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Properties of groups with points[*,†]

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Abstract

In this paper, we consider groups with points which were introduced by V.P. Shunkov in 1990. In Novikov-Adian's group, Adian's periodic products of finite groups without involutions and Olshansky's periodic monsters every non-unit element is a point. There exist groups without points. In this article we shall prove some properties of the groups with points.

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1 Introduction

Finiteness conditions in groups which are connected with finiteness of systems of subgroups were traditionally studied in Krasnoyarsk group theory School. An element in a group is a point if the sets of finite subgroups in special system of subgroups connected with this element are finite. More precisely, an element of finite order of a group G of the following types is called a *point* of G

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a) The identity element is a point if and only if the set of elements of finite orders of G is finite;

b) Non-identity element a of G is a point if for every non-identity finite subgroup K of G normalized by the element a, then the set of finite subgroups of $N_G(K)$ containing a is finite.

The definition of a point was introduced by V.P. Shunkov in 1990(see, for example [13]).

The concept of the points in groups give us the possibility of studying infinite groups. In particular, by using this concept, the sign of non-simplicity of an infinite group came to exist in [12]. In this article, we establish some properties of groups with points. We start by proving some properties of the common character (Lemmas and Theorems 1–5). Theorem 6 gives us the sign of placement of a point in an infinite group outside of infinite locally finite subgroups. Simultaneously, it will be proved that there are no points in an infinite locally finite group.

Theorems 7–10 have more special character. In Theorem 7, we construct an infinite subset of the set of finite subgroups with intersection that contains some points, such that every infinite subset of it has the same intersection. Theorem 8 describes a construction of an infinite subset of a set of finite subgroups with intersection contains a point of second order. Theorem 9 describes the centralizer of a point of second order if one more finiteness condition is valid for this point. Theorem 10 is about Sylow 2-subgroups of groups with point of second order.

Now, we recall some definitions, which we use frequently in this article.

A point a is called a *trivial point*, if the set of finite subgroups of G containing a is finite.

A group G is said to be *locally finite*, if any finite subset of G generates a finite subgroup.

A group G is called *Chernikov group*, if it is a finite group and a finite extension of direct product of a finite number of quasi-cyclic groups.

Let π' be the complement of the set of prime numbers π. The periodic group G is called a *π'–group*, if all the prime divisors of orders of non-unit elements of the group G belong to the set of π'.

An element of order two is called an *involution*.

A group G of the form $G = F\lambda H$ is called a *Frobenius group* with the kernel F and the complement H, if $H \cap H^g = 1$, for any $g \in G \setminus H$ and $F \setminus 1 = G \setminus \cup_{g\in G} H^g$, where H is a proper subgroup of G.

A group G is called *locally solvable*, if every finite set of its elements generates a solvable subgroup. A maximal normal p'-subgroup of G is denoted by $O_{p'}(G)$.

For the elements $a, b \in G$, the group G is carried out (a, b)*-finiteness condition*, if the subgroup $L_g = \langle a, b^g \rangle$ is finite, for almost all $g \in G$ (i.e., except may be finite number). An (a, b)-finiteness condition called *strong*, if L_g is a finite group for all $g \in G$.

An element a of G is called strictly real with respect to the involution i, if $iai = a^{-1}$.

A subgroup H of the group G is called *strongly embedded*, if H contains an involution and for any element $g \in G \setminus H$ there are no involutions in the subgroup $H \cap H^g$.

2 Examples of groups with points

Here, we give some examples of groups with different sets of points.

All finite groups are examples of groups, in which every element is considered as a point.

Novikov-Adian's group, Adian's periodic products of finite groups without involution [1] and Olshansky's periodic monster [11] are examples of groups, in which every non-unit element is a point.

Unit group and torsion-free group are groups with unique point.

Groups with a finite periodic part is a group, in which every element of finite order is a point.

Free product of a non-trivial finite group by any other non-trivial group is a group with infinite set of points.

