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An interactive algorithm for solving multiobjective optimization problems based on a general scalarization technique

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Abstract

The wide variety of available interactive methods brings the need for creating general interactive algorithms enabling the decision maker (DM) to apply freely several convenient methods which best fit his/her preferences. To this end, in this paper, we propose a general scalarizing problem for multiobjective programming problems. The relation between optimal solutions of the introduced scalarizing problem and (weakly) efficient as well as properly efficient solutions of the main multiobjective optimization problem (MOP) is discussed. It is shown that some of the scalarizing problems used in different interactive methods can be obtained from proposed formulation by selecting suitable transformations. Based on the suggested scalarizing problem, we propose a general interactive algorithm (GIA) that enables the DM to specify his/her preferences in six different ways with capability to change his/her preferences any time during the iterations of the algorithm. Finally, a numerical example demonstrating the applicability of the algorithm is provided.

Keywords: Multiobjective optimization; Interactive method; Scalarizing problem; Proper efficiency; Preference information.

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1 Introduction

The general goal of solving a multiobjective optimization problem (MOP) is to support the decision maker (DM) seeking the most preferred solution of many Pareto optimal solutions as the final one. Inasmuch as finding a most preferred solution needs some extra information from the DM, interactive approaches, based on the participation of the DM, have become popular.

In interactive methods, an iterative algorithm is proposed. Then, the steps of the algorithm are repeated where at each iteration, some information is given to the DM and he/she specifies his/her preferences. The process is repeated until the DM is satisfied with regard to the obtained solution. The benefits of using interactive approaches are that, the DM (*i*) does not need to have any global preference structure, (*ii*) has the possibility of learning about the interrelationship between the objectives, (*iii*) can learn about the feasibility of solutions during the solving process.

Heretofore, many interactive methods have been suggested in the literature [1, 13, 19, 23, 26, 30, 31]. As pointed out already, interactive methods are very useful and realistic to solve an MOP. However, since there have been many interactive methods available, it is not easy to choose an appropriate method conveniently. Therefore, creating global algorithms with an ability to accommodate different methods will be useful. By creating a global algorithm, it is possible for the DM to select freely an appropriate method (and the way of specifying preference information) as well as to switch between methods. To this end, it is necessary to design a general scalarizing problem yielding scalarizing problems used in different interactive methods.

Until now, some global algorithms have been proposed. For example, Gardiner and Steuer [7, 8] proposed a unified algorithm including nine to thirteen different methods. Romero [27] presented another general optimization structure, called extended lexicographic goal programming. Moreover, Vassileva [32] suggested a general scalarizing problem which incorporates different scalarizing problems. More recently, based on a global formulation (GLIDE), Luque et al. [21] proposed a global procedure which accommodates eight interactive methods of different types. Nevertheless, their formulation is unlikely to consider the computational efficiency, therefore Ruiz et al. [28] improved the computational efficiency of GLIDE by reformulating it.

In some of the mentioned publications, the authors have provided theorems concerning (weak) efficiency of the optimal solutions of their proposed general scalarizing problems [21, 28, 32] and as far as we know few results related to proper efficiency have been provided. Now, in this paper we suggest a general scalarizing problem which not only considers computational efficiency by reducing the number of added constraints, but also provides theorems concerning (weak) efficiency as well as proper efficiency of its optimal solutions. The provided results are established without any convexity assumption. Also, by setting suitable values for parameters and index sets of the proposed general scalarizing problem, we obtain many known scalarizing problems. Based on the mentioned problem, we propose a general interactive algorithm (GIA) to solve a given MOP, subsequently. In this algorithm, the DM has the ability to specify his/her preference information in six different ways.

The rest of this paper is organized as follows: Section 2 contains some preliminaries and basic definitions. In Section 3, we propose our general formulation and obtain some theorems. Section 4 gives some scalarizing problems used in different interactive methods which can be obtained from our general formulation. Section 5 contains our proposed interactive algorithm. In Section 6, some computational and theoretical advantages are mentioned. An example is presented in Section 7 and finally, in Section 8 conclusions are given.

2 Preliminaries and basic definitions

A general multiobjective optimization problem can be written as:

$$(MOP) \qquad \min f(\mathbf{x}) \quad s.t. \ \ \mathbf{x} \in \mathcal{X}, \tag{1}$$

where $\mathcal{X} \subseteq \mathbb{R}^n$ is a nonempty compact set, and $f(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_p(\mathbf{x}))^T : \mathcal{X} \to \mathbb{R}^p$ is a vector-valued function.

The set of all attainable outcomes or objective vectors is defined as the image of the feasible solutions $\mathbf{x} \in \mathcal{X}$ under f. In fact $\mathcal{Y} := f(\mathcal{X}) \subset \mathbb{R}^p$. For $\mathbf{y^1}$ and $\mathbf{y^2} \in \mathbb{R}^p$, $\mathbf{y^1} \leqq \mathbf{y^2}$ means that $y_i^1 \leq y_i^2$, for each $i = 1, \cdots, p$, also $\mathbf{y^1} \leq \mathbf{y^2}$ stands for $\mathbf{y^1} \leqq \mathbf{y^2}$ and $\mathbf{y^1} \neq \mathbf{y^2}$. Furthermore, $\mathbf{y^1} < \mathbf{y^2}$ means that $y_i^1 < y_i^2$, for each $i = 1, ..., p$. The Pareto cone is defined as $\mathbb{R}^p_{\geqq} = \{\mathbf{y} \in \mathbb{R}^p : \mathbf{y} \geqq 0\}$. $\mathbb{R}^p_{\geq}$ and $\mathbb{R}^p_{>}$ are defined, similarly. In this paper, we shall assume that $\mathcal{Y} := f(\mathcal{X})$ is bounded.

