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Research Article

Heuristic solutions for interval-valued games

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Abstract

When we design the payoff matrix of a game on the basis of the available information, then rarely the information is free from impreciseness, and as a result, the payoffs of the payoff matrix have a certain amount of ambiguity associated with them. In this work, we have developed a heuristic technique to solve two persons $m \times n$ zero-sum games $(m > 2, n > 2)$, with interval-valued payoffs and interval-valued objectives. Thus the game has been formulated by representing the impreciseness of the payoffs with interval numbers. To solve the game, a real coded genetic algorithm with interval fitness function, tournament selection, uniform crossover, and uniform mutation has been developed. Finally, our proposed technique has been demonstrated with a few examples and sensitivity analyses with respect to the genetic algorithm parameters have been done graphically to study the stability of our algorithm.

AMS subject classifications (2020): 45D05; 42C10; 65G99.

Keywords: Two persons zero sum game; Interval-valued payoffs; Genetic Algorithm; Order relations.

1 Introduction

In this paper, an effort has been made to solve two persons $m \times n$ zero-sum games $(m < 2, n < 2)$, with interval-valued payoffs and interval-valued ob-

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jectives. A payoff matrix with interval-valued payoffs has been considered. Normally, when we design the payoff matrix of a game on the basis of the available information, then rarely the information is free from impreciseness/ vagueness, and as a result, the payoffs of the payoff matrix have a certain amount of ambiguity associated with them [8]. Due to this reason, the payoff received by a player as the result of interaction between any of his strategies with any strategy of his competitor is assumed to be interval-valued. Out of the several types of researches done on games, a large fraction has been based on two-person games like Blackwell's work considering vector payoffs [4], Zeleny's investigation on games with multiple payoffs, and so on [32]. Nishizaki and Sakawa [25] worked on two persons zero sum games with multiple goals. Since the end of the last century, especially in the last two decades, researchers have given more emphasis on multiobjective games, considering impreciseness in payoffs and goals. In the majority of these researches, the impreciseness has been represented and dealt with the help of the fuzzy approach. Researchers like Aubin [1, 2], Butnariu [5, 6], Campos [7], Nishizaki and Sakawa [26], Bector and Chandra [3], Vijay et al. [31], Cunlin and Qiang [11], Dutta and Gupta [12], Gong and Hai [15], Chandra and Aggarwal [10], Li [18, 19], Roy and Mondal [28], Jiang et al. [17], Qiu et al. [27], Madandar et al. [21] have all significantly contributed to the researches on the fuzzy games.

However, in the case of fuzzy approach, the user arbitrarily defines the shapes of fuzzy numbers. It is assumed that the constraints, objectives, and parameters are fuzzy sets and that the membership functions are known to the user/decision-maker. However, sometimes the user or the decision-maker is unable to specify the membership function accurately and hence, has to resort to an arbitrary approach. On the other hand, while dealing with impreciseness using interval numbers, one can be absolutely sure that the interval results will always contain the exact result irrespective of whether its upper boundary and lower boundary are overestimated. No scope for subjectivity is present in this case. Thus in a way, the interval approach has some distinct advantages over the fuzzy approach. Because of this, in this paper, we have dealt with the impreciseness with the help of the interval approach; that is, the imprecise payoffs have been represented by interval numbers. The objective function of the game is also interval-valued. In this paper, a genetic algorithm (GA) based heuristic technique of solving two persons zero sum games with interval-valued payoffs and objectives has been proposed, that is, to solve the game a real coded GA with interval fitness function, tournament selection, uniform crossover, and uniform mutation has been developed. GA is a widely popular heuristic search and optimization technique, first developed by Prof. J. H. Holland, University of Michigan. Currently, there are several textbooks on GAs by authors like Goldberg [14], Michalewicz [24], Sakawa [29], Gen and Cheng [13] and others. While implementing GA to solve the game, for selection operation and also for finding the best chromosome in each generation, we have used the concepts of interval arithmetic and order relation of interval number. The contributions of researchers like Ishibuchi and Tanaka [16], Chanas and Kuchta [9], Sengupta and Pal [30], Mahato and Bhunia [22] in defining ordered relations of interval-valued numbers are worth mentioning.

