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Research Article

Investigating a claim about resource complexity measure

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Abstract

The utilization factor (UF) measures the ratio of the total resources' amount required to the availability of resources' amount during the life cycle of a project. In 1982, in the journal of Management Science, Kurtulus and Davis claimed that "If two resource-constrained problems for each type of resource have the same UF's value in each period of time, then each problem is subjected to the same amount of delay provided that the same sequencing rule is used (If different tie-breaking rules are used, a different schedule may be obtained)". In this paper, with a counterexample, we show that the claim of authors cannot be justified.

AMS subject classifications (2020): 90B35, 68M20.

Keywords: Scheduling scheme; Priority rule; Multi-project environment; Resource measure.

1 Brief description

For the resource-constrained project scheduling problem (RCPSP) in single-project environments, several resource measures and distributions have been proposed in the literature. Notable among them are resource factor, resource strength, resource density, and resource constrained-ness (see, e.g., [2, 3]). However, the related RCPSP's measures are not suitable for the multi-project environments [4]. Hence, for the multi-project environments, some resource measures such as average loading factor, average resource loading factor (ARLF), and average utilization factor (AUF) are proposed for resource-constrained multi-project scheduling problems (RCMPSPs).

The AUF is more widely used than the others, and first, we describe it as blow (for more details, see, e.g., [4, 5]):

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The AUF measures the degree of dependency for each of the required resources. As the AUF considers the ratio of the amount of resource required to the level of available resource, it is complementary to the ARLF.

To explain more, suppose that CP_i is the length of the critical path of project $i \quad (i = 1, \ldots, M)$ in the RCMPSP subject to

$$CP_1 \leq CP_2 \leq \cdots \leq CP_M.$$

Define

$$S_1 = CP_1, S_2 = CP_2 - CP_1, \ldots, S_M = CP_M - CP_{M-1}.$$

Then the total required resource of type k on Lth interval, that is, $[a, b] = [CP_{L-1}, CP_L]$ is defined as

$$W_{S_L,k} = \sum_{t=a}^{b} \sum_{i=1}^{M} \sum_{j=1}^{N_i} r_{ijk} X_{ijt},$$

where

$$X_{ijt} = \begin{cases} 1 & \text{if activity } j \text{ in project } i \text{ at time } t \text{ is active,} \\ 0 & \text{otherwise,} \end{cases}$$

the amount of required resource of type k for the activity j of the project i is denoted by r_{ijk}, and N_i is the number of activities in the project i.

The AUF for the resource k (AUF_k) is defined as

$$AUF_k = \frac{1}{M} \sum_{L=1}^{M} \frac{W_{S_L,k}}{R_k \times |S_L|},$$

where R_k is the total amount of renewable resource k per unit of time ($k \in \mathcal{R} = \{1, \ldots, K\}$). Hence, the value of AUF for the RCMPSP with K resources is determined by

$$AUF = \{\max\{AUF_1, \ldots, AUF_K\}\}.$$

When $AUF_k > 1$, it can be concluded that the resource k is constrained in the RCMPSP [4].

For addressing the AUF measure, a related measure, that is, the utilization factor (UF) is needed to discuss: The UF measures the ratio of the total amount of resources required to the amount of available resources during the life cycle of a project [1]. Hence, for a particular type of resource in a project, if $UF \leq 1$, then it is clear that there is no resource constraint and that the early schedule is optimal [4].

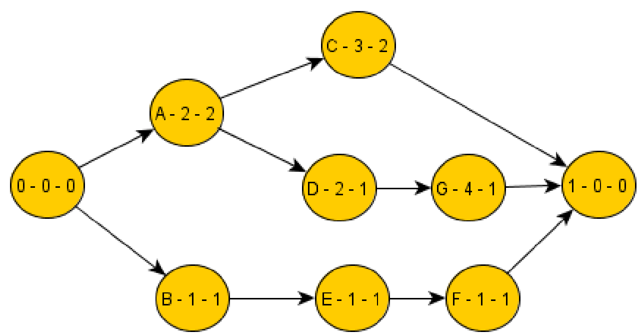
2 Main problem

In 1982, Kurtulus and Davis [4, p.163] claimed that "If two resource-constrained problems for each type of resource have the same UF's value in each period of time, then each problem is subjected to the same amount of delay provided that the same sequencing rule is used (If different tie-breaking rules are used, a different schedule may be obtained)".

