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Application of modified simple equation method to Burgers, Huxley and Burgers-Huxley equations

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Abstract

In this paper, modified simple equation method has been applied to obtain generalized solutions of Burgers, Huxley equations and combined forms of these equations. The new exact solutions of these equations have been obtained. It has been shown that the proposed method provides a very effective, and powerful mathematical tool for solving nonlinear partial differential equations.

Keywords: Modified simple equation method; Burgers equation; Huxley equation; Burger-Huxley equation.

1 Introduction

Mathematical modeling of many real phenomena leads to a non-linear partial differential equations in various fields of sciences and engineering. Many powerful methods have been presented for solving PDEs so far, such as tanh-function method [19] and [28], sine-cosine method [29], Homotopy Analysis method [17], Homotopy perturbation method [6], variational iteration method [9] and [10], Adomian decomposition method [1], Exp-function method [1], [11], [36] and [37], simplest equation method [7] and [4], and many others. Most recently, a modification of simplest equation method (MSE method) has been developed to obtain solutions of various nonlinear

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evolution equations [14], [15], [21], [31], [32], [33] and [34]. The present paper is motivated by the desire to extend the MSE method to obtain generalized solutions of Burgers, Huxley, and Burgers-Huxley. The procedure of this method, by the help of Matlab, Maple or any mathematical package, is of utter simplicity.

2 The MSE method

Consider a nonlinear partial equation in two independent variables, say x and t , in the form of

$$P(u, u_t, u_x, u_{tt}, u_{xx}, ...) = 0, \tag{1}$$

where $u = u(x,t)$ an unknown function, P is a polynomial in $u = u(x,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. This method consists of the following steps.

Step 1. Using the transformation

$$\xi = x + wt, \tag{2}$$

where w is constant, we can rewrite equation (1) as a following nonlinear ODE:

$$Q(u, u', u'', ...) = 0. \tag{3}$$

Where the superscripts denote the derivatives with respect to ξ .

Step 2. Suppose that the solution of equation (2) can be expressed as follows

$$u(\xi) = \sum_{i=0}^{m} a_i (\frac{F'(\xi)}{F(\xi)})^i. \tag{4}$$

Where a_i are constants to be determined later, with $a_m \neq 0$ and $F(\xi)$ is an unknown function to be determined later.

Step 3. The positive integer m can be determined by considering the homogeneous balance of the highest order derivatives and highest order nonlinear appearing in equation (2).

Step 4. Calculating all necessary derivatives $u', u'', u''', ...$, and substituting equation (3) into equation (2) yields a polynomial of $\frac{F'(\xi)}{F(\xi)}$ and its derivatives. Equating the coefficients of same power of $F^{-i}(\xi)$ to zero gives a system of equations which can be used to solve for determining unknown constants,$F(\xi)$

and $F'(\xi)$. By substituting obtained results into equation (3), solutions of the equation (1) can be obtained.

3 Application of the MSE method

In this section, the modified simple equation method has been applied to obtain generalized solutions of Burgers, Huxley, and Burgers-Huxley.

3.1 Application MSE method to Burgers equation

The Burgers equation is a nonlinear partial differential equation of second order of the form

$$u_t + uu_x = \nu u_{xx}. \tag{5}$$

Where ν is the viscosity coefficient [2], [22], [23] and [27]. Many problems can be modeled by the Burgers‘ equation. This equation is one of the very few nonlinear partial differential equations which can be solved exactly for the restricted set of initial function. The study of the general properties of the equation has drawn considerable attention due to its place of application in some fields such as gas dynamics, heat conduction, elastically, etc.
To apply MSE method on equation (4), lets introduce a variable ξ , defined as

$$\xi = x - wt. \tag{6}$$

So, equation (4) turns to the following system of ordinary different equation,

$$-wu' + uu' = \nu u''. \tag{7}$$

Where w is constant to be determined. Integrating (7) and considering the integral constant to be zero, we obtain

$$-wu + \frac{1}{2}u^2 = \nu u'. \tag{8}$$

Suppose that the solution of ODE equation (8) can be expressed by a polynomial in $\frac{F'}{F}$ as shown in (3). Balancing the terms u^2 and u' in equation (8), yields to $m = 1$. So we can write (3) as the following simple form