2 Concepts of interval numbers and their comparisons

Let $K = [k_L, k_R] = \{x : k_L \leq x \leq k_R, k_L, k_R, x \in R\}$ be an interval-valued number, where k_L and k_R are the left and right boundaries of K, respectively. Also, assume that $K = \langle kC, kr\rangle = \{x : k_C k_r \leq x \leq k_C + k_r, , x \in R\}$, where $k_C = (k_L + k_R)/2$ and $k_r = (k_R - k_L)/2$, are perspectively the center and radius of the interval, and R is the set of real numbers. If there are two closed interval numbers K and $G(= [g_L, g_R])$ and $\Delta \in (+, -, 0, 1)$ is a binary operation on the set of real numbers, then the binary operation on interval K and G is defined by $K\Delta G = \{k\Delta g : k \in K \ and \ g \in G\}$. In the case of division, it is assumed that $0 \notin G$. Thus,

(i) $K + G = [k_L + g_L, k_R + g_R]$,

(ii) $K - G = [k_L - g_R, k_R - g_L]$,

(iii) $\lambda K = \begin{cases} [\lambda k_L, \lambda k_R] \text{ if } \lambda \geq 0, \\ [\lambda k_R, \lambda k_L] \text{ if } \lambda < 0, \end{cases}$ where λ is a real number.

(iv) $$K \times G = [k_L, k_R] \times [g_L, g_R] = \begin{cases} [Min(k_L g_L, k_L g_R, k_R g_L, k_R g_R), Max(k_L g_L, k_L g_R, k_R, g_L, k_R g_R)], \\ [k_L g_L, k_R g_R], \text{only if } k_L \geq 0 \text{ and } g_L \geq 0. \end{cases}$$

Considering the optimistic decision making for maximization problems, the order relation $\geq_{o\max}$ between the intervals K and G is defined as

(v) $K \geq_{o\max} G$ if and only if $k_L \geq g_L$,

(vi) $K > o\max G$ if and only if $K \geq_{o\max} G$ and $K \neq G$.
Considering the pessimistic decision-making for minimization problems, the order relation $>_{p\max}$ between the intervals $K = [k_L, k_R] = \langle k_C, k_r\rangle$ and $G = [g_L, g_R] = \langle g_c, g_r\rangle$ may be defined as

(vii) $K >_{p\max} G$ if and only if $k_C > g_C$, when intervals K and G are either disjoint or partially overlapping.

(viii) $K >_{p\max} G$ if and only if $k_C \geq g_C$ and $k_r < g_r$ when interval K is contained in interval G.

3 Methodology and formulation

A popular and simple case of game theory is two persons zero sum game in which it is assumed that two players are involved, each having a finite number of strategies, and the algebraic sum of the gains and losses of those two players is equal to zero, that is, due to the interaction of any pair of strategies of the competitive players (say A and B), the amount received by one player is exactly equal to the losses of the other player. This can be illustrated with the following the $m \times n$ matrix:

$$M = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

where $M \in \mathbb{R}^{m\times n}$ is an $m \times n$ real payoff matrix of player A. Here $\mathbb{R}^U$ is the U-dimensional Euclidean space and a_{ij} is the payoff of player A, when player A plays the strategy i and player B plays the strategy j.
A mix strategy of player A is given by the vector x in R_+^m, that is, nonnegative orthant of R^m, such that

$$\underset{\sim}{x}^t e_m = 1 \ where \ e_m = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}_{m\times 1} \quad \text{and} \quad \underset{\sim}{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}_{m\times 1} \geq \underset{\sim}{0}.$$

