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Approximate Analytical Solution for Quadratic Riccati Differential Equation

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Abstract

In this paper, we introduce an efficient method for solving the quadratic Riccati differential equation. In this technique, combination of Laplace transform and new homotopy perturbation methods (LTNHPM) are considered as an algorithm to the exact solution of the nonlinear Riccati equation. Unlike the previous approach for this problem, so-called NHPM, the present method, does not need the initial approximation to be defined as a power series. Four examples in different cases are given to demonstrate simplicity and efficiency of the proposed method.

Keywords: Riccati differential equation; Laplace transform method; NHPM; LTNHPM.

1 Introduction

The quadraic Riccati differential equation, deriving its name from Jacopo Francesco, Count Riccati (1676-1754). These kinds of differential equations are a class of nonlinear differential equations of much importance, and play a significant role in many fields of applied science [1]. At an early stage, the occurrence of such differential equations in the study of Bessel functions led to its appearance in many related applications and, to the present time, the literature on the Riccati equation has been extensive. For several reasons, a Riccati equation comprises of a highly significant class of nonlinear ordinary differential equations. Firstly, this equation is closely related to ordinary linear homogeneous differential equation of the second order. Secondly, the solution of Riccati equation possesses a very particular structure in that the general solution is a fractional linear function of the constant of integration. Thirdly, the solution of an Ricaati equation is involved in the reduction of nth-order linear homogeneous ordinary differential equations. Fourthly, a one-dimensional Schrodinger equation is closely related to Riccati differential

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equation. Solitary wave solutions of a nonlinear partial differential equation can be expressed as a polynomial in two elementary functions satisfying a projective Riccati equation [3]. Moreover, such types of problems also arise in the optimal control literature, and can be derived in solving a second-order linear ordinary differential equation with constant coefficients [2]. In conformity with the general study of differential equations, much of the early works were concerned with the study of particular classes of Riccati differential equation with the aim of determining the solution in finite form. Many mathematicians such as Jame Bernouli, Jahn Bernouli (1667-1748), Leonhard Euler (1707-1783), Jean-le-Rondd'Alembert(1717-1783), and Adrian Marine Legendre (1752-1833) contributed to the study of such differential equations [1]. Deriving analytical solution for Riccati equation in an explicit form seems to be unlikely except for certain special situations. Of course, having known its one particular solution, its general solution can be easily derived. Therefore, one has to go for numerical techniques or approximate approaches for getting its solution. Recently, Adomian's decomposition method has been proposed for solving Riccati differential equations [7, 8]. HPM was introduced by He [4] and has been already used by many mathematicians and engineers to solve various functional equations [9, 10, 5, 6, 12, 11]. Abbasbandy solved a Riccati differential equation using He's VIM, homotopy perturbation method (HPM) and iterated He's HPM and compared the accuracy of the obtained solution to that derived by Adomian decomposition method [13, 14, 16]. Moreover, Homotopy analysis method (HAM) and a piecewise variational iteration method (VIM) are proposed for solving Riccati differential equations [16]. In [15], Liao has shown that HPM equations are equivalent to HAM equations when $\hbar = -1$. Aminikhah and Hemmatnezhad [6] proposed a new homotopy perturbation method (NHPM) to obtain the approximate solution of ordinary Riccati differential equation. In this work, we present the solution of Riccati equation by combination of placeLaplace transform and new homotopy perturbation methods. An important property of the proposed method, which is clearly demonstrated in examples, is that spectral accuracy is accessible in solving specific nonlinear Riccati differential equations which have analytic solution functions.