$$u(\xi) = a_1 \frac{F(\xi)}{F(\xi)} + a_0, a_1 \neq 0. \tag{9}$$

So

$$u' = a_1(\frac{F''}{F} - (\frac{F'}{F})^2). \tag{10}$$

Substituting (9) and (10) into equation (8) and equating each coefficient of $F^{-i}(\xi)$, $(i = 0, 1, 2)$ to zero, we derive

$$-wa_0 + \frac{1}{2}a_0^2 = 0, \tag{11}$$

$$(-w + a_0)F' - \nu F'' = 0, \tag{12}$$

$$(\frac{1}{2}a_1^2 + \nu a_1)(F')^2 = 0. \tag{13}$$

By solving equations (11) and (13), the following results will be obtained

$$a_0 = 0, 2w, a_1 = -2\nu.$$

Case 1. when equation (12) turns to

$$wF' + \nu F'' = 0.$$

So

$$F' = Ae^{\frac{-w}{\nu}\xi}. \tag{14}$$

Where A is a arbitrary constant. Integrating (13) with respect ξ, $F(\xi)$ will be obtained as fallows

$$F = \frac{-A\nu}{w}e^{\frac{-w}{\nu}\xi} + B,$$

where B is a constant of integration. Now, the exact solution of equation (4) has the form

$$u_1(x,t) = \frac{-2\nu Ae^{\frac{-w}{\nu}(x-wt)}}{\frac{-A\nu}{w}e^{\frac{-w}{\nu}(x-wt)} + B}.$$

Case 2. when $a_0 = 2w$, equation (12) yields to

$$wF' - \nu F'' = 0.$$

So

$$F' = Ae^{\frac{w}{\nu}\xi},$$

and

$$F = \frac{A\nu}{w}e^{\frac{w}{\nu}\xi} + B.$$

Now, the exact solution of equation (4) has the form

$$u_2(x,t) = 2w - \frac{2w\nu A e^{\frac{w}{\nu}(x-wt)}}{A\nu e^{\frac{w}{\nu}(x-wt)} + wB} = \frac{2w^2 B}{A\nu e^{\frac{w}{\nu}(x-wt)} + wB}.$$

Note that all obtained solutions have been checked with maple 13 by putting into the original equation and found correct.

3.2 Application MSE method to Huxley equation

Now we will bring to bear the MSE method to obtain exact solution to the Huxley equation [3], [7], [8], [12], [13], [18], [25] and [26] in the following form

$$u_t = u_{xx} + u(k-u)(u-1). \tag{15}$$

The Huxley equation is an evolution equation that describes the nerve propagation in biology from which molecular CB properties can be calculated. It also gives a phenomenological description of the behaviour of the myosin heads II. This equation has many fascinating phenomena such as bursting oscillation [3], interspike [18], bifurcation, and chaos [35]. A generalized exact solution can gain an insight into these phenomena. There is no universal method for nonlinear equations. In this part, the exact solution will be obtained by the MSE method.

By considering (6), equation (15) turns to the following ordinary differential equation,

$$wu' + u'' + u(k-u)(u-1) = 0. \tag{16}$$

Balancing the terms u'' and u^3 in equation (16), yields to $m = 1$. So we can rewrite (3) as the following simple form

$$u(\xi) = a_1 \frac{F(\xi)}{F(\xi)} + a_0, a_1 \neq 0. \tag{17}$$

Now by substituting 19) into equation (16) and equating each coefficient of $F^{-i}(\xi)$,$(i = 0,1,2,3)$ to zero, the following result will be obtained

$$a_0^3 - (k+1)a_0^2 + ka_0 = 0, \tag{18}$$

$$a_1 F''' + wa_1 F'' + (2(k+1)a_0 a_1 - 3a_0^2 a_1 - ka_1)F' = 0, \tag{19}$$

$$-3a_1 F' F'' + ((k+1)a_1 - wa_1 - 3a_0 a_1^2)(F')^2 = 0, \tag{20}$$

$$(2a_1 - a_1^3)(F')^3 = 0. \tag{21}$$

Solving equations (15) and (22), we drive

$$a_0 = 0, 1, k,$$

$$a_1 = \pm\sqrt{2}.$$

Case 1. when $a_0 = 0$, equations (16) and (17) yield

$$F''' + wF'' - kF' = 0, \tag{22}$$

$$3F'' + (w - k - 1)F' = 0. \tag{23}$$

By substituting equation (23) into (22), we obtain

$$F''' + (w + \frac{3k}{w-k-1})F'' = 0.$$

So

$$F'' = Ae^{-\alpha\xi}. \tag{24}$$

Where $\alpha = w + \frac{3k}{w-k-1}$ and A is a arbitrary constant. Therefore, we have

