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Research Article

Multicriteria allocation model of additional resources based on DEA: MOILP-DEA

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Abstract

We exploit the relationship between multiobjective integer linear problem (MOILP) and data envelopment analysis (DEA) to develop an approach to a resource reallocation problem. The general purpose of the mathematical formulation of this multicriteria allocation model based on DEA is to enable decision-makers to take into account the efficiency of units under control to allocate additional resources for a new period of operation. We develop a formal approach based on DEA and MOILP to find the most preferred allocation plan taken account additional resources. The mathematical model is given, and we illustrate it with a numerical example.

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Keywords: Multicriteria decision aiding; Data envelopment analysis; Multiobjective integer linear programming; Additional resource allocation; Efficient frontier.

1 Introduction

The use of the data envelopment analysis (DEA) approach [1, 4] provides useful information for monitoring and management in a production organization. The essential elements are knowledge reference units, sources of inefficient units, variations of productivity from one period to the next, and variable returns to scale.

Multicriteria decision aiding (MCDA) and DEA are two techniques in operations research, which attract the attention of many researchers. Some

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authors are interested in their relationship and how to exploit [2, 3, 7–9, 11, 13, 15, 16]. We propose a mathematical model of multiobjective optimization based on DEA that can help managers or decision-makers adjust investments in different units of the control system by additional assignments to bring.

The general purpose of the mathematical formulation of this multicriteria allocation model based on DEA is to enable decision-makers to take into account the performance of units under control to allocate additional resources for a new period of operation. So, it is necessary to determine information intermediaries:
– information on production or performance efficiencies;
– information on the returns to scale presented by each unit;
– optimal targets for each production unit.

The mathematical model, we propose, takes into account all this information intermediaries. The objective is, on the basis of additional quantity of resources and demands expressed by the different DMUs, to allocate optimally and efficiently these resources available to maximize estimates of product returns based on information gathered from performance measures and returns of the ladder. In a given production system, the reference units (the efficient units) are those that use resources (inputs) rationally, at best, to achieve optimal results. This transformation of inputs into outputs is supposed to follow a function the best of which is that used by efficient units. These efficient units give an idea of best practices and thus provide the best possible production plans. On the basis of information on the units, namely the identification of the efficient and the inefficient, it is important to consider how this information can be used to assist the decision-maker in allocating resources for a new production period with the assumption that the system will use the same process of transformation. One of the information that can be provided by the application of the DEA method is the identification of the variable returns to scale associated with each unit of the system on a production function basis. Efficient and inefficient units can be either of increasing returns to scale, constant returns to scale, or decreasing returns to scale.

In this paper, we present the modeling of the problem, the mathematical model and illustrate with a numerical example.

2 Issue and literature review

The DEA methodology is one of the most widely used decision aiding approaches in the literature. Emrouznejad and Yang (2017) [5] listed more than 10,300 published articles. The fields of application are very diverse and broad. “Resource allocation is the generic problem of assigning available resources to users in the best possible way. Usually the resources are limited,

thus sets of activities compete for these resources establishing explicit and implicit dependencies, which may be subject to uncertainty. The resulting decision problem on the best use of the resources can be decomposed in several problems that have been intensively studied in the field of operations research in the last few decades. Examples of such problems are [1]:
Scheduling problem. Scheduling is used to optimally allocate scarce resources to activities and includes problems like the allocation of plant and machinery resources, the organization of human resources, and the control of production processes and materials purchase.
Knapsack problem. The main idea is to consider the capacity of the knapsack as the available amount of resource and the item types as activities to which this resource can be allocated.
Cutting stock problem. It is a general resource allocation problem, where the objective is to cut pieces of stock material into pieces of specified sizes, minimizing the wasted material.
Power generation scheduling problem. Power generation scheduling is required in order to find the optimum allocation of energy such that the annual operating cost of a power system is minimized, or the obtained profit is maximized."