Thus if S^m is the strategy space of player A, then $S^m = \{\underset{\sim}{x} \in R_+^m, \underset{\sim}{x}^t\ e_m = 1\}$. Similarly, the strategy space of player B is given by $S^n = \{\underset{\sim}{y} \in R_+^n, \underset{\sim}{y}^t\ e_n = 1\}$.
Here, the vector $\underset{\sim}{y}$ denotes a mix strategy of player B such that $\underset{\sim}{y}^t\ e_n = 1$.
If player A plays the mix strategy $\underset{\sim}{x}$ and B plays the mix strategy $\underset{\sim}{y}$, then the expected payoff of player A is given by $x^t M y = \sum_{i=1}^m \sum_{j=1}^n a_{ij} x_i y_j$.
The player A's mix strategy $\underset{\sim}{x}^{\#}$ is said to be his optimal strategy if $\underset{\sim}{x}^t\ M\ \underset{\sim}{y}^{\#} \leq \underset{\sim}{x}^{\#}\ M\ \underset{\sim}{y}^{\#}$ for all $\underset{\sim}{x} \in S^m$, where $\underset{\sim}{y}^{\#}$ is B optimal strategy.
Similarly, the player B's strategy $\underset{\sim}{y}^{\#}$ is said to be optimal strategy if

$$\underset{\sim}{x}^{\#^t}\ M\ \underset{\sim}{y} \geq \underset{\sim}{x}^{\#^t}\ M\ \underset{\sim}{y}^{\#} \qquad \text{for all } \underset{\sim}{y} \in S^n.$$

Thus player A's objective is to determine the optimum values of x_i's in such a manner that it can maxi-min its expected payoff for any values of the elements of the vector y that player B chooses.
Hence from player A's point of view, the game can be expressed by the

following problem:

$$Max\ Z(\underset{\sim}{x}) = \overset{n}{\underset{j=1}{Min}} \sum_{i=1}^{m} a_{ij}x_i \tag{1}$$

subject to $\underset{\sim}{x}^t\ e_m = 1$

$$\underset{\sim}{x}^t \geq \underset{\sim}{0}.$$

From B's side, we have

$$Min\ U(\underset{\sim}{y}) = \overset{m}{\underset{i=1}{Max}} \sum_{j=1}^{n} a_{ij}y_j \tag{2}$$

subject to $\underset{\sim}{y}^t e_n = 1$.

However, as discussed in the introduction section in real-life situation, the payoffs are generally imprecise, and here we have represented them by interval-valued numbers, that is, $a_{ij} = [a_{ij_L}, a_{ij_R}]$.
Thus

$$\sum_{i=1}^{m} a_{ij}x_i = \sum_{i=1}^{m}[a_{ij_L}, a_{ij_R}]x_i = \left[\sum_{i=1}^{m} a_{ij_L}x_i, \sum_{i=1}^{m} a_{ij_R}x_i\right].$$

Hence (1) becomes

$$Max\ Z(\underset{\sim}{x}) = \overset{n}{\underset{j=1}{Min}} \left[\sum_{i=1}^{m} a_{ij_L}x_i, \sum_{i=1}^{m} a_{ij_R}x_i\right],$$

subject to $\underset{\sim}{x}^t e_m = 1$

$$\underset{\sim}{x}^t \geq \underset{\sim}{0}.$$

From B's point of view, the problem is

$$Min\ U(\underset{\sim}{y}) = \overset{m}{\underset{i=1}{Max}} \left[\sum_{j=1}^{m} a_{ij_L}y_j, \sum_{j=1}^{m} a_{ij_R}x_j\right]$$

subject to $\underset{\sim}{y}^t e_n = 1$

$$\underset{\sim}{y}^t \geq \underset{\sim}{0}.$$

4 Solution procedure

We have developed a real-coded GA for solving the interval-valued game. The main algorithm and the broad working principle of GA are widely pop-