Definition 1. *A feasible solution $\hat{x} \in \mathcal{X}$ is called:*

(i) weakly efficient (weakly Pareto optimal) solution to MOP (1) if there is no other $\mathbf{x} \in \mathcal{X}$ such that $f(\mathbf{x}) < f(\hat{x})$,

(ii) efficient (Pareto optimal) solution to MOP (1) if there is no other $\mathbf{x} \in \mathcal{X}$ such that $f(\mathbf{x}) \leq f(\hat{x})$,

(iii) properly efficient (properly Pareto optimal) solution to MOP (1) if it is efficient and there exists a real positive number M such that for each $i \in \{1, 2, \ldots, p\}$ and each $\mathbf{x} \in \mathcal{X}$ satisfying $f_i(\mathbf{x}) < f_i(\hat{\mathbf{x}})$, there exists an index $j \in \{1, 2, \ldots, p\}$ with $f_j(\hat{\mathbf{x}}) < f_j(\mathbf{x})$ and

$$\frac{f_i(\hat{\mathbf{x}}) - f_i(\mathbf{x})}{f_j(\mathbf{x}) - f_j(\hat{\mathbf{x}})} \leq M.$$

The set of all weakly efficient, efficient, and properly efficient solutions of MOP (1) will be denoted by $\mathcal{X}_{WE}$, $\mathcal{X}_E$ and $\mathcal{X}_{PE}$ respectively. The image $f(\mathbf{x}) \in \mathcal{Y}$ of an (weakly, properly) efficient solution $\mathbf{x} \in \mathcal{X}$, is called (weakly, properly) nondominated point.

Remark 1. Obviously, $\mathcal{X}_{PE} \subseteq \mathcal{X}_E \subseteq \mathcal{X}_{WE}$.

Remark 2. In this paper, we use definition of proper efficiency in the sense of Geoffrion [9]. There are other definitions of proper efficiency which are almost the same when using the Pareto cone as the order cone. For considering relationships between different definitions of proper efficiency one can refer to [4].

Definition 2. *The ideal point* $\mathbf{y^I} = (y_1^I, \ldots, y_p^I)$ *of MOP (1) is defined by* $y_i^I := min_{\mathbf{x}\in\mathcal{X}} f_i(\mathbf{x}), i = 1, \cdots, p.$

Definition 3. *The point* $\mathbf{y^U} := \mathbf{y^I} - \alpha$*, where* $\alpha \in \mathbb{R}^p_>$ *is a vector with small positive components, is called the utopia point of MOP (1).*

Definition 4. *The nadir point* $\mathbf{y^N} = (y_1^N, \ldots, y_p^N)$ *of MOP (1) is defined by* $y_i^N := max_{\mathbf{x}\in\mathcal{X}_E} f_i(\mathbf{x}), i = 1, \cdots, p.$

Definition 5. *The vector* $\bar{\mathbf{y}} = (\bar{y}_1, \ldots, \bar{y}_p) \in \mathbb{R}^p$*, consisting of the desired or aspiration values to the DM, is called a reference point. It should be noted that reference point may be achievable or not.*

One of the most popular approaches to solve a given MOP is scalarization, which involves formulating a single objective problem associated with the given MOP. Let us consider a single objective programming problem as follows:

$$\begin{aligned} &\min g(\mathbf{x}) \\ &s.t. \;\; \mathbf{x} \in \mathcal{S}, \end{aligned} \tag{2}$$

where $g : \mathcal{S} \to \mathbb{R}$.

Definition 6. *A feasible solution* $\hat{\mathbf{x}} \in \mathcal{S}$ *is said to be*
(i) an optimal solution of problem (2) if $g(\hat{\mathbf{x}}) \leq g(\mathbf{x})$ *for all* $\mathbf{x} \in \mathcal{S}$*,*
(ii) a strictly optimal solution of problem (2) if $g(\hat{\mathbf{x}}) < g(\mathbf{x})$ *for all* $\mathbf{x} \in \mathcal{S}\backslash\{\hat{\mathbf{x}}\}$.

3 A general scalarizing problem

In this section, we propose a general scalarizing problem associated with MOP (1), which is defined such that many scalarizing problems, used in different interactive methods, can be deduced from it by selecting suitable values of parameters and index sets. The general scalarizing problem is proposed as follows:

$$\begin{aligned} \min \ \max_{i \in I_1^k} \quad & \lambda_i^k \Big(f_i(\mathbf{x}) - r_i^k + \rho \sum_{t=1}^{p} w_t^k (f_t(\mathbf{x}) - r_t^k) \Big) \\ & s.t. \begin{cases} f_i(\mathbf{x}) \leq \delta_i^k & \forall i \in I_2^k, \\ \mathbf{x} \in \mathcal{X}, \end{cases} \end{aligned} \tag{3}$$

where $\lambda_i^k \geq 0$, $\rho \geq 0$, δ_i^k, r_i^k, and $w_t^k \geq 0$ are parameters specified depending on the information given by DM. Also, $I_1^k \neq \emptyset$ and I_2^k are index sets, which are subsets of $\{1, \cdots, p\}$. Notice that hereafter, we make the assumption that on the proposed scalarizing problem, the parameters $\delta_i^k, i \in I_2^k$ are selected such that problem (3) remains feasible. Let k be the current iteration. Then, the optimal solution obtained from scalarizing problem (3) is defined by $\hat{\mathbf{x}}^{k+1}$ and the corresponding objective vector by $f(\hat{\mathbf{x}}^{k+1})$.

According to [14, p. 305], or [23, p. 97], if we replace the max term by a new variable $z \in \mathbb{R}$, then problem (3) is equivalent to the following scalarizing optimization problem:

$$\begin{aligned} & \min \quad z \\ & s.t. \begin{cases} \lambda_i^k \Big(f_i(\mathbf{x}) - r_i^k + \rho \sum_{t=1}^{p} w_t^k (f_t(\mathbf{x}) - r_t^k) \Big) \leq z & \forall i \in I_1^k, \\ f_i(\mathbf{x}) \leq \delta_i^k & \forall i \in I_2^k, \\ \mathbf{x} \in \mathcal{X}. \end{cases} \end{aligned} \tag{4}$$

Notice that the scalarizing problem (3) is nondifferentiable, even if the main MOP (1) is differentiable (i.e., the objective functions and constraint functions are differentiable). Therefore, if the original MOP is the differentiable we propose to use formulation (4) since it preserves differentiability. In this case, the scalar optimization problem (4) can be solved with standard methods of (non)linear constraint optimization or using available single objective solvers. However, if the original MOP (1) is nondifferentiable, both scalarized problems (3) and (4) are nondifferentiable, too. In this case, the scalarized problem (3) is recommended since it has a reduced number of constraints.

It should be noted that, unlike the formulations proposed in [21, 28], the bounds on trade-offs generated by the suggested formulation are independent of parameters λ_i. For more details about bounds on trade-offs see [17, 18]. So far, many authors have provided theorems concerning weak efficiency and efficiency of the optimal solutions of the scalarized problems used in the interactive methods. Now, we prove some general theorems concerning weak efficiency, efficiency, as well as proper efficiency of (strictly) optimal solutions of problems (3) and (4). It is important to point out that the following theorems are general and many theorems concerning (weak, proper) efficiency [4, 23] can be resulted from them. Moreover, the theorems are provided with no convexity assumption. Since problems (3) and (4) are equivalent, we only provide theorems for the first one.