There is a large literature dealing with these problems and several solving techniques have been proposed. Several authors have been interested in the problem of resource allocation. Fang, [6] used a generalized DEA model for centralized resource allocation to uncover the sources of such total input contraction. Wu et al. [14] integrated DEA and MOLP to deal with the resource allocation problem. Several authors have been interested in the linkage of the two methods, the DEA method coupled with the MOILP method (rkc et al. [10] and Sueyoshi [12]). We looked at questions such as: how can we take into account the performance indices of the different units of a production system to reallocate additional resources. To answer this question, we have made certain assumptions. First, the observed efficient production boundary of the system provides the best production function of the moment. Second, the lower expected returns to scale of a unit is an indicator to minimize the risk of resource waste. Indeed, when a unit has a variable returns to scale, the methods give a forecast of the expected yield. This random aspect led us to focus on the lowest expected return. In developing countries, the problem of allocating additional resources concerns more than one. Donors and technical and financial partners, not only do they want to have measurable performance indicators, but they also want to know if an efficient unit is not going to lose performance if we add too many resources to it. With respect to inefficient units, the same questions arise. Our approach is to address these two types of concerns: to take into account the performance indices calculated by DEA and to maximize the expected returns to scale of the units taking into account the risks of waste. For example, if the minimum return to scale

[1] Dimitri Thomopulos, Models and Solutions of Resource Allocation Problems based on Integer Linear and Nonlinear Programming

is zero, then the risk of waste is high. As a precautionary measure, we have to wait for another evaluation period before adding additional resources.

3 Modeling of the problem

The Modeling of the problem requires intermediate steps to calculate, on the one hand, coefficients of improvement of efficiency, according to the increase inputs, and on the other hand, minimum values of variable returns to scale of the different units.

3.1 Determination of reference units and estimation of the "technical" function for improving returns

One of the major information in the process of modeling the problem is knowledge of reference units that are technically effective units of system. These units are determined using the Charnes, Cooper and Rhodes (CCR) model [4]. We note

$$\mathcal{E} = \{j \in \{1, \ldots, N_E\} \text{ such as Decision Making Unit (DMU) : } j \text{ is CCR efficient}\},$$

which contains the technical efficient DMUs. To obtain $\mathcal{E}$, we use the DEA techniques [1,4]. Once the set $\mathcal{E}$ is determined, we use the Banker, Charnes and Cooper (BCC) model [1] with input orientation analyzing the technical efficient units by taking turns, individually each output factor associated with all input factors. This step consists of measuring the performance of technical efficient units relative to each output factor obtained with input factors. It is a question of measuring the performance of the N_E units in $\mathcal{E}$ with a single output k and the m input factors. Thus, we estimate the possible maximum efficiency for each output factor in relation to the observed data. This mathematically translates into resolution for each unit $d \in \mathcal{E}$ and for each output factor $k \in \{1, \ldots, s\}$ the following problem.

$$\max\, ft(k,d) = \mu_k^d y_{kd} + u^{k,d}$$
$$\text{subject to the constraints (s.c)} \begin{cases} \mu_k^d y_{kj} - \sum\limits_{i=1}^{m} \nu_i^{k,d} x_{ij} + u^{k,d} \leq 0, \quad j = 1, \ldots, n, \\ \sum\limits_{i=1}^{m} \nu_i^{k,d} x_{id} = 1, \\ \mu_k^d \geq \varepsilon, \quad \nu_i^{k,d} \geq \varepsilon, \quad \text{for all } i, u^{k,d} \in \mathbb{R}, \end{cases} \tag{1}$$

Remark: Although the units under evaluation are all technically effective, therefore using the full BCC model, they are not necessarily efficient here since only one output factor is considered and the unit may be less efficient for this single factor. In other words, the optimal value simply checks $\tilde{f}t(k,d) \leq 1$.

To avoid problems related to situations of multiple optimal solutions, we use the following min max model (2).

$$\min \Delta_k^d$$
$$\text{s.c.} \begin{cases} \mu_k^d(y_{kj} - y_{kd}) - \sum\limits_{i=1}^{m} \nu_i^{k,d}(x_{ij} - x_{id}) - \delta_j^{k,d} \leq 0, j = 1,\ldots,n, \\ \sum\limits_{i=1}^{m} \nu_i^{k,d} x_{id} = 1, \\ \Delta_k^d - \delta_j^{k,d} \geq 0, \quad j = 1,\ldots,n, \\ \mu_k^d \geq \varepsilon, \quad \nu_i^{k,d} \geq \varepsilon, \qquad i = 1,\ldots,m. \end{cases} \tag{2}$$

The above formulation indicates that for a given output factor k, the hyperplane representing the maximum efficiency of that factor with the reference unit d is defined by

$$\tilde{\mu}_k^d(y_k - y_{kd}) - \sum_{i=1}^{m} \tilde{\nu}_i^{k,d}(x_i - x_{id}) = \tilde{\Delta}_k^d.$$