2 LTNHPM for quadratic Ricatti equation

Consider the nonlinear Riccati differential equation as the following form

$$\begin{cases} u'(t) = A(t) + B(t)u(t) + C(t)u^2(t), \; 0 \le t \le T, \\ u(0) = \alpha. \end{cases} \tag{1}$$

where $A(t)$, $B(t)$ and $C(t)$ are continuous and α is an arbitrary constant. By the new homotopy technique, we construct a homotopy $U : \Omega \times [0, 1] \to \mathbb{R}$, which satisfies

$$H(U(t),p) = U'(t) - u_0(t) + pu_0(t) - p[A(t) + B(t)U(t) + C(t)U^2(t)] = 0, \quad (2)$$

where $p \in [0,1]$ is an embedding parameter, $u_0(t)$ is an initial approximation of solution of equation (1). Clearly, we have from equation (2)

$$H(U(t),0) = U'(t) - u_0(t) = 0, \quad (3)$$

$$H(U(t),1) = U'(t) - A(t) - B(t)U(t) - C(t)U^2(t) = 0 \quad (4)$$

By applying Laplace transform on both sides of (2), we have

$$\mathrm{L}\left\{U'(t) - u_0(t) + pu_0(t) - p[A(t) + B(t)U(t) + C(t)U^2(t)]\right\} = 0 \quad (5)$$

Using the differential property of Laplace transform we have

$$s\mathrm{L}\{U(t)\} - U(0) = \mathrm{L}\left\{u_0(t) - pu_0(t) + p[A(t) + B(t)U(t) + C(t)U^2(t)]\right\} \quad (6)$$

or

$$\mathrm{L}\{U(t)\} = \frac{1}{s}\left\{U(0) + \mathrm{L}\left\{u_0(t) - pu_0(t) + p[A(t) + B(t)U(t) + C(t)U^2(t)]\right\}\right\} \quad (7)$$

By applying inverse Laplace transform on both sides of (7), we have

$$U(t) = \mathrm{L}^{-1}\{\frac{1}{s}\{U(0) + \mathrm{L}\{u_0(t) - pu_0(t) + p[A(t) + B(t)U(t) + C(t)U^2(t)]\}\}\} \quad (8)$$

According to the HPM, we use the embedding parameter p as a small parameter, and assume that the solutions of equation (8) can be represented as a power series in p as $U(t) = \sum_{n=0}^{\infty} p^n U_n$. Now let us write the Eq. (8) in the following form

$$\sum_{n=0}^{\infty} p^n U_n(t) = \mathrm{L}^{-1}\left\{\frac{1}{s}\Big(U(0) + \mathrm{L}\Big\{u_0(t) - pu_0(t) + p\Big[A(t) + B(t)\sum_{n=0}^{\infty} p^n U_n(t) + C(t)(\sum_{n=0}^{\infty} p^n U_n(t))^2\Big]\Big\}\Big)\right\} \quad (9)$$

Comparing coefficients of terms with identical powers of p leads to

$$p^0: \ U_0(t) = \mathrm{L}^{-1}\left\{\tfrac{1}{s}\left(U(0) + \mathrm{L}\left\{u_0(t)\right\}\right)\right\}$$
$$p^1: \ U_1(x) = \mathrm{L}^{-1}\left\{-\tfrac{1}{s}\left(\mathrm{L}\left\{u_0(t) - A(t) - B(t)U_0(t) - C(t)U_0^2(t)\right\}\right)\right\}$$
$$p^j: \ U_j(x) = \mathrm{L}^{-1}\left\{\tfrac{1}{s}\left(\mathrm{L}\left\{B(t)U_{j-1}(t) + C(t)\textstyle\sum_{k=0}^{j-1} U_k(t)U_{j-k-1}(t)\right\}\right)\right\}, \quad j = 2,3,\ldots \quad (10)$$

Suppose that the initial approximation has the form $U(0) = u_0(t) = \alpha$, therefore the exact solution may be obtained as following

$$u(t) = \lim_{p \to 1} U(t) = U_0(t) + U_1(t) + \cdots \tag{11}$$

3 Examples

Example 1. Consider the following quadratic Riccati differential equation taken from [1]

$$\begin{cases} u'(t) = 16t^2 - 5 + 8tu(t) + u^2(t), \\ u(0) = 1 \end{cases} \tag{12}$$