Theorem 3. *Let $\lambda_i^k > 0\ \forall i \in I_1^k$. If $\hat{\mathbf{x}}^{k+1} \in \mathcal{X}$ is an optimal solution of problem (3), then $\hat{\mathbf{x}}^{k+1}$ is a weakly efficient solution of MOP (1).*

Proof. Let $\hat{\mathbf{x}}^{k+1} \in \mathcal{X}$ be an optimal solution of problem (3) and suppose that $\hat{\mathbf{x}}^{k+1} \notin \mathcal{X}_{WE}$. Then, there exists $\mathbf{x} \in \mathcal{X}$ such that $f(\mathbf{x}) < f(\hat{\mathbf{x}}^{k+1})$. Therefore, $f_i(\mathbf{x}) < f_i(\hat{\mathbf{x}}^{k+1}) \leq \delta_i^k \ \ \forall i \in I_2^k$, which means $\mathbf{x} \in \mathcal{X}$ is a feasible solution for problem (3). Also, we have

$$f_i(\mathbf{x}) - r_i^k < f_i(\hat{\mathbf{x}}^{k+1}) - r_i^k \quad \forall i \in I_1^k,$$

and

$$\rho \sum_{t=1}^{p} w_t^k (f_t(\mathbf{x}) - r_t^k) \leq \rho \sum_{t=1}^{p} w_t^k (f_t(\hat{\mathbf{x}}^{k+1}) - r_t^k).$$

Therefore,

$$\max_{i \in I_1^k} \lambda_i^k \Big(f_i(\hat{\mathbf{x}}^{k+1}) - r_i^k + \rho \sum_{t=1}^{p} w_t^k (f_t(\hat{\mathbf{x}}^{k+1}) - r_t^k) \Big) >$$

$$\max_{i \in I_1^k} \lambda_i^k \Big(f_i(\mathbf{x}) - r_i^k + \rho \sum_{t=1}^{p} w_t^k (f_t(\mathbf{x}) - r_t^k) \Big),$$

which is a contradiction with optimality of $\hat{x}^{k+1}$. Thus, $\hat{\mathbf{x}}^{k+1} \in \mathcal{X}_{WE}$. □

In the following theorem, utilizing the general formulation (3), a sufficient condition for efficiency is provided.

Theorem 4. *If $\hat{\mathbf{x}}^{k+1} \in \mathcal{X}$ is a strictly optimal solution of problem (3), then $\hat{\mathbf{x}}^{k+1} \in \mathcal{X}_E$.*

Proof. The proof is similar to that of Theorem 3. □

It is found out from part *(ii)* of Definition 1, that in an efficient solution it is not possible to improve any criterion without deterioration of at least one other criterion. Sometimes, these trade-offs may be unbounded and it is obvious that efficient solutions with bounded trade-offs (called properly efficient) are desirable. Until now, many scholars have considered relationships between optimal solutions of the scalarizing problem used in their proposed interactive methods and (weakly) efficient solutions of the related MOP [21, 28, 32], but there are fewer results concerning proper efficiency. In the following theorem, we provide a sufficient condition concerning properly efficient solutions of MOP (1).

Theorem 5. *If $\hat{\mathbf{x}}^{k+1} \in \mathcal{X}$ is an optimal solution for problem (3) with $\lambda_i^k > 0\ \forall i \in I_1^k$, $\rho > 0$ and $w^k \in \mathbb{R}^p_>$, then $\hat{\mathbf{x}}^{k+1} \in \mathcal{X}_{PE}$.*

Proof. We show that $\hat{x}^{k+1} \in \mathcal{X}_E$. Let $\hat{\mathbf{x}}^{k+1} \notin \mathcal{X}_E$. Then, there exists $\mathbf{x} \in \mathcal{X}$ with $f_i(\mathbf{x}) \leq f_i(\hat{\mathbf{x}}^{k+1})$, $\forall i \in \{1, \cdots, p\}$ and $f_j(\mathbf{x}) < f_j(\hat{\mathbf{x}}^{k+1})$ for some $j \in \{1, \cdots, p\}$. Hence, $f_i(\mathbf{x}) \leq f_i(\hat{\mathbf{x}}^{k+1}) \leq \delta_i^k\ \forall i \in I_2^k$. Thus, $\mathbf{x} \in \mathcal{X}$ is a feasible solution of (3). Using the assumptions and the definition of efficiency, it follows that:

$$\max_{i \in I_1^k} \lambda_i^k \Big(f_i(\hat{\mathbf{x}}^{k+1}) - r_i^k + \rho \sum_{t=1}^{p} w_t^k (f_t(\hat{\mathbf{x}}^{k+1}) - r_t^k) \Big) >$$

$$\max_{i \in I_1^k} \lambda_i^k \Big(f_i(\mathbf{x}) - r_i^k + \rho \sum_{t=1}^{p} w_t^k (f_t(\mathbf{x}) - r_t^k) \Big).$$

This is a contradiction with optimality of $\hat{x}^{k+1}$ and therefore $\hat{x}^{k+1} \in \mathcal{X}_E$. Now, we show that $\hat{x}^{k+1}$ is a properly efficient solution to MOP (1). To this end, we define:

$$M = \max_{i \in \{1, \cdots, p\}} \{ \frac{1 + \rho \sum_{t=1}^{p} w_t^k}{\rho w_i^k} \},$$

and consider an index $i \in \{1, \cdots, p\}$ and $\overline{\mathbf{x}} \in \mathcal{X}$ such that $f_i(\overline{\mathbf{x}}) < f_i(\hat{x}^{k+1})$. To prove the proper efficiency of $\hat{x}^{k+1}$, we must show that there exists an index $j \in \{1, 2, \ldots, p\}$ with $f_j(\hat{\mathbf{x}}^{k+1}) < f_j(\overline{\mathbf{x}})$ such that

$$\frac{f_i(\hat{\mathbf{x}}^{k+1}) - f_i(\overline{\mathbf{x}})}{f_j(\overline{\mathbf{x}}) - f_j(\hat{\mathbf{x}}^{k+1})} \leq M.$$