It can therefore be seen that, for the output factor k and for the DMU d, $\tilde{\mu}_k^d(y_k - y_{kd})$ is provided by poor performance

$$\sum_{i=1}^{m} \tilde{\nu}_i^{k,d}(x_i - x_{id}) + \tilde{\Delta}_k^d.$$

The contribution of the various input factors to this increase can therefore be measured by the expression $\sum\limits_{i=1}^{m} \tilde{\nu}_i^{k,d} x_i$, where x_i represents the increase in factor input i. In order to take into account all the reference units $d \in \mathcal{E}$, we consider the average of these values by introducing the quantity

$$\sum_{i=1}^{m} \alpha_i^k x_i, \quad \text{where} \quad \alpha_i^k = \frac{1}{N_E} \sum_{d=1}^{N_E} \tilde{\nu}_i^{k,d}. \tag{3}$$

The coefficients $\alpha_i^k \geq 0$ reflect the relationship between the increase of the input factor i and the maximum average return to which it contributes for the improvement of the output k.

3.2 Determination of returns to scale values

However, it is also necessary to take into account the information on the variable returns to scale of the different units in order to allocate the additional amount of available input optimally. In order to do this, we determine the minimum values of the returns to scale that each unit can present. These values are calculated for projections obtained on the efficient frontier, that is, if the unit is not efficient, we project it on the efficient border before calculating the minimum value of return to scale it can present. We use the input-oriented BCC model to determine the values of the projections and the minimum values of the returns to scale. We note $\widehat{X}_j$ and $\widehat{Y}_j$ the vectors of the inputs and outputs, respectively, corresponding to the projection of unit j on the efficient frontier. For each unit $d, \quad d = 1, \ldots, n$, we calculate $\rho^{\min,d}$ the finite minimum value of the present returns to scale by solving the following problem:

$$u^{\min,d} = \min u^d$$
$$\text{s.c.} \quad \begin{cases} \sum_{r=1}^{s} \mu_r^d y_{rj} - \sum_{i=1}^{m} \nu_i^d x_{ij} + u^d \leq 0, \quad j = 1, \ldots, n, \quad j \neq d, \\ \sum_{r=1}^{s} \mu_r^d \widehat{y}_{rd} + u^d = 1, \\ \mu_r^d \geq 0, \quad \nu_i^d \geq 0, \quad \text{for all} r, i, \quad u^d \in \mathbb{R}, \end{cases} \tag{4}$$

According to Banker and Thrall [1],

$$\rho^{\min,d} = \frac{1}{1 - u^{\min,d}} \geq 0.$$

Note $\gamma_j = \rho^{\min,j}, \quad j = 1, \ldots, n$. We note that the objective is to determine the smallest theoretically observable scale performance.

4 Mathematical model

We now describe the formulation of the model, starting with the constraints and then the objective functions.

4.1 Constraints on resource availability

We consider the following:
–that for each input factor i, a b_i quantity is available to be distributed between the different units j.
–that each DMU j expresses a request a_{ij} units for input i (we assume $a_{ij} \leq b_i, \quad$ for all j.) by introducing binary variables $t_{i,j}$ by $t_{i,j} = 1$, if the request for input i of the DMU j is retained and $t_{i,j} = 0$, otherwise. The constraints of the MOILP-DEA model are then written

$$\sum_{j=1}^{n} a_{ij} t_{i,j} \leq b_i, \quad i = 1, \ldots, m. \tag{5}$$

4.2 Criteria

In the multicriteria model, we introduce s objective functions $z_k, k = 1, \ldots, s$ corresponding to the s output factors. z_k will translate the additional return (expected) of the factor k due to the increase of inputs i granted to the different DMUs j. In order to calculate this increase it is necessary to take into account both the coefficients α_i^k determined in 3.1 and the coefficients γ_j determined in 3.2.
Also the z_k function is written

$$\begin{aligned} & z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k t_{i,j}, \\ & \text{where} \quad c_{ij}^k = \alpha_i^k \gamma_j a_{ij}. \end{aligned} \tag{6}$$

4.3 MOILP-DEA

The multicriteria problem that we propose to allocate additional resources to DMUs is thus finally formulated by

$$\begin{aligned} \text{“max”} \ z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^k t_{i,j}, \quad k = 1, \ldots, s \\ \text{s.c.} \ \begin{cases} \sum_{j=1}^{n} a_{ij} t_{i,j} \leq b_i, \quad i = 1, \ldots, m, \\ t_{i,j} \in \{0,1\} \quad \text{for all } i, \quad \text{for all } j. \end{cases} \end{aligned} \tag{7}$$