Let $T_1, T_2, ..., T_n, ...$ be infinite sequence of finite Frobenius groups with the same complement H, where $T_n = F_n \lambda H,\ n = 1, 2, 3, ...$ Then, the free product G of groups in this sequence by the joined subgroup H is a group with a non-trivial point.

3 Some properties of groups with points

In this section, we study some properties of groups with points.

All the necessary known results are listed in Section 4 at the end of the article. We refer to these results with the appropriate numbers.

Lemma 3.1 *If a is a point of the group G, then a is a point of any subgroup of G, containing a.*

Proof. Let a be a point of the group G, H be an arbitrary subgroup of G containing a and L be a non-trivial finite subgroup of H. By the definition of a point, the set of finite subgroups of normalizer $N_G(L)$ containing a point a is finite as the set of finite subgroups of normalizer $N_H(L)$ containing a is also finite. Hence, the element a is a point of the subgroup H.

Proposition 3.2 *No group G can contain simultaneously an infinite set of finite subgroups with non-trivial intersection containing a point a and a non-trivial finite normal subgroup.*

Proof. Let the group G contain a non-trivial finite normal subgroup K and an infinite set of finite subgroups with non-trivial intersection L containing a point a. Then, the group G contains infinite number of elements of finite orders and $a \neq e$, by the definition of a point. As K is a normal subgroup of the group G, then $N_G(K) = G$ and the set of finite subgroups in $N_G(K)$ containing a is infinite. Thus a is not a point of the group G. This proves the proposition.

Proposition 3.3 *If a group G contains a point a, then for every element b of finite order of the normalizer $N_G(a)$, the intersection $N_G((a)) \cap C_G(b)$ has finite index in $N_G(a)$.*

Proof. By the way of contradiction, we assume that there is an element b of the normalizer $N_G(a)$, such that the index of the intersection $N_G((a)) \cap C_G(b)$ in the normalizer $N_G(a)$ is infinite. We then consider two cases: $a = e$ and $a \neq e$ in G.

If $a = e$, then $N_G((a)) = G$ and by the assumption $|G : C_G(b)| = \infty$. It means that the number of elements, conjugate with b in the group G is infinite. This is a contradiction to the definition of points.

Now we consider the second case and assume a is a non-identity element of G. By the assumption, the intersection $N_G((a)) \cap C_G(b)$ has infinite index in the normalizer $N_G(a)$. Then, the number of elements conjugated with b in the normalizer $N_G(a)$ is infinite. Hence, the normalizer $N_G(a)$ contains infinite number of finite subgroups of the form $< a, b^c >, c \in N_G(a)$, which contradicts Proposition 3.2. So, the result holds.

Proposition 3.4 *No group may have simultaneously an infinite set of finite subgroups containing a point a and a finite non-trivial invariant set of elements of finite orders.*

Proof. Let the group G have a finite non-trivial invariant set of elements of finite orders. By Ditsman's Lemma (see Theorem 1), this set generates a finite normal subgroup in G. However, the group G can not have infinite set of finite subgroups containing the point a, by Proposition 3.2.

Theorem 3.5 *Infinite Chernikov's group has no points.*

Proof. By the properties of Chernikov's groups, in infinite Chernikov's group, every element is contained in an infinite set of finite subgroups. As every infinite Chernikov's group has a finite normal subgroup, then the statement follows from Proposition 3.2.

The following lemma is already proved in [8].