The exact solution of above equation was found to be of the form

$$u(t) = 1 - 4t. \tag{13}$$

To solve equation (12) by the LTNHPM, we construct the following homotopy

$$U'(t) - u_0(t) + p\left[u_0(t) + 5 - 16t^2 - 8tU(t) - U^2(t)\right] = 0 \tag{14}$$

Applying Laplace transform on both sides of (14), we have

$$\mathrm{L}\left\{U'(t) - u_0(t) + p\left[u_0(t) + 5 - 16t^2 - 8tU(t) - U^2(t)\right]\right\} = 0 \tag{15}$$

Using the differential property of Laplace transform we have

$$s\mathrm{L}\{U(t)\} - U(0) = \mathrm{L}\left\{u_0(t) - p\left[u_0(t) + 5 - 16t^2 - 8tU(t) - U^2(t)\right]\right\} \tag{16}$$

or

$$\mathrm{L}\{U(t)\} = \frac{1}{s}\left\{U(0) + \mathrm{L}\left\{u_0(t) - p\left[u_0(t) + 5 - 16t^2 - 8tU(t) - U^2(t)\right]\right\}\right\} \tag{17}$$

By applying inverse Laplace transform on both sides of (17), we have

$$U(t) = \mathrm{L}^{-1}\left\{\frac{1}{s}\left(U(0) + \mathrm{L}\left\{u_0(t) - p\left[u_0(t) + 5 - 16t^2 - 8tU(t) - U^2(t)\right]\right\}\right)\right\} \tag{18}$$

Suppose the solution of equation (18) to have the following form

$$U(t) = U_0(t) + pU_1(t) + p^2U_2(t) + \cdots , \tag{19}$$

where $U_i(t)$ are unknown functions which should be determined. Substituting equation (19) into equation (18), collecting the same powers of p and equating each coefficient of p to zero, results in

$$
\begin{aligned}
&p^0: \ U_0(t) = \mathrm{L}^{-1}\left\{\tfrac{1}{s}\left(U(0) + \mathrm{L}\left\{u_0(t)\right\}\right)\right\} \\
&p^1: \ U_1(x) = \mathrm{L}^{-1}\left\{-\tfrac{1}{s}\left(\mathrm{L}\left\{u_0(t) + 5 - 16t^2 - 8tU_0(t) - U_0^2(t)\right\}\right)\right\} \\
&p^j: \ U_j(x) = \mathrm{L}^{-1}\left\{\tfrac{1}{s}\left(\mathrm{L}\left\{-8tU_{j-1}(t) + \textstyle\sum_{k=0}^{j-1} U_k(t)U_{j-k-1}(t)\right\}\right)\right\} \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad j = 2, 3, \ldots
\end{aligned} \tag{20}
$$

Assuming $u_0(t) = U(0) = 1$, and solving the above equation for $U_j(t)$, $j = 0, 1, \cdots$ leads to the result

$$
\begin{aligned}
&U_0(t) = 1 + t, \\
&U_1(t) = \tfrac{5t}{3}\left(5t^2 + 3t - 3\right), \\
&U_2(t) = \tfrac{5t^2}{3}\left(10t^3 + 10t^2 - 8t - 3\right), \\
&U_3(t) = \tfrac{5t^3}{63}\left(425t^4 + 595t^3 - 399t^2 - 399t + 63\right), \\
&U_4(t) = \tfrac{t^4}{567}\left(38750t^5 + 69750t^4 - 36000t^3 - 72135t^2 + 7938t + 8505\right), \\
&\vdots
\end{aligned}
$$

Therefore we gain the solution of Eq. (12) as

$$u(t) = U_0(t) + U_1(t) + U_3(t) + \cdots = 1 - 4t$$

which is exact solution.