$$F' = -\frac{A}{\alpha}e^{-\alpha\xi} + B. \tag{25}$$

where A and B are arbitrary constants. By substituting (25) into equations (22) and (23), we get

$$w = \frac{k+1}{4} \pm \frac{3}{4}\sqrt{k^2 - 6k + 1}, B = 0.$$

Thus, (25) can be rewritten as follows

$$F' = -\frac{A}{\alpha}e^{-\alpha\xi}. \tag{26}$$

Integrating (26) with respect ξ , $F(\xi)$ will be obtained as follows

$$F = \frac{A}{\alpha^2}e^{-\alpha\xi} + C,$$

where C is a constant of integration. Substituting the value of F and F' into equation (19), the following exact solution of equation (16) has been obtained

$$u_1(x,t) = \frac{\pm\sqrt{2}A\alpha e^{-\alpha\xi}}{Ae^{-\alpha\xi} + \alpha^2 C}.$$

Case 2. when $a_0 = 1$, equations (16) and (17) turns to

$$F''' + wF'' + (k-1)F' = 0, \tag{27}$$

$$3F'' + ((w - k - 1 \pm 3\sqrt{2})F' = 0. \tag{28}$$

By substituting equation (8) into (27), we obtain

$$F''' + (w - \frac{3(k-1)}{w-k-1 \pm 3\sqrt{2}})F'' = 0.$$

So

$$F'' = Ae^{-\alpha\xi}, \tag{29}$$

where $\alpha = w - \frac{3(k-1)}{w-k-1\pm3\sqrt{2}}$ and A is a arbitrary constant. Therefore, we have

$$F' = -\frac{A}{\alpha}e^{-\alpha\xi} + B, \tag{30}$$

where A and B are arbitrary constants. By substituting (30) into equations (27) and (28), we get

$$w = \frac{k+1}{4} - \frac{3}{4}\sqrt{2} \pm \frac{3}{4}\sqrt{k^2 - 6k - 6k\sqrt{2} + 27 - 6\sqrt{2}}, B = 0,$$

or

$$w = \frac{k+1}{4} + \frac{3}{4}\sqrt{2} \pm \frac{3}{4}\sqrt{k^2 - 6k + 6k\sqrt{2} + 27 + 6\sqrt{2}}, B = 0.$$

Thus, (30) can be rewritten as follows

$$F' = -\frac{A}{\alpha}e^{-\alpha\xi}. \tag{31}$$

Integrating (31) with respect , will be obtained as follows

$$F = \frac{A}{\alpha^2}e^{-\alpha\xi} + C,$$

where C is a constant of integration. Substituting the value of F and F' into equation (19), the following exact solution of equation (16) has been obtained

$$u_2(x,t) = 1 + \frac{\pm\sqrt{2}A\alpha e^{-\alpha\xi}}{Ae^{-\alpha\xi} + \alpha^2 C}.$$

Case 3. when $a_0 = k$, equations (16) and (17) turn to

$$F''' + wF'' + k(1-k)F' = 0,$$

$$3F'' + ((w - 1 + k(-1 \pm 3\sqrt{2}))F' = 0.$$

By using the same method applied in case 1, the following solution will be obtained

$$u_3(x,t) = k + \frac{\pm\sqrt{2}A\alpha e^{-\alpha\xi}}{Ae^{-\alpha\xi} + \alpha^2 C},$$

where

$$\alpha = w - \frac{3k(1-k)}{w - 1 + k(-1 \pm 3\sqrt{2})},$$

and

$$w = \frac{1 - k(-1 \pm 3\sqrt{2})}{4} \pm \frac{3}{4}\sqrt{1 - 2k(-1 \pm 3\sqrt{2}) + k(-1 \pm 3\sqrt{2})^2 - 8k^2 + 8k}.$$

3.3 Application MSE method to Burgers-Huxley equation

The analysis presented in this part is based on the generalized nonlinear Burgers-Huxley equation,

$$u_t = u_{xx} + uu_x + u(k-u)(u-1), \tag{32}$$

which models the interaction between reaction mechanisms, convection effects and diffusion transports [20], and some special cases of the equation, which usually appear in mathematical modelling of some real world phenomena. It also gives a phenomenological description of the behaviour of the myosin heads II [30] and Fitzhugh-Nagoma equation, an important nonlinear reaction-diffusion equation used in circuit theory, biology and population genetics [5].