Lemma 3.6 *Every group has no infinite locally finite subgroup containing a point a.*

Proposition 3.7 *Let a be a point of a group G, $\mathfrak{M}$ be an infinite set of finite subgroups of G and $a \in \cap_{H \in \mathfrak{M}} H$. Then, $\mathfrak{M}$ contains an infinite subset $\mathfrak{B}$ such that for any infinite subset $\mathfrak{U}$ of $\mathfrak{B}$, $\cap_{H \in \mathfrak{U}} H = \cap_{H \in \mathfrak{B}} H$.*

Proof. Let $T = \cap_{H \in \mathfrak{M}} H$ and assume that the claim is not true. Then, $\mathfrak{M}$ has an infinite subset $\mathfrak{M}_1$ with intersection $T_1 = \cap_{H \in \mathfrak{M}_1} H \neq T$, $\mathfrak{M}_1$ has a subset $\mathfrak{M}_2$ with intersection $T_2 = \cap_{H \in \mathfrak{M}_2} H \neq T_1$ and etc. As a result of such choices of subsets $\mathfrak{M}_n (n = 1, 2, \dots)$ from $\mathfrak{M}$, we obtain a strictly ascending chain of finite subgroups $T < T_1 < T_2 < ... < T_n < ....$

Clearly the union V of this chain is an infinite locally finite subgroup containing the point a, which contradicts Lemma 2.6. Hence, the chain breaks off after finite number of steps. This proves the result.

Theorem 3.8 *Any infinite set $\mathfrak{M}$ of finite subgroups of a group G with intersection $T = \cap_{H \in \mathfrak{M}} H$, where i is a point of the second order, almost all (for exception, may be, of finite number) consists of subgroups isomorphic to Frobenius groups with complements containing T or subgroups isomorphic to groups $Sz(Q), SL_2(Q)$, where Q is a field of characteristic two, $T = P\lambda(c)$ and P is some Sylow 2-subgroup of such subgroups.*

Proof. In view of Proposition 3.7 and without loss of generality, the statement is valid for $\mathfrak{M}$.

1) If $\mathfrak{B}$ is an infinite subset of $\mathfrak{M}$ such that $T = \cap_{H \in \mathfrak{B}} H$.

Assume that for some infinite subset $\mathfrak{N}$ of $\mathfrak{M}$ and for some (i)-invariant subgroup $K \neq 1$ of T we have $N_H(K) \not\leq T (H \in \mathfrak{N})$.

The set $\{N_H(K) | H \in \mathfrak{N}\}$ can not be infinite, as in this case we come to the contradiction of conditions $K \neq 1$, $i \in N_H(K)$ and the involution i is a point of G. Hence, $\{N_H(K) | H \in \mathfrak{N}\}$ is finite and by statement 1) $\mathfrak{N}$ has such infinite subset $\mathfrak{U}$, that $N_H(K) \leq T (H \in \mathfrak{U})$ contrary to the definition of the set $\mathfrak{N}$. The contradiction means, that the condition $N_H(K) \not\leq T$ can be only valid for finite number of subgroups $H \in \mathfrak{M}$. Therefore without loss of generality, we may suppose that

2) $N_H(K) \leq T \neq H$, for any non-trivial (i)-invariant subgroup K of T and any subgroup H of $\mathfrak{M}$.

Let M be some subgroup of $\mathfrak{M}$ and $O_{2'}(M) \neq 1$. Then, we are able to prove that

3) M is a Frobenius group with complement $C_M(i)$, containing T.

Let R be a nilpotent radical of $O_{2'}(M)$, then by Theorem 3.7, $R \neq 1$. If $T \cap R \neq 1$, then using the normalizer condition for nilpotent groups ([9], Theorem 17.1.4) and statement 2), we show that $R \leq T$ and $M \leq T$ contrary to the condition $T \neq M$ from statement 2). Hence, $T \cap R = 1$ and, in particular, $C_M(i) \cap R = 1$. If $C_M(R)$ has an involution k. Clearly, R can be chosen so that $k \in C_G(i)$. Now by statement 2), $R < C_M(K) \leq T$ and we obtain a contradiction to the above, $R \cap T = 1$. From here we have, that $C_M(R)$ does not contain any involutions, and as $C_M(R) \triangleleft M$, that $C_M(R) \leq O_{2'}(M)$. Furthermore, in view of Theorems 11 and 12, $C_M(R) = R$ and $M = RC_M(i)$. Using this and statement 2), it obviously follows that $C_M(i)$ is a complement of Frobenius group M. Hence the statement 3) is proved.