ular and are available in several books and journal papers. Hence, we do not present it here anymore. However, the finer details of its basic components like "representation of chromosome", "initialization of population", "evaluation function", "Selection process", and "Genetic operators (crossover and mutation)" are explained below in brief. Furthermore, here we explain the GA developed for solving the problem from player A's side. For solving the problem from player B's side, an almost similar approach is adopted.
In the problem formulated by us, from player A's side, there are "m" (≥ 2) continuous decision variables (each representing the probabilities with which each of the "m" strategies is played by player A). Hence the ith chromosome is represented by a real row matrix $X_i(A) = [X_{i1}, X_{i2}, \ldots, X_{im},]$, where $X_{i1}, X_{i2}, \ldots, X_{im}$ (such that $\sum_{j=1}^{m} X_{ij} = 1$ & $0 \leq X_{ij} \leq 1$ for all $j = 1, 2, \ldots m,$) represent the decision variables, $x_1, x_2, \ldots, x_m$, respectively, of the problem.
After representing the chromosomes, the population size (popsize) numbers of chromosomes have been initialized.
In this problem, for each chromosome, the first component (gene), that is, X_{i1}, is initialized by randomly generating a real number between 0 and 1. Then, X_{i2} is initialized by randomly generating a real number between 0 and $(1 - X_{i1})$. Similarly, X_{i3} is initialized by randomly generating a real number between 0 and $1 - (X_{i1} + X_{i2})$, and so on, until the value of the last component is taken as $X_{im} = \{1 - \sum_{j=1}^{m-1} X_{ij}\}$. For each component of each chromosome, random numbers have been selected by using uniform distribution.

In the evaluation function, the fitness value for each chromosome (i.e., a potential solution) is calculated. In our work, the fitness value for each chromosome is considered to be the value of the objective function corresponding to it. Using the selection operation, the below-average solutions are eliminated from the population for the next generation. In this work, the tournament selection scheme of size two with replacement has been implemented. In this selection scheme, at first, two chromosomes are randomly selected. Then out of these two chromosomes, the better chromosome (i.e., one having the better fitness value) is finally selected for the next generation. The selection of a better chromosome is made on the basis of definitions (*vii*) and (*viii*) (discussed under section 2) of order relations between two interval numbers as the objective function of optimization problem be interval valued. Once the selection process is complete, the surviving chromosomes become eligible to take part in crossover operation, in which at the time two parent chromosomes get involved to generate offspring. These offspring possess the features of both the parent chromosomes. Here we have denoted the probability of crossover by pobcross. In this work, the crossover operation (as shown in Figures 1 and 2) is done in the following manner:
At first, *pobcross* popsizeis* is found and its integral value is stored in variable V.

(i) Then two chromosomes $X_i(A)$ and $X_q(A)$ are randomly selected from the population for crossover. For creating the rth ($r = 1, 2, \ldots, m$) component $X^{/}_{ir}$ and $X^{/}_{qr}$ of the two offspring, the following procedure is adopted.

(ii) A real number "h_r" is randomly generated between 0 and $|X_{ir} - X_{qr}|$.

(iii) If $X_{ir} > X_{qr}$, then $X^{/}_{ir} = X_{ir} - h_r$ and $X^{/}_{qr} = X_{qr} + h_r$.

Otherwise, if $X_{ir} < X_{qr}$, then $X^{/}_{ir} = X_{ir} + h_r$ and $X^{/}_{qr} = X_{qr} - h_r$.
The above steps [(i) –(iii)] are repeated for $V/2$ times. Let the parent chromosomes be $X_i(A)$ and $X_q(A)$, as shown below in Figure 1: Now if it is assumed