By considering (6), equation (32) turns to the following ordinary differential equation,

$$wu' + u'' + uu' + u(k-u)(u-1) = 0. \tag{33}$$

Balancing the terms u'' and u^3 in Eq. (33), yields to $m = 1$. So we can rewrite (3) as the following simple form

$$u(\xi) = a_1(\frac{F'}{F}) + a_0, a_1 \neq 0. \tag{34}$$

Now by substituting (34) into equation (33) and equating each coefficient of $F^{-i}(\xi)$, $(i = 0, 1, 2, 3)$ to zero, the following result will be obtained

$$a_0^3 - (k+1)a_0^2 + ka_0 = 0, \tag{35}$$

$$a_1F''' + (wa_1 + a_0a_1)F'' + (2(k+1)a_0a_1 - 3a_0^2a_1 - ka_1)F' = 0, \tag{36}$$

$$(-3a_1 + a_1^2)F'F'' + ((k+1)a_1 - wa_1 - a_0a_1 - 3a_0a_1^2)(F')^2 = 0, \tag{37}$$

$$(2a_1 - a_1^2 - a_1^3)(F')^3 = 0. \tag{38}$$

Solving equation (35) and (38), we get

$$a_0 = 0, 1, k,$$

$$a_1 = 1, -2.$$

Case 1. when $a_0 = 0$ and $a_1 = 1$ equations (36) and (37) yield

$$F''' + wF'' - kF' = 0, \tag{39}$$

$$2F'' + (w - k - 1)F' = 0. \tag{40}$$

By substituting equation (40) into (39), we obtain

$$F''' + (w + \frac{2k}{w - k - 1})F'' = 0.$$

So

$$F'' = Ae^{-\alpha\xi} \tag{41}$$

where $\alpha = w + \frac{2k)}{w-k-1}$ and A is a arbitrary constant. Therefore, we have

$$F' = -\frac{A}{\alpha}e^{-\alpha\xi} + B, \tag{42}$$

where A and B are arbitrary constants. By substituting (42) into equations (39) and (40), we get

$$w = \pm(k - 1), B = 0,$$

So

$$\alpha = -1, -k.$$

Thus, (42) can be rewritten as follows

$$F' = -\frac{A}{\alpha}e^{-\alpha\xi}. \tag{43}$$

Integrating (43) with respect ξ , $F(\xi)$ will be obtained as follows

$$F' = \frac{A}{\alpha^2}e^{-\alpha\xi} + C,$$

where C is a constant of integration. Substituting the value of F and F' into equation (34), the following exact solutions of equation (33) has been obtained

$$u_1(x, t) = \frac{Ae^{x-(k-1)t}}{Ae^{x-(k-1)t} + C},$$

$$u_2(x, t) = \frac{AKe^{k(x+(k-1)t)}}{Ae^{k(x+(k-1)t)} + k^2C}.$$

Case 2. when $a_0 = 0$, and $a_1 = -2$ equations (36) and (37) yield

$$F''' + wF'' - kF' = 0, \tag{44}$$

$$-5F'' + (w - k - 1)F' = 0. \tag{45}$$

By substituting equation (45) into (44), we obtain

$$F''' + (w + \frac{5k)}{w - k - 1})F'' = 0.$$

By using the same method applied in case 1, the following solution will be obtained

$$u_{3,4}(x,t) = \frac{2A\alpha e^{-\alpha\xi}}{Ae^{-\alpha\xi} + \alpha^2 C},$$

where

$$\alpha = w + \frac{5k}{w - k - 1},$$

and

$$w = \frac{3}{8}(k+1) \pm \frac{5}{8}\sqrt{k^2 - 14k + 1}.$$

Case 3. when $a_0 = 1$ and $a_1 = 1$, equations (31) and (32) turn to

$$F''' + (w + 1)F'' + (k - 1)F' = 0, \tag{46}$$

$$2F'' + (w + 3 - k)F' = 0. \tag{47}$$

By substituting equation (47) into (46), we obtain

$$F''' + (w + 1 - \frac{2(k-1)}{w + 3 - k})F'' = 0.$$

So

$$F'' = Ae^{-\alpha\xi},$$

where $\alpha = w + 1 - \frac{2(k-1)}{w+3-k}$ and A is an arbitrary constant. Therefore, we have

$$F' = -\frac{A}{\alpha}e^{-\alpha\xi} + B, \tag{48}$$

where A and B are arbitrary constants.
By substituting (48) into equations (46) and (47), we get

$$w = -1 \pm k, B = 0.$$

Thus, the following exact solutions of equation (33) has been obtained.