It should be noted that, since no interaction is involved in the constraints between the variables relating to two different inputs, the problem can be broken down into separate problems, considering each input separately.
For a fixed input i, the problem can be written

$$\begin{aligned} \text{“max”} \ z_{k,i} = \sum_{j=1}^{n} c_{ij}^k t_{i,j}, \quad k = 1, \ldots, s \\ \text{s.c.} \ \begin{cases} \sum_{j=1}^{n} a_{ij} t_{i,j} \leq b_i, \\ t_{i,j} \in \{0,1\} \quad \text{for all } j. \end{cases} \end{aligned} \tag{8}$$

These m problems are each presented as a multicriteria "Knapsack problem" for which many methods of resolution exist. We have of course

$$z_k = \sum_{i=1}^{m} z_{k,i}.$$

5 Illustration

We consider the illustrative example with 3 input and 3 output factors, the data of which are presented in Table 1.

Table 1: Data and Results of the Example

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3	score CCR	$\rho^{\min,d}$
1	8	4	6	5	4	8	1.0000	0.6748
2	7	8	4	3	7	4	0.9960	0.6772
3	11	6	4	11	11	5	1.0000	0.3423
4	10	10	2	9	14	1	1.0000	0.0732
5	2	7	3	4	1	4	1.0000	0.9582
6	6	10	10	8	10	13	1.0000	0.0000
7	11	7	11	11	2	16	1.0000	0.0000
8	5	14	8	6	13	9	0.8885	0.2899
9	10	11	9	9	5	12	0.9101	0.8969
10	1	8	2	9	13	2	1.0000	0.0388
11	6	5	3	12	5	5	1.0000	0.1574
12	5	9	1	10	8	1	1.0000	0.0845
13	9	12	8	14	17	10	1.0000	0.0000
14	13	13	7	15	12	9	0.8099	0.0000

Using the CCR model, we get all the technically efficient DMUs

$$\mathcal{E} = \{1, \ldots, 14\} \setminus \{2, 8, 9, 14\}.$$

Using the approach proposed by Banker and Thrall in [1] (see problem (4) above), we obtain the minimum values of the variable returns to scale $\rho^{\min,d}$ of the 14 DMUs considered (see the last column of Table 1). To analyze each $d \in \mathcal{E}$ and each output factor k, we use the problem (2) and we obtain the following results (see Table 2). The results in Table 3 allow the calculation of the coefficients α_i^k.

Let us consider that the requests a_{ij} expressed in units i for the different DMUs j are those shown in Table 4.

Table 2: Results

DMU d	$\tilde{\Delta}_1^d$	$\tilde{\Delta}_2^d$	$\tilde{\Delta}_3^d$	$\tilde{\mu}_1^d$	$\tilde{\mu}_2^d$	$\tilde{\mu}_3^d$	$\tilde{\nu}_1^{1,d}$	$\tilde{\nu}_1^{2,d}$	$\tilde{\nu}_1^{3,d}$	$\tilde{\nu}_2^{1,d}$	$\tilde{\nu}_2^{2,d}$	$\tilde{\nu}_2^{3,d}$	$\tilde{\nu}_3^{1,d}$	$\tilde{\nu}_3^{2,d}$	$\tilde{\nu}_3^{3,d}$
1	0.000	0.000	0.000	0.001	0.001	0.001	0.060	0.061	0.062	0.129	0.126	0.124	0.001	0.001	0.001
3	0.185	0.000	0.185	0.001	0.030	0.001	0.001	0.001	0.001	0.137	0.141	0.135	0.043	0.036	0.045
4	0.218	0.000	0.218	0.001	0.182	0.001	0.001	0.001	0.001	0.071	0.087	0.071	0.142	0.062	0.142
5	0.000	0.000	0.000	0.001	0.001	0.001	0.056	0.056	0.053	0.107	0.109	0.105	0.047	0.042	0.054
6	0.385	0.345	0.000	0.005	0.031	0.050	0.046	0.012	0.065	0.072	0.092	0.060	0.001	0.001	0.001
7	0.312	0.365	0.000	0.019	0.001	0.148	0.001	0.038	0.089	0.140	0.081	0.001	0.001	0.001	0.001
10	0.000	0.000	0.000	0.001	0.090	0.001	0.019	0.010	0.112	0.083	0.001	0.111	0.158	0.491	0.001
11	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.090	0.091	0.090	0.181	0.180	0.181
12	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.017	0.001	0.001	0.084	0.001	0.986	0.158	0.986
13	0.000	0.000	0.157	0.238	0.220	0.082	0.048	0.109	0.021	0.047	0.001	0.001	0.001	0.001	0.100

