \leq d_i \quad i = 1, 2, \dots, n \quad (25)$$

$$d'_i \geq d''_i \quad i = 1, 2, \dots, n \quad (26)$$

$$P_t \geq 0 \quad t = T, 2T, \dots, mT \quad (27)$$

$$x_{it} \in \{0, 1\} \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \quad (28)$$

Constraints (25) and (26) ensure that activity's compressed duration is within its corresponding bounds. We now linearize the objective function in (20).

To that end, we replace  $d'_i x_{it}$  with the new variable,  $F_{it}$ . This is a standard substitution requiring additional constraints to link  $x_{it}$ ,  $d'_i$  and  $F_{it}$ . The values of  $F_{it}$  are determined by the following additional constraints.

$$F_{it} - x_{it} d_i \leq 0 \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \tag{29}$$

$$F_{it} - d'_i \leq 0 \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \tag{30}$$

$$F_{it} \geq d'_i - (1 - x_{it})d_i \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \tag{31}$$

$$F_{it} \geq 0 \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \tag{32}$$

Now the second part of (20) becomes

$$\sum_{i=1}^n \sum_{t=0}^D ((c_i + c'_i d_i) x_{it} - c'_i F_{it}) c_t \tag{33}$$

The resulting model is a mixed integer linear program which we call *linear crash max NPV<sub>1</sub><sup>PP</sup>*. The optimum solution for the example problem using the above model is shown in Fig.4. Note that the project NPV has increased to 3036.982 when compared with the case where no crashing of activities was allowed. This shows that even when the contractor pays for the extra cost of activity crashing, it can increase his NPV. As can be seen, activities 5, 6, 7, and 8 have been compressed resulting in a larger volume of work being performed up to review point of 30. Activity 11 which has been crashed by 2 days has caused activities 9 and 10 to end 2 days later and as a result their corresponding costs accrued in later time.

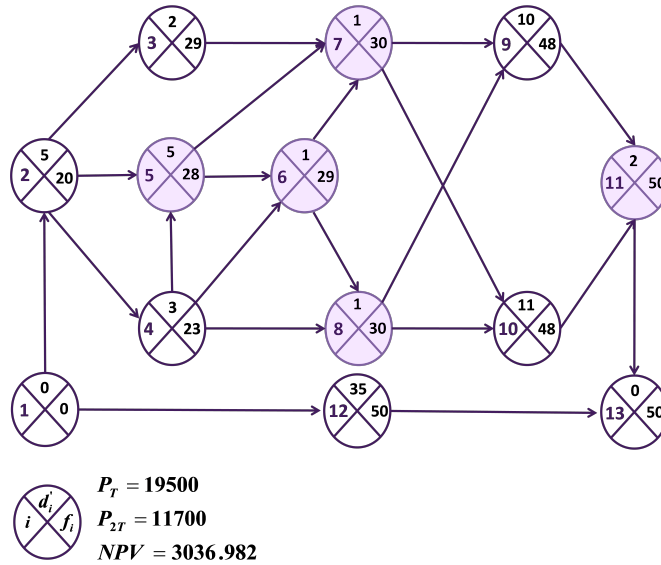


Figure 4: The *linear crash max NPV<sub>1</sub><sup>PP</sup>* model - the optimal payment schedule.

### 3.2 crash max NPV<sup>PP</sup> Model based on Approach 2

The following model is based on the max NPV<sub>2</sub><sup>PP</sup>.

$$\text{Max} \sum_{t=T, 2T, \dots, mT} P_t e^{-\alpha t} - \sum_{i=1}^n (c_i + c'_i(d_i - d'_i)) e^{-\alpha f_i} \quad (34)$$

s.to.

$$f_1 = 0 \quad (35)$$

$$f_j - d'_j \geq f_i \quad (i, j) \in A \quad (36)$$

$$f_i \leq t + (1 - \delta_{it})D \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \quad (37)$$

$$f_i \geq t - \delta_{it}D \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \quad (38)$$

$$W_{it} = \delta_{it} + (1 - \delta_{it}) \max\left\{0, \frac{t - (f_i - d'_i)}{d'_i}\right\} \quad (39)$$

$$i = 1, 2, \dots, n; t = T, 2T, \dots, mT$$

$$\sum_{s=T}^t P_s \leq \sum_{i=1}^n (1 + \gamma_i) W_{it} c_i \quad t = T, 2T, \dots, mT \quad (40)$$

$$d'_i \leq d_i \quad i = 1, 2, \dots, n \quad (41)$$

$$d'_i \geq d''_i \quad i = 1, 2, \dots, n \quad (42)$$

$$0 \leq W_{it} \leq 1 \quad i = 1, 2, \dots, n; t = T, 2T, \dots, mT \quad (43)$$

$$f_i \geq 0 \quad i = 1, 2, \dots, n \quad (44)$$

$$P_t \geq 0 \quad t = T, 2T, \dots, mT \quad (45)$$

$$\delta_{it} \in \{0, 1\} \quad i = 1, 2, \dots, n; t = 0, 1, \dots, D \quad (46)$$

The resulting model is a mixed integer non-linear program which we call *crash max NPV<sub>2</sub><sup>PP</sup>*. An acceptable solution for the example problem using above model is the same as Fig.4. However the project's NPV has increased to 3405.28. This is because at a review point 30 part of activity 12 which is partially completed has also been accounted for.