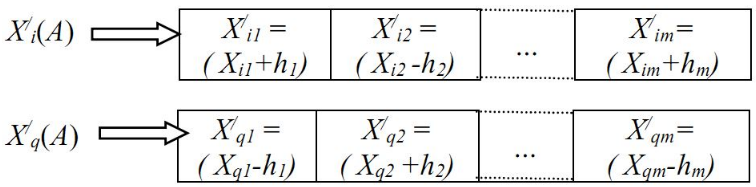

Figure 1: The parent chromosomes

that $X_{i1} < X_{q1}$, $X_{i2} > X_{q2}$, and $X_{im} < X_{qm}$, then the child chromosomes $X^{/}_1(A)$ and $X^{/}_q(A)$ will be given by (as shown below in Figure 2): By apply-

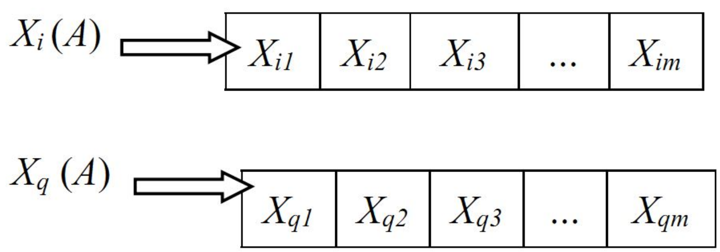

Figure 2: The child chromosomes

ing the mutation operation to a single chromosome, random variations have been injected into the population. The objective of mutation is to push the population slightly towards a better path. Here uniform mutation has been used and the probability of mutation has been denoted by pobmute.
If the elements (genes) X_{iw} and X_{it} of chromosome X_i are selected for mutation, then $|X_{iw} - X_{it}| = b(say)$ is first calculated. Then the modified value of X_{iw} and X_{it} are given by

$$X'_{iw} = \begin{pmatrix} X_{iw} + \Delta(b), & \text{if } X_{it} > X_{iw}, \\ X_{iw} - \Delta(b), & \text{if } X_{it} < X_{iw}, \end{pmatrix} \text{and } X'_{it} = \begin{pmatrix} X_{it} + \Delta(b), & \text{if } X_{iw} > X_{it}, \\ X_{it} - \Delta(b), & \text{if } X_{iw} < X_{it}, \end{pmatrix}$$

where $w \in \{1,2,\dots,m\}$, $t \in \{1,2,\dots,m\}$, and $\Delta(b) = a$ is a random real number between $[0,b]$.

5 Numerical example

We have considered three examples and solved them with the proposed GA. Except for Example 3, in the other two examples, the payoffs are considered as interval-valued. However, in Example 3, they are considered as fixed (by taking identical values for the left boundaries and right boundaries of the intervals).The values of the parameters considered in these examples are all feasible, although they have not been selected from any case study. In each of the examples, 20 independent runs have been performed by the proposed GA, of which the best value of the game has been taken and the corresponding mix strategies of both the players are determined and displayed in Tables 1–6. The following values of GA parameters are used in this work: $popsize = 100$, $pobcross = 0.95$, $pobmute = 0.15$, $maxgen = 1000$.

Example 1.

$$M_1 = \begin{bmatrix} [2,7,3,3] & [0,8] & [-3,-1] \\ [-5,-1] & [0,0] & [0,2] \\ [-1.5,-0.5] & [-6,-2] & [0.25,3.75] \end{bmatrix}$$

Mixed Strategy of player A i.e.$\underset{\sim}{x}$	Value of the game in interval form [Obj L, Obj R]	Expected value of the game, i.e., center value of [Obj L, Obj R]
$\underset{\sim}{x} = \begin{bmatrix} 0.403846 \\ 0.230769 \\ 0.365385 \end{bmatrix}$	$[-1.120192, 1.427885]$ $[-2.192308, 2.500000]$ $[-0.611539, 0.919231]$	0.153846

Table 1: Solution for player A

Mixed strategy of player B, i.e., $\underset{\sim}{y}$	Value of the game in interval form [Obj L, Obj R]	Expected value of the game, i.e., center value of [Obj L, Obj R]
$\underset{\sim}{y} = \begin{bmatrix} 0.153846 \\ 0.230769 \\ 0.615385 \end{bmatrix}$	$[-1.1738462, 1.430769]$ $[-1.769231, 1.461538]$ $[-0.076923, 0.769231]$	-0.153846