From efficiency of $\hat{x}^{k+1}$, we conclude that there exists an index $t \in \{1, \cdots, p\}$ such that $f_t(\hat{\mathbf{x}}^{k+1}) < f_t(\overline{\mathbf{x}})$. We define

$$f_j(\hat{\mathbf{x}}^{k+1}) - f_j(\overline{\mathbf{x}}) = \min_{m \in \{1, \cdots, p\}} (f_m(\hat{\mathbf{x}}^{k+1}) - f_m(\overline{\mathbf{x}})). \tag{5}$$

It is obvious that $f_j(\hat{\mathbf{x}}^{k+1}) - f_j(\overline{\mathbf{x}}) < 0$.
Moreover, optimality of $\hat{x}^{k+1}$ for problem (3), concludes

$$\max_{m \in I_1^k} \lambda_m \Big(f_m(\overline{\mathbf{x}}) - r_m^k + \rho \sum_{t=1}^{p} w_t^k (f_t(\overline{\mathbf{x}}) - r_t^k) \Big) \geq$$

$$\max_{m \in I_1^k} \lambda_m \Big(f_m(\hat{\mathbf{x}}^{k+1}) - r_m^k + \rho \sum_{t=1}^{p} w_t^k (f_t(\hat{\mathbf{x}}^{k+1}) - r_t^k) \Big).$$

Now, let

$$\lambda_l \Big(f_l(\overline{\mathbf{x}}) - r_l^k + \rho \sum_{t=1}^{p} w_t^k (f_t(\overline{\mathbf{x}}) - r_t^k) \Big) =$$

$$\max_{m\in I_1^k} \lambda_m\Big(f_m(\overline{\mathbf{x}}) - r_m^k + \rho\sum_{t=1}^{p} w_t^k(f_t(\overline{\mathbf{x}}) - r_t^k)\Big).$$

Hence,

$$\lambda_l\Big(f_l(\overline{\mathbf{x}}) - r_l^k + \rho\sum_{t=1}^{p} w_t^k(f_t(\overline{\mathbf{x}}) - r_t^k)\Big) \geq$$

$$\max_{m\in I_1^k} \lambda_m\Big(f_m(\hat{\mathbf{x}}^{k+1}) - r_m^k + \rho\sum_{t=1}^{p} w_t^k(f_t(\hat{\mathbf{x}}^{k+1}) - r_t^k)\Big) \geq$$

$$\lambda_l\Big(f_l(\hat{\mathbf{x}}^{k+1}) - r_l^k + \rho\sum_{t=1}^{p} w_t^k(f_t(\hat{\mathbf{x}}^{k+1}) - r_t^k)\Big).$$

Then,

$$0 \geq (f_l(\hat{\mathbf{x}}^{k+1}) - f_l(\overline{\mathbf{x}})) + \rho\sum_{t=1}^{p} w_t^k(f_t(\hat{\mathbf{x}}^{k+1}) - f_t(\overline{\mathbf{x}})). \tag{6}$$

Now, from (5) and (6), we have:

$$0 \geq (f_j(\hat{\mathbf{x}}^{k+1}) - f_j(\overline{\mathbf{x}})) + \rho\sum_{t=1}^{p} w_t^k(f_t(\hat{\mathbf{x}}^{k+1}) - f_t(\overline{\mathbf{x}})).$$

That is,

$$\rho w_i^k(f_i(\hat{\mathbf{x}}^{k+1}) - f_i(\overline{\mathbf{x}})) \leq f_j(\overline{\mathbf{x}}) - f_j(\hat{\mathbf{x}}^{k+1}) + \rho\sum_{\substack{t=1\\ t\neq i}}^{p} w_t^k(f_t(\overline{\mathbf{x}}) - f_t(\hat{\mathbf{x}}^{k+1})) \leq$$

$$(1 + \rho\sum_{\substack{t=1\\ t\neq i}}^{p} w_t^k)(f_j(\overline{\mathbf{x}}) - f_j(\hat{\mathbf{x}}^{k+1})).$$

Hence

$$\frac{f_i(\hat{\mathbf{x}}^{k+1}) - f_i(\overline{\mathbf{x}})}{f_j(\overline{\mathbf{x}}) - f_j(\hat{\mathbf{x}}^{k+1})} \leq \frac{1 + \rho\sum_{\substack{t=1\\ t\neq i}}^{p} w_t^k}{\rho w_i^k} \leq M,$$

which completes the proof. □

It should be noted that, using suitable values for parameters in (3), we can provide necessary conditions related to (weakly, properly) efficient solutions of MOP (1). For example, if we choose $I_1^k = \{1, \cdots, p\}$, $I_2^k = \emptyset$, $r_i^k = y_i^U$ and $w_i^k = 1\ \forall i \in \{1, \ldots, p\}$, then we have the modified weighted Tchebycheff method [15] and, by Theorem 4.2 in [16], for every properly efficient solution of MOP (1) we can find suitable parameters $\lambda_i^k > 0,\ \forall i \in \{1, \cdots, p\}$ and $\rho > 0$ such that this properly efficient solution be an optimal solution of (3).

4 Achieving different scalarizing problems from the general formulation

The general formulations (3) and (4) are generalizations of already known scalarizing problems. In this section, we are going to show that how many famous scalarizing problems (used in different interactive methods) can be attained from (3) and (4) by choosing appropriate values of parameters and index sets. We obtain the scalarizing problems from (3). By a similar method it is possible to obtain them from (4).

4.1 GUESS method and STOM

The GUESS method is one of the simplest interactive methods, proposed by Buchanan [2]. In this method, the DM has to determine the components of the reference point ($\bar{y}_i^k$) as preference information. At the kth iteration, the scalarizing problem used in this method is formulated as follows:

$$\begin{aligned} \min \max_{i=1,\dots,p} \quad & \frac{f_i(\mathbf{x}) - \bar{y}_i^k}{y_i^N - \bar{y}_i^k} \\ s.t. \quad & \mathbf{x} \in \mathcal{X}. \end{aligned} \tag{7}$$

Notice that the reference vector specified by the DM, must be strictly lower than the nadir objective vector, that is, $\bar{\mathbf{y}} < \mathbf{y^N}$. This scalarizing problem can be achieved from (3) by considering the following replacements:

(1) $I_1^k = \{1, \dots, p\}$ and $I_2^k = \emptyset$;

(2) $w_i^k = 0$, $\lambda_i^k = \frac{1}{y_i^N - \bar{y}_i^k}$, and $\rho = 0$;

(3) $r_i^k = \bar{y}_i^k$ and $i = 1, \dots, p$.