## 4 Discussion of Results

The results obtained by our proposed four models with respect to the project example are given in Table 1. In this example, payments are done at the end of the review points 30 and 50. Since the time and the total amount of payments are known, increasing the amount of payments in review points nearer to the beginning of the project will increase the contractors profit. Therefore, in the example project the compression of activities is used to increase the amount of payments at review point 30. As expected, the amount

of payment at review point 30 when some activities are crashed has increased from 16250 to 19500 under Approach 1 and from 18478.57 to 21728.57 under Approach 2. The schedule obtained by the  $max\ NPV^{PP}$  model and the

Table 1: Models and results.

Model	Amount of Payments	NPV
$linear\ max\ NPV_1^{PP}$	$P_T = 16250$ $P_{2T} = 14950$	2611.944
$linear\ crash\ max\ NPV_1^{PP}$	$P_T = 19500$ $P_{2T} = 11700$	3036.982
$max\ NPV_2^{PP}$	$P_T = 18478.57$ $P_{2T} = 12721.43$	2980.275
$crash\ max\ NPV_2^{PP}$	$P_T = 21728.57$ $P_{2T} = 9471.43$	3405.280

$crash\ max\ NPV^{PP}$  model is shown in Fig.5. Observe that in both generated schedules, the activities are scheduled as near to the review points as possible. This is done so that the costs are incurred as near as possible to the locations where payments are due. In addition, the payment of costs has also been delayed as much as possible. For instance, in  $crash\ max\ NPV^{PP}$  model, activities 9 and 10 can start at time instance of 30. Note that, this does not affect volume of the work performed up to this point of time. Should these activities start at time instance 30, the payment of their costs that is now advanced will affect the NPV adversely. Therefore, the model has not scheduled them to start at time 30. The activities in  $crash\ max\ NPV^{PP}$  model are compressed such that: (i) the volume of work performed up to the time instant 30 is increased as much as possible; and (ii) activities in the interval (30, 50] are scheduled as near as possible to the review point 50. Although compression of activity 11 has no effect on the volume of work performed, but it allows activities 9 and 10 to be completed 2 days later and as such delay the payment of their costs by the same amount. This will improve the NPV. Note that, the cost of compressing activity 11 is compensated by the profit yielded by delaying the payments of activities 9 and 10 costs. It is obvious that the crashing costs, the marginal profit and the discount rate are significant parameters that affect the decision regarding the compression of project activities to increase the contractor's NPV. For example, high compression costs and low marginal profit may convince the contractor not to proceed with activity crashing. In fact, the contractor pays for the cost of crashing some of the activities by the profit yielded from increasing the volume of work up to each review point and also by delaying some other activities. Therefore, if crashing of activities does not yield any profit for the contractor, he has no motivation to proceed with activity crashing. In the example project, if we consider the profit margin as 15% and the crashing

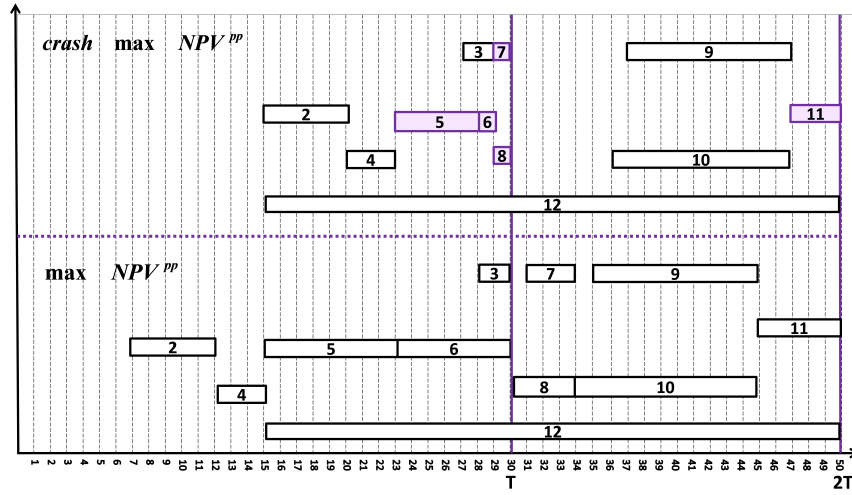


Figure 5: Comparison of normal and compressed schedules.

costs of activities as 0, 2700, 1800, 1200, 5800, 4000, 1400, 2400, 4500, 1500, 1800, 5900 and 0, no activity will be crashed.