Table 2: Solution for player B

Example 2.

$$M_2 = \begin{bmatrix} [1,5] & [-2,6] & [7,9] \\ [3,7] & [7,7] & [3,5] \\ [4,8] & [0,1] & [3,3] \end{bmatrix}$$

Mixed strategy of player A, i.e., $\underset{\sim}{x}$	Value of the game in interval form [Obj L, Obj R]	Expected value of the game, i.e., center value of [Obj L, Obj R]
$\underset{\sim}{x} = \begin{bmatrix} 0.225490 \\ 0.598039 \\ 0.176471 \end{bmatrix}$	$[2.725490, 6.725490]$ $[3.901961, 5.549020]$	4.725490

Table 3: Solution for player A

Mixed strategy of player B, i.e., $\underset{\sim}{y}$	Value of the game in interval form [Obj L, Obj R]	Expected value of the game, i.e., center value of [Obj L, Obj R]
$\underset{\sim}{y} = \begin{bmatrix} 0.607843 \\ 0.039216 \\ 0.352941 \end{bmatrix}$	$[-6.450980, -3.000000]$ $[-6.294117, -3.156863]$	-4.725490

Table 4: Solution for player B

Example 3.

$$M_3 = \begin{bmatrix} [5,5] & [7,7] & [4,4] & [10,10] \\ [4,4] & [3,3] & [7,7] & [2,2] \\ [7,7] & [2,2] & [5,5] & [6,6] \end{bmatrix}$$

Mixed strategy of player A, i.e., $\underset{\sim}{x}$	Value of the game in interval from [Obj L, Obj R]	Expected value of the game, i.e., center value of [Obj L, Obj R]
$\underset{\sim}{x} = \begin{bmatrix} 0.548387 \\ 0.290323 \\ 0.161290 \end{bmatrix}$	$[5.032258, 5.032258]$	5.032258

Table 5: Solution for player A

Mixed strategy of player B, i.e., $\underset{\sim}{y}$	Value of the game in interval from [Obj L, Obj R]	Expected value of the game, i.e., center value of [Obj L, Obj R]
$\underset{\sim}{y} = \begin{bmatrix} 0.354839 \\ 0.225806 \\ 0.413955 \\ 0.000000 \end{bmatrix}$	$[-5.032258, -5.032258]$	-5.032258

Table 6: Solution for player B

6 Sensitivity analysis

The outcome of this work is heavily dependent on the stability, convergence, and efficiency of the algorithm proposed by us. Hence, to study this stability, by considering Example 1, the sensitivity of the "expected value of the game" (from player A's point of view) have been analyzed graphically with respect to GA parameters like pobcross, pobmute, maxgen and pobsize separately, keeping the other parameters at their original values. Figures 3–6 reveal that the result of Example 1, obtained by the GA proposed by us, is stable over a large range of the GA parameters mentioned above.