The satisficing trade-off method (STOM) [24] can be obtained from (3), similar to the GUESS method, by setting $\lambda_i^k = \frac{1}{\bar{y}_i^k - y_i^U}$ and $r_i^k = y_i^U$ $(i = 1, \dots, p)$. Other parameter values and index sets are the same as those of GUESS method. In this method, $\bar{\mathbf{y}}$ must be chosen such that $\bar{\mathbf{y}} > \mathbf{y}^U$.

4.2 Reference direction approach

In this method, a vector from the current iteration point to the reference point (a reference direction) is projected onto the efficient set [20]. To obtain the points along the reference direction at the kth iteration, the following scalarizing problem needs to be solved:

$$\begin{aligned} \min \max_{i=1,\ldots,p} \quad & \frac{f_i(\mathbf{x}) - (f_i^k + td_i^k)}{\mu_i} \\ & s.t. \quad \mathbf{x} \in \mathcal{X}, \end{aligned} \tag{8}$$

where, f^k is the current nondominated objective vector, $d^k = \bar{\mathbf{y}}^k - f^k$, t has different discrete nonnegative values, and μ is a weighting vector that can be either a reference point presented by the DM or defined as $\mathbf{y^N} - \mathbf{y^U}$. This problem can be obtained from (3) by considering the following replacements:

(1) $I_1^k = \{1, \ldots, p\}$ and $I_2^k = \emptyset$;

(2) $w_i^k = 0$, $\lambda_i^k = \frac{1}{\mu_i}$ and $\rho = 0$;

(3) $r_i^k = f_i^k + td_i^k$ and $i = 1, \ldots, p$.

4.3 Step method

The step method is one of the first known interactive methods [1]. Eschenauer et al. [6] extended this method to nonlinear problems. In this method, based on the current objective vector (f^k), the DM can improve some unacceptable objective functions f_i ($i \in J_1^k$) by relaxing some other objective function(s) f_i ($i \in J_2^k$) such that $J_1^k \cup J_2^k = \{1, \ldots, p\}$. In this regard, the DM must specify upper bounds $\varepsilon_i^k > f_i^k$ for functions f_i ($i \in J_2^k$). In this case, the scalarizing problem is formulated as follows:

$$\begin{aligned} & \min \max_{i=1,\ldots,p} \left(\frac{e_i}{\sum_{j=1}^p e_j} (f_i(\mathbf{x}) - y_i^I) \right) \\ & s.t. \quad \begin{cases} f_i(\mathbf{x}) \le f_i^k & \forall i \in J_1^k, \\ f_i(\mathbf{x}) \le \varepsilon_i^k & \forall i \in J_2^k, \\ \mathbf{x} \in \mathcal{X}, \end{cases} \end{aligned} \tag{9}$$

where $e_i = \frac{1}{y_i^I}(\frac{y_i^N - y_i^I}{y_i^N})$, $i = 1, \ldots, p$ (the denominators are not allowed to be zero). We can obtain (9) from (3) using the following replacements:

(1) $I_1^k = \{1, \ldots, p\}$ and $I_2^k = J_1^k \cup J_2^k$;

(2) $w_i^k = 0$, $\forall i \in \{1, \ldots, p\}$, $\lambda_i^k = \frac{e_i}{\sum_{j=1}^p e_j}$ $\forall i \in \{1, \ldots, p\}$ and $\rho = 0$;

(3) $r_i^k = y_i^I$, $\forall i \in \{1, \ldots, p\}$;

(4) $\delta_i^k = f_i^k$ for $i \in J_1^k$ and $\delta_i^k = \varepsilon_i^k$ for $i \in J_2^k$.

4.4 SPOT method

In the SPOT method, given the current objective vector f^k, the DM is asked to select a reference objective function f_l and then compare each objective function f_i $(i = 1, \ldots, p,\ i \neq l)$ with f_l by providing the marginal rates of substitutions (MRSs) m_{li}^k $(i = 1, \ldots, p,\ i \neq l)$ [29]. The MRSs can be approximated as $m_{li}^k \simeq \frac{\Delta f_l^k}{\Delta f_i^k}$, $i = 1, \ldots, p$, where Δf_i^k is the amount of improvement, provided by the DM, on the value of the objective function f_i that can exactly compensate for the given amount Δf_l^k to be deteriorated of the reference objective f_l. The intermediate single objective optimization problem, used in this method, can be formulated as follows:

$$\begin{aligned} &\min \quad f_l(\mathbf{x}) \\ &s.t. \begin{cases} f_i(\mathbf{x}) \leq f_i^k + \alpha(\mu_{li}^k - m_{li}^k) & \forall i \in \{1, \ldots, p\},\ i \neq l, \\ \mathbf{x} \in \mathcal{X}, \end{cases} \end{aligned} \tag{10}$$

where $\mu_{li}^k, i \neq l$ are K.K.T multipliers, corresponding to the current non-dominated objective vector [21] and several values for α are set and in this way different solutions are obtained. This problem is achieved from (3), by considering the following transformations:

(1) $I_1^k = \{l\}$ and $I_2^k = \{1, \ldots, p\} \backslash \{l\}$;

(2) $\lambda_l^k = 1, \rho = 0$ and $r_l^k = 0$;

(3) $\delta_i^k = f_i^k + \alpha(\mu_{li}^k - m_{li}^k), i = 1, \ldots, p$ and $i \neq l$.

4.5 Modified reference point method

This method is an interactive reference direction method for solving convex nonlinear integer problems [31]. Here, the DM is asked to set his/her preferences as aspiration levels of the objective functions at each iteration. Let J_1^k be the set of indices of the objective functions which the DM wants to improve and J_2^k denotes the set of indices which can worsen and J_3^k contains the indices that are satisfactory to the DM. The scalarizing problem used in this method is formulated as follows:

$$\begin{aligned} \min \max_{i \in J_1^k,\ j \in J_2^k} &\left\{ \frac{f_i(\mathbf{x}) - \bar{y}_i^k}{f_i^k - \bar{y}_i^k},\ \frac{f_j(\mathbf{x}) - f_j^k}{\bar{y}_j^k - f_j^k} \right\} \\ s.t. &\begin{cases} f_i(\mathbf{x}) \leq f_i^k & \forall i \in J_3^k, \\ \mathbf{x} \in \mathcal{X}, \end{cases} \end{aligned} \tag{11}$$

where the denominators must be positive. By the following replacements, problem (11) can be resulted from (3):

(1) $I_1^k = J_1^k \cup J_2^k$ and $I_2^k = J_3^k$;

(2) $w_i^k = 0$, $\lambda_i^k = \frac{1}{f_i^k - \bar{y}_i^k}$ $\forall i \in J_1^k$, $\lambda_i^k = \frac{1}{\bar{y}_i^k - f_i^k}$ $\forall i \in J_2^k$, $\rho = 0$ and $i = 1, \dots, p$;

(3) $r_i^k = \bar{y}_i^k$ $\forall i \in J_1^k$, $r_i^k = f_i^k$ $\forall i \in J_2^k$ and $\delta_i^k = f_i^k$ $\forall i \in J_3^k$.

