Iranaian J. Numerical Analysis and Optimization
Vol. 3, No. 1, (2013), 9-12

Block-Coppels chaos in set-valued discrete systems

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Abstract

Let (X, d) be a compact metric space and $f : X \to X$ be a continuous map. Consider the metric space $(K(X), H)$ of all non empty compact subsets of X endowed with the Hausdorff metric induced by d. Let $\bar{f} : K(X) \to K(X)$ be defined by $\bar{f}(A) = \{f(a) : a \in A\}$. We show that Block-Coppels chaos in f implies Block-Coppels chaos in $\bar{f}$ if f is a bijection.

Keywords: Chaos; Discrete system; Dynamical system.

1 Introduction

Let (X, d) be a compact metric space with metric d and $f : X \to X$ be a continuous map. For every positive integer n, we define f^n inductively by $f^n = f \circ f^{n-1}$, where f^0 is the identity map on X. A map f is called to be Block-Coppels chaotic [3] if there exist disjoint non-empty compact subsets J, K of X and a positive integer n such that $J \cup K \subseteq f^n(J) \cap f^n(K)$.
Roman-Flores and Chalco-Cano investigated Robinsons chaos in set-valued discrete systems [5]. Gu investigated Katos chaos in set-valued discrete systems [2]. Devaneys chaos in set-valued discrete systems has been studied in several papers. For example see [4], [1].
In this paper, we investigate the relationships between Block-Coppels chaoticity of (X, f) and Block-Coppels chaoticity of $(K(X), \bar{f})$.

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2 Preliminaries

Let (X, d) be a compact metric space with metric d. The distance of a point x from a set A in X is defined by $d(x, A) = \inf\{d(x, a) : a \in A\}$ if $A \neq \varnothing$, and $d(x, \varnothing) = 1$. Let $K(X)$ be the family of all non-empty compact subsets of X.

The Hausdorff metric on $K(X)$ is defined by $H(A, B) = \max\{\sup\{d(a, B) : a \in A\}, \sup\{d(b, A) : b \in B\}\}$ for $A, B \in K(X)$. It is easy to see that $(K(X), H)$ is a compact metric space.

Let τ_d be the topology of X induced by the metric d. The topology τ_H of $K(X)$ induced by the Hausdorff metric H coincides with the topology τ_v generated by the basis β_v consisting of all sets of the form $G_0^u \cap G_1^l \cap \cdots \cap G_k^l$ where $G_0, G_1, \ldots, G_k \in \tau_d, G_0^u = \{A \in K(X) : A \subseteq G_0\}$ and

$$G_i^l = \{A \in K(X) : A \cap G \neq \varnothing\}, i = 1, 2, \ldots, k.$$

The topology τ_ν is also called the Vietoris topology or the exponential topology on $K(X)$.

If $f : X \in X$ is a continuous map then one can define a continuous map $\bar{f} : K(X) \to K(X)$ by letting $\bar{f}(A) = \{f(a) : a \in A\}$ for every $A \in K(X)$.

3 Block-Coppels Chaoticity

In this section, we show that Block-Coppels chaoticity of (X, f) implies Block-Coppels chaoticity of $(K(X), \bar{f})$ if f is bijection.

Definition 3.1. Let A be a subset of X, the extension of A to $K(X)$ is defined by $e(A) = \{K \in K(X) : K \subseteq A\}$.

Remark. It is clear that $e(A) = \varnothing$ if and only if $A = \varnothing$.

Lemma 3.1. *Let A be a non-empty compact subset of X. Then, $e(A)$ is a non-empty compact subset of $K(X)$.*

Proof. It is sufficient to show that $(e(A))^c$ is open because in this case $e(A)$ is closed and a closed subset of a compact space, is compact. If $K \in (e(A))^c$ then $K \notin e(A)$ which means $K \subsetneq A$. Therefore $K \cap A^c \neq \varnothing$, and hence $K \in (A^c)^l$. So that

$$(e(A))^c \subseteq (A^c)^l \tag{1}$$

On the other hand if $K \in (A^c)^l$ then $K \cap A^c \neq \varnothing$, therefore $K \subsetneq A$ and hence $K \notin e(A)$. So that $K \in (e(A))^c$ and therefore

$$(A^c)^l \subseteq (e(A))^c \tag{2}$$

These two relations show that $(e(A))^c = (A^c)^l$ and the proof is completed. □

The following lemma is obvious from definition.

Lemma 3.2. *Let A be a subset of X. Then,*

i) $e(A \cap B) = e(A) \cap e(B)$;
ii) $\bar{f}(e(A)) \subseteq e(f(A))$;
iii) $\overline{f^n} = \bar{f}^n$.

Lemma 3.3. *Let $f : X \to X$ be a continuous bijection and A be a subset of X.*
Then $\bar{f}(e(A)) = e(f(A))$.

Proof. According to previous Lemma $\bar{f}(e(A)) \subseteq e(f(A))$. Conversely if $K \in e(f(A))$, then $f^{-1}(K) \subseteq f^{-1}(f(A))$. Also $f^{-1}(f(A)) = A$ because f is a bijection. Therefore $f^{-1}(K) \subseteq A$ and $f(f^{-1}(K)) \in f(e(A))$. Also $f^{-1}(f(K)) = K$. Hence $K \in f(e(A))$ and therefore $e(f(A)) \subseteq \bar{f}(e(A))$. □

Theorem 3.1. *Let X be a compact metric space and $f : X \to X$ be a continuous bijection. If f is chaotic in the sense of Block-Coppel's, then so is $\bar{f}$.*

Proof. Let f be Block-Coppel's chaotic, then there exist disjoint non-empty compact subsets J, K of X and a positive integer n such that $J \cup K \subseteq f^n(J) \cap f^n(K)$. We claim that

$$e(J) \cup e(K) \subseteq \bar{f}^n(e(J)) \cap \bar{f}^n(e(K)).$$

Since $J \subseteq f^n(J)$, then $e(J) \subseteq e(f^n(J))$. According to Lemma 3.3 $e(f^n(J)) = \bar{f}^n(e(J))$. Therefore $e(J) \subseteq \bar{f}^n(e(J))$. In a similar way

$$e(J) \subseteq \bar{f}^n(e(K)), e(K) \subseteq \bar{f}^n(e(K)) \text{ and } e(K) \subseteq \bar{f}^n(e(J)).$$

Therefore

$$e(J) \cup e(K) \subseteq \bar{f}^n(e(J)) \cap \bar{f}^n(e(K))$$

On the other hand $e(J) \cap e(K) = e(J \cap K) = \varnothing$ and according to Lemma 3.1 $e(J), e(K)$ are compact, and the proof is completed. □

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