Now we show that:

4) If H is a subgroup of $\mathfrak{M}$, then all involutions of T are conjugate with i in H.

Let j be an involution from T. If $V = \langle \{j\}^H \rangle \leq T$, then by statement 2) $H \leq N_H(V) \leq T$ and $T = H$, but this is impossible, as in view of statement 2) $T \neq H$. Hence, $k = j^g \notin T$ for some element $g \in H$. If the element ik has even order, then by Theorem 13 and statement 2), it follows that $k \in T$ contrary to the above that $k \notin T$. This contradiction means that the element ik has odd order and so by Theorem 13, i and $k = j^g$ are conjugate in H. Hence i and j are conjugate in H and thus the statement 4) is proved.

Finally, we shall prove that:

5) If $H \in \mathfrak{M}$, then T is strongly embedded subgroup in H. By statement 4), every involution of T is a point and therefore statement 2) is valid for every involution of T. Using this remark, it is easy to show that if, for some $g \in H$, the intersection $T \cap T^g$ contains an involution, then it contains also some Sylow 2-subgroup S of T. Then in view of Sylow Theorem [9] $tg \in N_H(S)$, where t is some element of T. By the above remark and statement 2), $tg \in N_G(S) \leq T$ and $g \in T$. So, the statement 3) is stablished.

Now, having applied the statements 2) – 5) and Theorems 14 and 15 to every subgroup of the set $\mathfrak{M}$, we obtain the following theorem.

Theorem 3.9 *Let G be a group with infinite set of elements of finite orders and i its point of the second order satisfying (i,i)-finiteness condition. Then, $H = C_G(i)$ is a strongly embedding subgroup in G and H has a finite periodic part that is not contained in any larger subgroup with such a property.*

Proof. By Proposition 3.4, $C_G(i)$ has finite periodic part and $|G : C_G(i)|$ is infinite. The set $\mathfrak{M}$ of all subgroups with periodic part containing $C_G(i)$ is partially ordered and obviously, the union of any chain of $\mathfrak{M}$ belongs to it. By Zorn's Lemma, $\mathfrak{M}$ has a maximal element, i.e., there exists a subgroup H of $\mathfrak{M}$ which is not contained in any larger subgroup of $\mathfrak{M}$. Let V be a periodic part of H. As $i \in V$, Proposition 3.7 implies that V is a finite subgroup. It is obvious that V is normal in H and V is automorphic permissible in H. In view of maximality of H in $\mathfrak{M}$, it follows that $N_G(V) = N_G(H) = H$.

Take an involution $k \in V$. If $\langle \{k^g | g \in G\} \rangle \leq V$, then we would obviously arrive to a contradiction with the definition of point i and (i,i)-finiteness condition. Hence, for some $c \in G$, the involution $t = k^c \notin H$. Now, we consider the dihedral subgroup $L = \langle i, t \rangle$ and assume that L is not a finite Frobenius group with complement (i) and kernel (d), where $d = it$. In this case $|d| = \infty$, or $|d|$ is even. The case $|d| = \infty$ is impossible in view of (i,i)-finiteness condition and Theorem 13. If $|d|$ is even, then by Theorem 13, (d) contains an involution j where $j \in C_G(i) \cap C_G(t)$. Obviously, $|H : C_G(j) \cap H|$ is finite and as i is a point and (i,i)-finiteness condition is valid in $C_G(j)$, then $C_G(j)$ has a finite periodic part R (using Theorem 16 and Proposition 3.4). The intersection $H \cap C_G(j)$ contains such subgroup X, so that $|H : X| < \infty$, $X \triangleleft H$ and $V, R < C_G(X) \leq N_G(X)$. But $t \in R$, and therefore $t \in N_G(X)$. On the other hand, $t \notin H$ and $H < N_G(X)$.