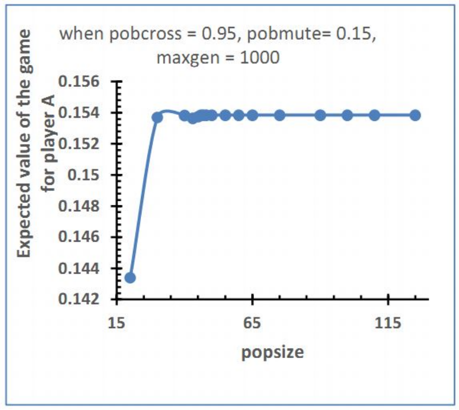

Figure 3: Expected value of the game for player A vs popsize

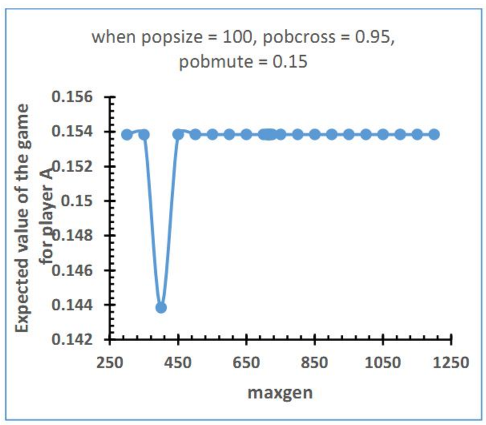

Figure 4: Expected value of the game for player A vs maxgen

Figure 3 shows that when *pobcrossis* fixed at 0.95, *pobmuteat* 0.15 and *maxgenat* 1000, then the expected value of the game is fairly stable when the *popsizeis* equal to or above 43. Similarly, it is evident from Figure 4 that when *pobcross, pobmute* and *pobsize* are kept at their original values, then once the value of *maxgen* goes above 452 mark. Moreover, the expected value of the game becomes perfectly horizontal and thus confirms that it is perfectly stable. Similarly, from Figures 5 and 6, it is clear that the expected value of the game is stable when the value of *pobcrossis* greater or equal to 0.72 and the value of *pobmuteis* greater or equal to 0.1.

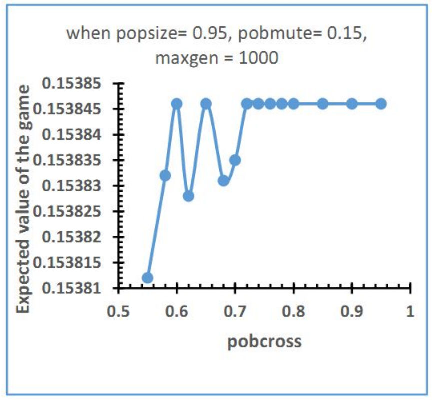

Figure 5: Expected value of the game for player A vs pobcross

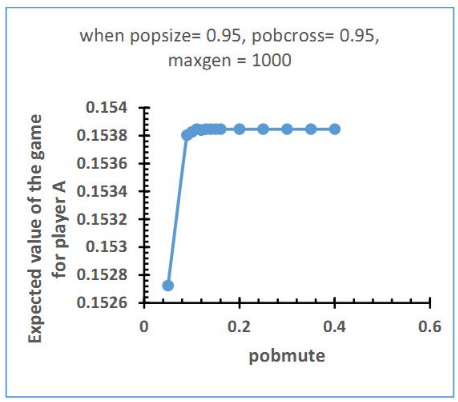

Figure 6: Expected value of the game for player A vs pobmute

7 Conclusion

In this paper, a heuristic technique has been developed to solve two persons $m \times n$ zero sum games $(m > 2, n > 2)$, with interval-valued payoffs and interval-valued objectives. It is a well-known fact that during the design phase of the payoff matrix of a game rarely the available information is free from impreciseness. Because of this impreciseness, the payoffs of the payoff matrix have a certain degree of ambiguity present in them. To address this impreciseness-driven ambiguity in payoffs and goals, since the end of the last century, several types of research have been done in two persons zero sum games with the help of fuzzy approach. However, as discussed in the "Introduction" section of this paper, the interval approach has some distinct advantages over the fuzzy approach. While dealing with impreciseness using the interval approach, one can be absolutely sure that the interval result will always contain the exact result irrespective of whether its upper boundary and lower boundary are overestimated. Hence, in this work, we have addressed the said impreciseness with the help of interval-valued numbers and thus formulated the game with interval-valued payoffs and objectives. To solve the game, a real coded GA with interval fitness function, tournament selection, uniform crossover, and uniform mutation has been developed. Finally, a few numerical examples have been solved with the help of the developed GA and to study the stability of the GA, sensitivity analyses with respect to different GA parameters have been done and shown graphically.

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