4.6 RD method

The reference direction (RD) method was proposed in [25]. At the kth iteration, the DM is asked to specify a reference point $\bar{\mathbf{y}}^k$. Specifying a reference point is equivalent to classifying the objective functions in three classes J_1^k , J_2^k and J_3^k, where these index sets are the same as those defined before. The scalarizing problem related to the RD method is as follows:

$$
\begin{aligned}
&\min \max_{i \in J_1^k} \quad \frac{f_i(\mathbf{x}) - f_i^k}{f_i^k - \bar{y}_i^k} \\
&s.t. \begin{cases} f_i(\mathbf{x}) \le f_i^k & \forall i \in J_3^k, \\ f_i(\mathbf{x}) \le \bar{y}_i^k + \alpha(f_i^k - \bar{y}_i^k) & \forall i \in J_2^k, \\ \mathbf{x} \in \mathcal{X}, \end{cases}
\end{aligned} \tag{12}
$$

where $0 \le \alpha < 1$ and the denominators must be positive. The general formulation (3) can be transformed to RD problem (12) by the following replacements:

(1) $I_1^k = J_1^k$ and $I_2^k = J_2^k \cup J_3^k$;

(2) $w_i^k = 0$ $\forall i \in \{1, \dots, p\}$, $\lambda_i^k = \frac{1}{f_i^k - \bar{y}_i^k}$ $\forall i \in J_1^k$ and $\rho = 0$;

(3) $r_i^k = f_i^k \forall i \in J_1^k$, $\delta_i^k = f_i^k$ $\forall i \in J_3^k$ and $\delta_i^k = \bar{y}_i^k + \alpha(f_i^k - \bar{y}_i^k)$ $\forall i \in J_2^k$.

4.7 $\epsilon-$Constraint method

In this method, one of the objective functions is minimized, while the other objectives are transformed into constraints by setting an upper bound [4,23]. The problem to be solved has the following form:

$$
\begin{aligned}
&\min \quad f_l(\mathbf{x}) \\
&s.t. \begin{cases} f_j(\mathbf{x}) \le \varepsilon_j^k & \forall j \in \{1, \dots, p\}, j \ne l, \\ \mathbf{x} \in \mathcal{X}. \end{cases}
\end{aligned} \tag{13}
$$

By the following transformations, problem (13) can be attained from (3):

(1) $I_1^k = \{l\}$ and $I_2^k = \{1, \ldots, p\} \backslash \{l\}$;

(2) $\lambda_l^k = 1,\ \rho = 0,\ r_l^k = 0,\ \delta_j^k = \varepsilon_j^k,\ j = 1, \ldots, p$ and $j \neq l$.

4.8 The weighted sum method

In this method, a weighting coefficient is associated with each objective function and then the weighted sum of the objectives is minimized [4, 23]. Accordingly, solutions are obtained by solving the following problem:

$$\begin{aligned} \min \quad & \sum_{i=1}^{p} \mu_i^k f_i(\mathbf{x}) \\ s.t. \quad & \mathbf{x} \in \mathcal{X}, \end{aligned} \tag{14}$$

with $\mu_i^k \geq 0\ \forall i \in \{1, \ldots, p\}$ and $\sum_{i=1}^{p} \mu_i^k = 1$. By the following replacements, we can obtain this problem from (3):

(1) $I_1^k = \{l\}$, where l is an index with $\mu_l^k \neq 0$ and $I_2^k = \emptyset$;

(2) $\lambda_l^k = \frac{\mu_l^k}{2}$, $\rho = \frac{2}{\mu_l^k}$, $w_i^k = \mu_i^k\ \forall i \neq l,\ w_l^k = \frac{\mu_l^k}{2}$ and $r_i^k = 0\ \forall i \in \{1, \ldots, p\}$.

4.9 Hybrid method

The hybrid method is a combination of the weighted sum method and the $\epsilon-$constraint method [4, 23]. This problem has the following form:

$$\begin{aligned} & \min \sum_{i=1}^{p} \mu_i^k f_i(\mathbf{x}) \\ s.t. \ & \begin{cases} f_j(\mathbf{x}) \leq \varepsilon_j^k, & \forall j \in \{1, \ldots, p\}, \\ \mathbf{x} \in \mathcal{X}, \end{cases} \end{aligned} \tag{15}$$

where $\mu_i^k \geq 0\ \forall i,\ \sum_{i=1}^{p} \mu_i^k = 1$ and $\varepsilon^k = (\varepsilon_1^k, \ldots, \varepsilon_p^k)$ is an upper bound vector. One can find this problem from (3) by the following transformations:

(1) $I_1^k = \{l\}$, where l is an index with $\mu_l^k \neq 0$ and $I_2^k = \{1, \ldots, p\}$;

(2) $\lambda_l^k = \frac{\mu_l^k}{2}$, $\rho = \frac{2}{\mu_l^k}$, $w_i^k = \mu_i^k\ \forall i \neq l$ and $w_l^k = \frac{\mu_l^k}{2}$;

(3) $r_i^k = 0$ and $\delta_i^k = \varepsilon_i^k,\ \forall i \in \{1, \ldots, p\}$.

Remark 3. Using similar procedure, we can obtain some other single objective problems used in different interactive approaches. For example, the intermediate problems of the interactive surrogate worth trade-off (ISWT) method [3] and the PROJECT method [22] can be obtained easily from our formulations. In addition, the weighted Tchebycheff scalarizing problem [4] and the modified weighted Tchebycheff problem [15] are resulted from the proposed general scalarizing problem.

5 General interactive algorithm

Based on the general formulations given in Section 3, we present a general interactive algorithm (GIA). The proposed algorithm allows the DM to specify his/her preference information in six different ways. Moreover, he/she will be able to change his/her preference information in each iteration. In addition to widely used ways (reference point specification, classification of the objective functions, and specification of marginal rate of substitution) for specifying preference information ([21, 28]), GIA allows the DM to specify his/her preferences as criteria weights, $\varepsilon-$constraint (choosing a reference objective function and setting upper bounds for the other objective functions), or criteria weights and upper bounds for objective functions, simultaneously. The main steps of the GIA are given in Algorithm 1.