Hence, $H \neq N_G(X)$ and in view of the definition of H a subgroup $M = N_G(X)$ has no periodic part. Furthermore, $X \leq C_G(i) \leq H$, $|H : X| < \infty$ and X has a finite periodic part. But then $i \in X$ would mean that $|M : C_G(i)| < \infty$ and hence M would have a finite periodic part which contradicts the above. Hence, $i \notin X$ and obviously in $\bar{M} = M/X$ the centralizer $C_{\bar{M}}(iX)$ is finite and (iX, iX)-

finiteness condition is valid. By Theorem 18, $\bar{M}$ is a locally finite group. Now, as $H/X \leq \bar{M}$ and H/X is a finite subgroup of $\bar{M}$, H/X is contained in a larger finite subgroup K/X of $\bar{M}$, where K is a subgroup of M and $X < H < K$. Obviously, $|K : C_G(i)| < \infty$, means that K has a finite periodic part. But $K \neq H$ and $H < K$. Hence, we obtain a contradiction to the definition of the subgroup H. This contradiction means that d is an element of odd order and the involutions i and k are conjugate in G (Theorem 13), so k and i are also conjugate in G.

Now we prove that H is a strongly embedding subgroup in G and we assume that it is not so. Then $H \neq H^g$, for some $g \in G$ and $H \cap H^g$ has an involution k. As it is proved above, k is a point of G and, besides, $|H : C_G(k) \cap H|$, $|H^g : C_G(k) \cap H^g|$ are finite. Again as proved above, $C_G(k) \leq H \cap H^g$ and $H = H^g$, i.e. $g \in N_G(H) = H$. Hence, H is a strongly embedding subgroup in G. If H has more than one involution, then by Theorem 17 and in view of (i,i)-finiteness condition in H there would be a non-unit element c of finite order, strictly real concerning to some involution $j \in G \setminus H$. By Theorem 17, i and j are conjugate in G and therefore j is a point. Now consider a subgroup $M = C_G(c)\lambda(j)$. As j is a point of M and M is satisfied to (j,j)-finiteness condition, then by Proposition 3.4, M has a finite periodic part. It is obvious that $|H : M \cap H| < \infty$ and as proved above, we obtain a contradiction to $j \notin M$. Hence, H has a unique involution and so the theorem is proved.

Theorem 3.10 *Let G be a group with infinite set of elements of finite orders and i be its point of the second order satisfying (i,i)-finiteness condition. Then, all Sylow 2-subgroups of $G = C_G(i)$ are cyclic or generalized quaternion groups.*

Proof. By Theorem 3.9 and Theorem 10, it follows that all Sylow 2-subgroups of H are cyclic or generalized quaternion groups. By Theorem 17, they are also Sylow subgroups in G, so they are conjugate in G. This completes the proof.

4 Known results

In this final section, we have collected some known results, which were used in proving our results and we referred to them as theorems with their appropriate numbers.

1. **Ditsman's Lemma.** Let M be a finite invariant set of elements of finite orders in a group, then the subgroup generated by this set is finite [10].

2. **Remak's Theorem**. Let G be a group, $H_i, i \in I$, be its normal subgroups and H be their intersection. Then the factor-group G/H is isomorphic to some sub cartesian product of the factor-groups G/H_i [9].

3. **Feit-Thompson Theorem**. Any finite group of odd order is solvable [5].

4. Let H be a periodic locally solvable group and k an element of prime order p of H such that $C_G(k)$ is finite. Then all Sylow p-subgroups of H are Chernikov groups [16].

5. Let H be a periodic locally solvable group with Chernikov Sylow p-subgroups for some $p \in \pi(H)$. Then $H/O_{p'}(H)$ is a Chernikov group [4].

6. **Blackburn Theorem**. If G is a locally finite p-group and the centralizer of some finite subgroup of G is a Chernikov group, then G is also a Chernikov group [3].