$$u_5(x,t) = 1 - \frac{Ae^{-x+(1-k)t}}{Ae^{-x+(1-k)t} + C},$$

$$u_6(x,t) = 1 - \frac{A(1-k)e^{-(1-k)(x+(1+k)t)}}{Ae^{-(1-k)(x+(1+k)t)} + (1-k)^2 C}.$$

Case 4. when $a_0 = 1$ and $a_1 = -2$, by using the same method applied in case 1, the following solution will be obtained.

$$u_{7,8}(x,t) = 1 + \frac{2A\alpha e^{-\alpha\xi}}{Ae^{-\alpha\xi} + \alpha^2 C}.$$

Where

$$\alpha = w + 1 - \frac{5(k-1)}{w-6-k},$$

and

$$w = \frac{13+3k}{8}(k+1) \pm \frac{5}{8}\sqrt{k^2+30k+33}.$$

Case 5. when $a_0 = k$ and $a_1 = 1$, equations (31) and (32) turn to

$$F''' + (w+k)F'' + k(1-k)F' = 0, \quad (49)$$

$$2F'' + (w+3k-1)F' = 0. \quad (50)$$

By substituting equation (50) into (49), we obtain

$$F''' + (w + k - \frac{2k(1-k)}{w+3k-1})F'' = 0.$$

So

$$F'' = Ae^{-\alpha\xi},$$

where $\alpha = w + k - \frac{2k(1-k)}{w+3k-1}$ and A is an arbitrary constant. Therefore, we have

$$F' = -\frac{A}{\alpha}e^{-\alpha\xi} + B, \quad (51)$$

where A and B are arbitrary constants. By substituting (51) into equations (49) and (50), we get

$$w = -k \pm 1, B = 0.$$

Thus, the following exact solutions of equation (32) has been obtained

$$u_9(x,t) = k - \frac{Ake^{-k(x+(k-1)t)}}{Ae^{-k(x+(k-1)t)} + k^2 C},$$

$$u_{10}(x,t) = k - \frac{A(k-1)e^{-(k-1)(x+(k+1)t)}}{Ae^{-(k-1)(x+(k+1)t)} + (k-1)^2C}.$$

Case 6. when $a_0 = k$ and $a_1 = -2$, by using the same method applied in case 5, the following solution will be obtained.

$$u_{11,12}(x,t) = k + \frac{2A\alpha e^{-\alpha\xi}}{Ae^{-\alpha\xi} + \alpha^2 C}.$$

Where

$$\alpha = w + k - \frac{5k(1-k)}{w-1-6k},$$

and

$$w = \frac{13+3k}{8}(k+1) \pm \frac{5}{8}\sqrt{33k^2+30k+1}.$$

4 Conclusion

In this paper, modified simple equation method has been applied to obtain the generalized solutions of some nonlinear partial differential equation. The results show that modified simple equation method is a powerful tool for obtaining the exact solutions of nonlinear differential equations. It may be concluded that, the method can be easily extended to all kinds of nonlinear equations. The advantage of this method over other methods is that in most methods applied for the exact solution of partial differential equations such as Exp-function method, $\frac{G'}{G}$ - expansion method, tanh-function method, and so on, the solution is presented in terms of some pre-defined functions, but in the MSE method, $F(\xi)$ is not pre-defined or not a solution of any pre-defined equation. Therefore, some new solutions might be found by this method.The computations associated in this work were performed by Maple 13.

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کاربرد روش معادله ساده توسعه یافته برای معادلات برگرز ، هوکسلی و هوکسلی برگرز

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چکیده : در این مقاله، روش معادله ساده توسعه یافته برای به دست آوردن جواب‌های معادلات برگرز، هوکسلی و شکل ترکیب شده آنها به کار رفته است. جوابهای دقیق جدیدی از این معادلات به دست آمده است.نشان داده شده است که روش ارائه شده یک ابزار ریاضی قوی و بسیار موثر برای حل معادلات با مشتقات جزئی می باشد.

کلمات کلیدی : روش معادله ساده توسعه یافته؛ معادله برگرز؛ معادله هوکسلی؛ معادله برگرز–هوکسلی.