As pointed out in Step 4, the values of parameters and index sets depend on the type of preference information given by DM in Step 3. For example, if DM specifies his/her preferences as reference point, we should set parameters and index sets in (3) or (4) so that one of the reference based on scalarizing points problems (see, for example, (7) and (8)) be attained.

6 Computational and theoretical advantages

The GIA and the proposed scalarizing formulation has a number of potential advantages both in theoretical and computational points of view. Here, we indicate only some key potential advantages, with special attention to those not shared by other competing algorithms.

(a) Taking the special characteristics of the problem into account, GIA allows using more efficient optimization methods. In the first step of the GIA, the type of problem (differentiable or nondifferentiable) is specified. This step provides some advantages. For example, if the proposed problem is differentiable, corresponding scalarizing formulation preserves differentiability and can be solved using available single objective solvers. On the other hand, for a nondifferentiable problem, *Algorithm 1. General interactive algorithm (GIA)*

Step 1- Determine type of the MOP being solved (differentiable or nondifferentiable).

Step 2- Compute ideal and nadir points. Set $k = 1$. Determine an initial solution (can be specified by the DM or by solving an arbitrary scalarizing problem). Denote this initial solution by $\hat{\mathbf{x}}^k$ and corresponding objective vector by $f(\hat{\mathbf{x}}^k)$. If the DM is satisfied with this solution, go to Step 6.

Step 3- Ask the DM to provide his/her preference information based on $f(\hat{\mathbf{x}}^k)$. The DM can specify his/her preference information in one of the following ways:

3.1. Specifying the desired objective function values as components of the reference point ($\bar{y}_i^k,\ i = 1, \ldots, p$);

3.2. Classifying the objective functions into two classes J_1^k and J_2^k or three classes J_1^k , J_2^k and J_3^k, described in the text;

3.3. Specifying the marginal rates of substitutions (MRSs);

3.4. Determining the criteria weights;

3.5. Providing preferences with the help of $\epsilon-$constraint;

3.6. Defining preferences with the help of criteria weights and selecting the upper bounds for all objective functions, simultaneously.

Step 4- Based on the preference information, given by the DM in Step 3, set appropriate values for parameters and index sets in formulation (3) (for nondifferentiable MOP) or formulation (4) (for differentiable MOP), and solve it.

Step 5- Present the obtained (weakly, properly) efficient solution(s) and the corresponding objective function vector(s) to the DM. Let DM chooses one of them. In this case, different states can occur:

5.1. If the DM approves this solution as the most preferred one, denote this solution by $\hat{\mathbf{x}}^{k+1}$ and go to Step 6.

5.2. If the DM wants to obtain other solutions with the same preference information, go to Step 4. Note that, in this case, Step 4 should be executed with other values for parameters and index sets.

5.3. If the DM wants to provide new preference information, denote this solution by $\hat{\mathbf{x}}^{k+1}$, set $k := k + 1$ and go to Step 3.

Step 6- Stop.

corresponding scalarizing formulation (3) has a reduced number of constraints which causes a decrease in solving time.

(b) Unlike the algorithms proposed in [21,28,32], the GIA allows the DM to specify his/her preference information in six different ways. Since the satisfaction of DM is an important factor in the interactive algorithms, this aspect of the GIA will increase the satisfaction of DM.

(c) To propose a general algorithm for solving an MOP, it is necessary to convert the MOP problem to a general scalarized problem with perhaps some additional constraints. It is obvious that the number of constraints added to the general scalarized problem has a major effect on the computational time. In Table 1 (for nondifferentiable MOPs) we compare the number of constraints added to the suggested general formulation (3) with those added to some general formulations as GLIDE [21], GLIDE-II [28] and GENWS [32].

Table 1: Number of additional constraints in each formulation (in nondifferentiable case)

Methods	GENWS	GLIDE	GLIDE-II nondiff	Our formulation
GUESS	$card(J_2^k)$	$2p$	0	0
Reference direction approach	−	−	0	0
STOM	p	−	0	0
SPOT	−	$2p$	$p-1$	$p-1$
Modified reference point	$card(J_2^k)+card(J_3^k)$	−	$card(J_3^k)$	$card(J_3^k)$
RD	−	−	−	$card(J_2^k)+card(J_3^k)$
$\epsilon-$ constraint	−	−	−	$p-1$
Weighted sum	−	−	0	0
Hybrid	−	−	−	p
ISWT	−	−	$p-1$	$p-1$
PROJECT	−	$2p$	0	0
Weighted Tchebycheff	−	−	0	0

(d) One of the most important theoretical advantages of the proposed general formulations is that, Theorem 5 enables us to provide results concerning proper efficiency. Unboundedness of the trade-offs means, practically, ignoring at least one of the objective functions when the DM wants to improve another objective function, which is not satisfactory to the DM. Since properly efficient solutions have bounded trade-offs, the DM can improve some unacceptable objective functions with no concern.

(e) All the provided theorems where established without convexity assumptions. In fact, the main MOP can be convex or nonconvex.

7 A numerical example

In this section, we illustrate the procedure mentioned in the GIA on an engineering example of designing a four-bar plane truss, studied in [5]. This problem, has two conflicting objective functions. We should minimize the volume of the truss (f_1), and its joint displacement (f_2), subject to given physical restrictions on the feasible cross-sectional areas x_1, x_2, x_3, and x_4 of the four bars. The stress on the truss structure is caused by several forces of magnitude F, and $2F$. The length L of each bar and the elasticity constants E and σ of the materials involved are modelled as constants. The mathematical model of this example is as follows:

$$\begin{aligned} Minimize \Big\{ f_1(\mathbf{x}) &= L(2x_1 + \sqrt{2}x_2 + \sqrt{2}x_3 + x_4), \\ f_2(\mathbf{x}) &= \frac{FL}{E}\left(\frac{2}{x_1} + \frac{2\sqrt{2}}{x_2} - \frac{2\sqrt{2}}{x_3} + \frac{1}{x_4}\right)\Big\} \\ s.t. \quad & \begin{cases} \frac{F}{\sigma} \le x_1 \le \frac{3F}{\sigma}, \\ \sqrt{2}(\frac{F}{\sigma}) \le x_2 \le \frac{3F}{\sigma}, \\ \sqrt{2}(\frac{F}{\sigma}) \le x_3 \le \frac{3F}{\sigma}, \\ \frac{F}{\sigma} \le x_4 \le \frac{3F}{\sigma}, \end{cases} \end{aligned}$$

where, the constant parameters are chosen as $F = 10\ kN$, $E = 2 \times 10^5\ kN/cm^2$, $L = 200\ cm$ and $\sigma = 10\ kN/cm^2$.