7. **Higman-Thompson Theorem.** Any finite group with regular automorphism of the prime order p is a nilpotent group. The length of its upper central series is also terminated after a finite number of steps, which only depends on p [7, 17].

8. Subgroups of a Chernikov group are Chernikov [9].

9. Extension of Chernikov group by a Chernikov group is also a Chernikov group [15].

10. A 2-group with only one involution is either a locally cyclic group (cyclic or quasi-cyclic), or a generalized quaternion group (finite or infinite) [16].

11. Let G be a finite group and H be its subgroup with $H \cap H^g = 1$ (for all $g \in G \setminus H$). Then

a) $G = F\lambda H$, where $F \setminus 1 = G \setminus \cup_{g\in G} H^g$ (Frobenius Theorem);

b) $(|F|, |H|) = 1$;

c) Sylow p-subgroups of H are cyclic or generalized quaternion groups;

d) If H has involution i, then $H = C_G(i)$, F is an abelian subgroup and $i = f^{-1} (f \in F)$;

e) If H has odd order, then all elements of prime orders of H generate a cyclic subgroup;

f) F is a nilpotent subgroup (Thompson Theorem);

g) If $p \in \pi(H)$, then the nilpotent length of subgroups F is only limited to a number depends on p (Higman Theorem);

h) If $h \in H$ and $f \in F$, then the elements h, fh are conjugate by some element of F [13].

12. let G be a finite solvable group and L its nilpotent radical. Then $C_G(L) < L$ [2].

13. Let $G = \langle i, k\rangle$ and i, k be involutions of G. Then a) $G = (c)\lambda(i) = (c)\lambda(k)$, where $c = ik$; b) $i^{-1}ci = ici = c^{-1}$, $k^{-1}ck = kck = c^{-1}$; c) i, ic^{2m} (or k, kc^{2m}) are conjugate in G, where m is an integer; d) if c is an element of odd order, then i and k are conjugate in G; e) if c is an element of even order and t is an involution of $\langle c\rangle$, then G is an elementary Abelian group of 4-th order or $Z(G) = \langle t\rangle$ [15].

14. **Bender Theorem**. Let G be a finite group and H be its strongly embedded subgroup. Then $G/O_{2'}(G) = T$ has a unique involution or normal subgroup of an odd index in T, which is isomorphic to one of the groups of type $SL(2, Q)$, $S_Z(Q)$ or $PSU(3, Q)$, where Q is a finite field of characteristic two [2].

15. Let $G \simeq PSU(3, Q)$, where Q is a finite field of characteristic two, S be a Sylow 2-subgroup of G and $H = N_G(S)$. Then H is a strongly embedded subgroup in G and H has a non-trivial element b such that $C_G(b) \not\leq H$ and $C_G(b) \cap S \neq 1$ [2].

16. If some involution $i \in G$ satisfies the (i, i)-finiteness condition, then every involution $k \in G$ is carried out strong (k, i)-finiteness condition [13].

17. Let G be a group, H be its strongly embedded subgroup, and i be

an involution of H satisfying the condition that for almost all elements $g^{-1}ig$ ($g \in G \setminus H$), the subgroups $\langle i^g \rangle$ are finite, then

a) if k is an involution of $G \setminus H$, then $|ki|$ is finite and odd number;

b) all involutions of H are conjugate in H;

c) all involutions of G are conjugate in G;

d) any element g of $G \setminus H$ has the form $g = h_g j_g$, where $h_g \in H$ and j_g is an involution of $G \setminus H$;

e) for every involution j of $G \setminus H$, the set of elements of H strictly to j, have the same power as the set of involutions in H [13].

18. Let G be a group and i an involution of it with finite centralizer $C_G(i)$. If G satisfies the (i,i)-finiteness condition, then G is a locally finite and almost solvable group [13].

19. If G has a locally finite group containing an element with finite centralizer, then G has locally soluble normal subgroup of finite index [6].

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