## 5 Conclusions

In this paper, we addressed the problem of project payment scheduling when a project deadline is specified and activities are allowed to be compressed with the purpose of maximizing the contractor's *net present value* (NPV). The cost of activity compression is paid by the contractor. We assumed that the payments are made to the contractor based on the volume of work performed, at some pre-specified points of time. We also assumed that the costs of activities are incurred at the completion of activities. We developed four mathematical models based on two different approaches to account for the volume of work performed. In the first approach, only those activities that by the review time have been completed were considered as the volume of work performed. In the second approach, we considered any section of activities that were completed by the review time. Through a small illustrative example, it was shown that the contractor may in fact increase his profit by crashing activities.

## Appendix A

A project with 11 activities is considered. All the relevant information is given in Table 2. The earliest time that the project can be delivered is 42, whereas the deadline is 50. Marginal profit with respect to each activity is 30%, the discount rate is 1.5% and the review period is considered to be 30. Therefore, review points are at  $T = 30$ ,  $2T = 50$ .

Table 2: The example project.

Activity	Predecessor(s)	Normal Duration	Crashed Duration	Normal Cost	Crashed Cost
1	—	0	0	0	0
2	1	5	1	2000	2700
3	2	2	2	1800	1800
4	2	3	3	1200	1200
5	2,4	8	5	5000	5150
6	4,5	7	2	2500	3000
7	3,5	3	1	1000	1100
8	4,6	4	1	1500	1600
9	7	10	1	3000	4000
10	7,8	11	5	1000	1500
11	9,10	5	2	1000	1150
12	1	35	20	4000	5500
13	11,12	0	0	0	0

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## فشرده‌سازی پروژه در مسئله زمانبندی پرداخت براساس پیشرفت پروژه

مرضیه مرتضوی نژاد ، حامد رضا طارقیان و زینب ساری

دانشگاه فردوسی مشهد، دانشکده علوم ریاضی، گروه ریاضی کاربردی

دریافت مقاله ۹ اردیبهشت ۱۳۹۴، دریافت مقاله اصلاح شده ۱۸ آبان ۱۳۹۴، پذیرش مقاله ۱۳ آذر ۱۳۹۵

**چکیده :** در این مقاله، دیدگاه جدیدی در زمینه زمانبندی پرداخت پروژه مطرح می‌شود. در این دیدگاه فعالیت‌های پروژه برای بیشینه کردن خالص ارزش جاری پیمانکار مجاز به فشرده‌سازی هستند. فرض می‌شود پرداخت به پیمانکار در پایان دوره‌های زمانی مشخص که به آن نقاط بازبینی گفته می‌شود، به ازای حجم کار انجام شده و هزینه‌ها در زمان اتمام فعالیت‌ها رخ دهند. دو رویکرد متفاوت برای تعیین حجم کار در نقاط بازبینی استفاده می‌شود. در رویکرد اول فعالیت‌هایی که در نقطه بازبینی به طور کامل تکمیل شده باشند، در نظر گرفته می‌شوند. در رویکرد دوم هر بخش از فعالیت‌هایی که اجرا شده‌اند، نیز در نظر گرفته می‌شوند. برای افزایش حجم کار در نقاط بازبینی، ممکن است پیمانکار تصمیم بگیرد برخی از فعالیت‌ها را برای افزایش خالص ارزش جاری خود فشرده کند. این در حالی است که هزینه‌های فشرده‌سازی توسط پیمانکار پرداخت می‌شود. براساس هر یک از رویکردها، دو مدل برنامه‌ریزی ریاضی یکی در حالتی که فعالیت‌ها در زمان عادی زمانبندی شوند و دیگری در حالتی که فعالیت‌ها امکان فشرده‌سازی داشته باشند، ارائه می‌شود. این مدل‌ها ابزاری برای تصمیم‌گیری بهتر پیمانکار درباره فشرده‌سازی فعالیت‌های پروژه هستند. نشان داده می‌شود که پیمانکار می‌تواند مقدار خالص ارزش جاری خود را با فشرده‌سازی فعالیت‌ها افزایش دهد حتی اگر هزینه‌های فشرده‌سازی را خود پیمانکار پرداخت کند. در پایان هر یک از مدل‌ها بر روی یک مثال پیاده‌سازی و نتایج حاصل تجزیه و تحلیل شده است.

**کلمات کلیدی :** زمانبندی پرداخت‌ها؛ پرداخت براساس پیشرفت پروژه؛ فشرده‌سازی پروژه؛ خالص ارزش جاری پیمانکار.