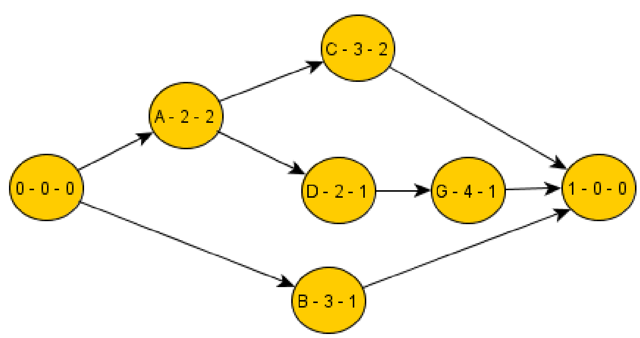
It is worth noting that the above statement is the main basis of many types of research in multi-project environments.

In the following example, we show that such a claim cannot be justified.

Example 1. Consider two projects P_1 and P_2 in Figure 1. They have the same UF distribution during their life cycles (see Figures 2 and 3). In the following, we show that even if the priority rule and the tie-breaking rule in the scheduling of these projects are the same, then the yielded schedules are not necessarily identical.



(a) Project P_1



(b) Project P_2

Figure 1: Two example project networks

Without loss of generality, we consider the case where $|\mathcal{R}| = 1$. Let $R_1 = 3$. Moreover, assume that the priority rule and the tie-breaking rule are the GRD[1] and the LPT[2], respectively. In other word, they are formulated as (see, e.g., [6])

$$\text{GRD:} \quad \max_j = d_j \sum_{k \in \mathcal{R}} r_{jk}$$

and

$$\text{LPT:} \quad \max_j = d_j,$$

where d_j indicates the processing time of activity j and r_{jk} is the amount of required resource k in the activity j.

The resource distribution for projects P_1 and P_2 is shown in Figures 2a and 2b, respectively.

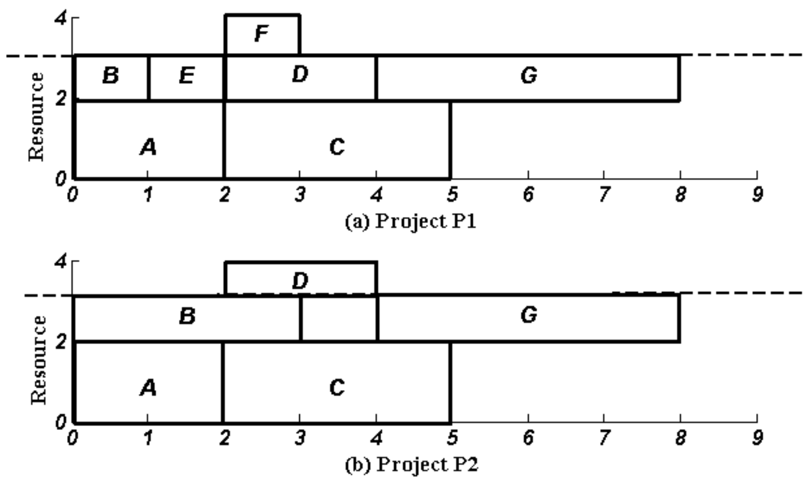


Figure 2: Resource distribution for projects P_1 and P_2

Since the amount of available resources and resource distribution for both P_1 and P_2 are the same, then the corresponding amount of the UFs is the same. Now, by applying the same priority rule (i.e., the GRD rule) and the same tie-breaking rule (i.e., the LPT rule), from Figure 3a, it is observed that the schedule for P_1 is optimal (T=8), while for P_2, as Figure 3b shows, the schedule's makespan is not optimal and equals 9 (i.e., T=9). Hence, the authors claim in [4] cannot be verified.

[1] Greatest Resource Demand (GRD)

[2] Longest Processing Time (LPT)

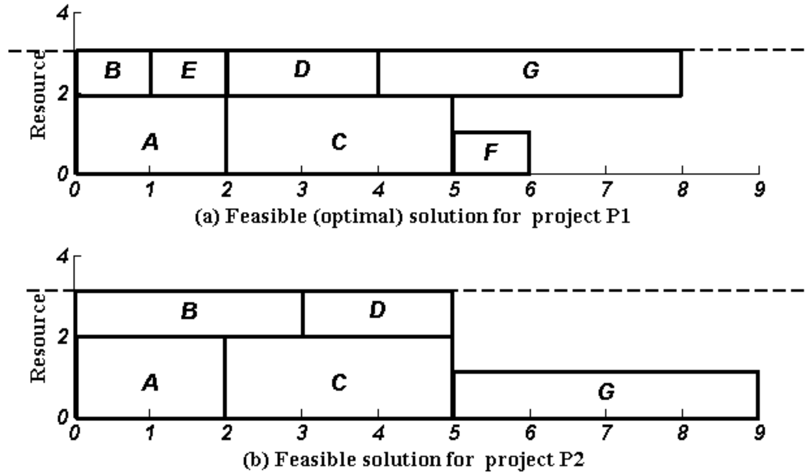


Figure 3: Feasible solution for projects P_1 and P_2

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