The ideal and nadir values for objective functions of this problem are obtained as $\mathbf{y^I} = (y_1^I, y_2^I) = (1400, -5.7191 \times 10^{-4})$ and $\mathbf{y^N} = (y_1^N, y_2^N) = (3.4971 \times 10^3, 0.0406)$. Now, based on the GIA, at first, we should find an initial solution. To this end, the $\epsilon-$constraint scalarizing problem (13) is used, which can be obtained from the proposed formulations by parameters and index sets given in Subsection 4.7, with $l = 2$ and $\epsilon_1 = 1800$. By solving the obtained problem, we find $(1.3906, 1.9963, 1.4142, 1.3957)$ for variables, and $(1800, 0.0157)$ for objective functions. As it can be seen, the values of the objective functions are between ideal and nadir values. Let $\hat{x}^1 = (1.3906, 1.9963, 1.4142, 1.3957)$ and $f(\hat{x}^1) = (1800, 0.0157)$ are shown to DM.

Suppose, the DM wishes to express his/her preference information as the reference point $\bar{\mathbf{y}}^1 = (\bar{y}_1^1, \bar{y}_2^1) = (1600, 0.01)$. Based on this preference given by DM, one of the reference point based scalarizing problems can be selected. Here, we set parameters in our formulation, such that the GUESS scalarizing problem is obtained, and by solving it, $(1.4613, 2.0666, 1.4142, 1.4613)$ is obtained for variables and $(1861.3, 0.0142)$ is attained for the objective function values. At this iteration, the volume of the truss has increased and its joint displacement has slightly decreased. Now, assume that the DM wants to change the type of his/her preference information. According to Step 5 of the GIA, set $\hat{x}^2 = (1.4613, 2.0666, 1.4142, 1.4613)$, $f(\hat{x}^2) = (1861.3, 0.0142)$ and $k = 2$. Now, Step 3 is executed.

Assume that the DM wants to classify the objective functions in two classes $J_1^2 = \{1\}$, and $J_2^2 = \{2\}$. This means, the DM wants to improve f_1 by somewhat relaxing f_2. Assume that the DM gives us $\bar{\mathbf{y}}^2 = (1500, 0.03)$. Now, the RD scalarizing problem is used with $\alpha = 0.5$. By solving the problem, $(1.1876, 1.6796, 1.4142, 1.1876)$ is obtained for variables and $(1587.6, 0.0221)$ is attained for the objective functions. Assume that the DM wants to provide new preferences by selecting weights for the objective functions. To this end, let $\hat{x}^3 = (1.1876, 1.6796, 1.4142, 1.1876)$, $f(\hat{x}^3) = (1587.6, 0.0221)$, and $k = 3$. Then, Step 4 is executed by $(w_1^3, w_2^3) = (\frac{3}{4}, \frac{1}{4})$ as weights given by the DM. By solving the weighted sum problem (14), $(1, 1.4142, 1.4142, 1)$ is obtained for variables and $(1400, 0.03)$ is obtained for the objective values. As it can be seen, the volume of the truss is in its ideal value, and this satisfies the DM. It is important to point out that by Theorem 5, the obtained objective vector is a properly nondominated point.

8 Conclusions

In this article, we suggested a general scalarizing formulation to obtain a global interactive algorithm for multiobjective optimization problems. We proposed the formulation in two versions; one of them for differentiable and the other for nondifferentiable MOPs. By selecting suitable values for parameters, we proved that optimal solutions of the suggested general scalarizing problem are (weakly, properly) efficient solutions for the main multiobjective problem. Moreover, it was shown that many scalarizing problems used in different interactive methods as GUESS, reference direction approach, Step, STOM, SPOT, modified reference point, and RD methods can be obtained from the proposed general formulation, by selecting suitable transformations. Some of the popular scalarizing problems such as, weighted sum, $\epsilon-$constraint, and hybrid problems derived from our general scalarizing problem. In addition, we proposed a general interactive algorithm. In the proposed algorithm, the DM could express his/her preference information in six different ways, and based on the kind of information given by the DM, a suitable scalarizing problem, by selecting appropriate values for parameters and index sets in the general formulation, was selected. Finally, by a numerical example we illustrated that how the proposed general interactive algorithm can be used.

However, for the future investigation, developing a software based on the suggested general interactive algorithm can be worthwhile. Also, proposing a general interactive procedure for approximate efficient solutions of an MOP can be worth studying. To this end, studying three recently published papers by Ghaznavi-ghosoni[1] and Khorram [10], Ghaznavi-ghosoni et al. [11] and Ghaznavi [12] is recommended.

[1] Previous name of the first author

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چکیده : تنوع روش‌های تعاملی موجود، نیاز به ارایه الگوریتم‌های تعاملی عمومی که تصمیم گیرنده را قادر می‌سازند که آزادانه چندین روش مناسب را که مورد ترجیح او است را انتخاب کند را نشان می‌دهد. برای این منظور، در این مقاله، یک مسئله اسکالرسازی عمومی برای مسائل برنامه‌ریزی چند هدفه پیشنهاد می‌دهیم. رابطه بین جواب های بهینه مسئله اسکالرسازی معرفی شده و جواب های کارا (ضعیف) و کارای سره مسئله بهینه‌سازی چند هدفه اصلی بررسی می‌شود. نشان می‌دهیم که با انتخاب تبدیل‌های مناسب، برخی از مسائل اسکالرسازی به کار رفته در روش‌های تعاملی مختلف می‌توانند از فرمول پیشنهاد شده بدست آیند. بر اساس مسئله اسکالرسازی پیشنهاد شده، یک الگوریتم تعاملی عمومی پیشنهاد می‌دهیم که تصمیم گیرنده را قادر می کند ترجیحاتش را با شش روش مختلف و با قابلیت تغییر در ترجیحات در هر زمان در طول تکرارهای الگوریتم مشخص کند. سرانجام، یک مثال عددی که بیانگر کاربردی بودن الگوریتم است ارایه می‌گردد.

کلمات کلیدی : بهینه‌سازی چندهدفه؛ روش تعاملی؛ مسئله اسکالرسازی؛ کارایی سره؛ اطلاعات ترجیحی.