Table 3: Results of the coefficients α_i^k

k	α_1^k	α_2^k	α_3^k
1	0.0233	0.0876	0.1560
2	0.0306	0.0812	0.0973
3	0.0406	0.0698	0.1513

Table 4: Quantities requested a_{ij}

DMU	1	2	3	4	5	6	7	8	9	10	11	12	13	14	b_i
Inputs i															
1	1	2	0	3	0	1	1	0	2	1	1	0	2	0	10
2	1	1	1	0	1	1	0	1	1	1	0	2	0	1	5
3	2	0	0	1	2	1	1	1	0	0	2	1	1	2	8

With this example, the MOILP-DEA model is equivalent to solving the 3 following multicriteria "knapsack problems".

$$\text{“max”} \begin{cases} \mathbf{z_{1,1}} = \alpha_1^1 \sum\limits_{j=1}^{14} a_{1j}\gamma_j t_{1,j} \\ \mathbf{z_{2,1}} = \alpha_1^2 \sum\limits_{j=1}^{14} a_{1j}\gamma_j t_{1,j} \\ \mathbf{z_{3,1}} = \alpha_1^3 \sum\limits_{j=1}^{14} a_{1j}\gamma_j t_{1,j} \end{cases}$$

$$\text{s.c.} \begin{cases} \sum\limits_{j=1}^{14} a_{1,j} t_{1j} \leq 10, \\ t_{1,j} \in \{0,1\} \quad j = 1, \dots, 14. \end{cases}$$

$$\text{“max”} \begin{cases} \mathbf{z_{1,2}} = \alpha_2^1 \sum\limits_{j=1}^{14} a_{2j}\gamma_j t_{2,j} \\ \mathbf{z_{2,2}} = \alpha_2^2 \sum\limits_{j=1}^{14} a_{2j}\gamma_j t_{2,j} \\ \mathbf{z_{3,2}} = \alpha_2^3 \sum\limits_{j=1}^{14} a_{2j}\gamma_j t_{2,j} \end{cases}$$

$$\text{s.c.} \begin{cases} \sum\limits_{j=1}^{14} a_{2j} t_{2,j} \leq 5, \\ t_{2,j} \in \{0,1\} \quad j = 1, \dots, 14. \end{cases}$$

$$
\text{“max”} \begin{cases} \mathbf{z_{1,3}} = \alpha_3^1 \sum_{j=1}^{14} a_{3j}\gamma_j t_{3,j} \\ \mathbf{z_{2,3}} = \alpha_3^2 \sum_{j=1}^{14} a_{3j}\gamma_j t_{3,j} \\ \mathbf{z_{3,3}} = \alpha_3^3 \sum_{j=1}^{14} a_{3j}\gamma_j t_{3,j} \end{cases}
$$

$$
\text{s.c.} \begin{cases} \sum_{j=1}^{14} a_{3j} t_{3,j} \leq 8, \\ t_{3,j} \in \{0,1\} \quad j = 1,\ldots,14. \end{cases}
$$

Individual maximizations of the 3 criteria give:

– $k = 1, t_{1,1} = t_{1,2} = t_{1,4} = t_{1,9} = t_{1,10} = t_{1,11} = 1$, and $t_{1,j} = 0$ otherwise

– $k = 2, t_{2,1} = t_{2,2} = t_{2,3} = t_{2,5} = t_{2,9} = 1$, and $t_{2,j} = 0$ otherwise

– $k = 1, t_{3,1} = t_{3,5} = t_{3,8} = t_{3,11} = 1$, and $t_{3,j} = 0$ otherwise.

6 Conclusion

In this article, using the relationship between DEA efficiency and MOILP, we have proposed a mathematical model, a multicriteria linear problem for additional resources allocation, and we have solved an illustrative sample. The formulation described here can provide an efficient framework for optimizing investment taking into account analyses of the performance efficiency of a production system. On the one hand, the determination of the units of reference can facilitate the planning of the development programs of a given unit (efficient or inefficient); on the other hand, the minimum values of variable returns to scale make it possible to analyze the minimum need for additional input factors to improve returns. This different information provided by the use of DEA makes it possible to efficiently allocate additional resources so, to search and performance efficiency, and stability of efficiency of the units of the system. We then hope, for validation purposes, to apply these models to specific cases relating to public sectors such as education. In the case of aid and financing projects for farmers, it is important to note that this is the case in the case of health and a private sector such as agriculture. This would then allow validation of this initial model proposal and could be re-designed if needed.

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