Example 2. Consider the following quadratic Riccati differential equation taken from [13, 7, 14, 6, 8]

$$\begin{cases} u'(t) = 1 + 2u(t) - u^2(t), \\ u(0) = 0. \end{cases} \tag{21}$$

The exact solution of above equation was found to be of the form

$$u(t) = 1 + \sqrt{2}\tanh\left[\sqrt{2}t + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right] \tag{22}$$

The Taylor expansion of $u(t)$ about $t = 0$ gives

$$u(t) = t + t^2 + \frac{1}{3}t^3 - \frac{1}{3}t^4 - \frac{7}{15}t^5 - \frac{7}{45}t^6 + \frac{53}{315}t^7 + \frac{71}{315}t^8 + \cdots . \tag{23}$$

To solve equation (21), by the LTNHPM, we construct the following homotopy

$$U'(t) = u_0(t) - p\left[u_0(t) - 1 - 2U(t) + U^2(t)\right] \tag{24}$$

Applying Laplace transform, we have

$$\mathrm{L}\left\{U'(t) - u_0(t) + pu_0(t) - p[1 + 2U(t) - U^2(t)]\right\} = 0 \tag{25}$$

Using the differential property of Laplace transform we have

$$sL\{U(t)\} - U(0) = L\left\{u_0(t) - p\left[u_0(t) - 1 - 2U(t) + U^2(t)\right]\right\} \tag{26}$$

or

$$L\{U(t)\} = \frac{1}{s}\left\{U(0) + L\left\{u_0(t) - p\left[u_0(t) - 1 - 2U(t) + U^2(t)\right]\right\}\right\} \tag{27}$$

By applying inverse Laplace transform on both sides of (27), we have

$$U(t) = L^{-1}\left\{\frac{1}{s}\left(U(0) + L\left\{u_0(t) - p\left[u_0(t) - 1 - 2U(t) + U^2(t)\right]\right\}\right)\right\} \tag{28}$$

Suppose the solution of equation (28) to have the following form

$$U(t) = U_0(t) + pU_1(t) + p^2U_2(t) + \cdots, \tag{29}$$

where $U_i(t)$ are unknown functions which should be determined. Substituting equation (29) into equation (28), collecting the same powers of p , and equating each coefficient of p to zero, results in

$$\begin{aligned} &p^0: \ U_0(t) = L^{-1}\left\{\tfrac{1}{s}\left(U(0) + L\left\{u_0(t)\right\}\right)\right\} \\ &p^1: \ U_1(x) = L^{-1}\left\{-\tfrac{1}{s}\left(L\left\{u_0(t) - 1 - 2U_0(t) + U_0^2(t)\right\}\right)\right\} \\ &p^j: \ U_j(x) = L^{-1}\left\{\tfrac{1}{s}\left(L\left\{2U_{j-1}(t) + \textstyle\sum_{k=0}^{j-1} U_k(t)U_{j-k-1}(t)\right\}\right)\right\}, \ j = 2, 3, \ldots \end{aligned} \tag{30}$$

Assuming $u_0(t) = U(0) = 0$, and solving the above equation for $U_j(t)$, $j = 0, 1, \cdots$ leads to the result

$$\begin{aligned} &U_0(t) = 0, \\ &U_1(t) = t, \\ &U_2(t) = t^2, \\ &U_3(t) = \tfrac{t^3}{3}, \\ &U_4(t) = -\tfrac{t^4}{3}, \\ &U_5(t) = -\tfrac{7t^5}{15}, \\ &\vdots \end{aligned} \tag{31}$$

Therefore we gain the solution of equation (21) as

$$\begin{aligned} u(t) &= U_0(t) + U_1(t) + U_3(t) + \cdots \\ &= t + t^2 + \tfrac{1}{3}t^3 - \tfrac{1}{3}t^4 - \tfrac{7}{15}t^5 - \tfrac{7}{45}t^6 + \tfrac{53}{315}t^7 + \cdots, \end{aligned} \tag{32}$$

and this in the limit of infinitely many terms, yields the exact solution of (21).

Example 3. Consider the following quadratic Riccati differential equation taken from [1]

$$\begin{cases} u'(t) = e^t - e^{3t} + 2e^{2t}u(t) - e^t u^2(t), \\ u(0) = 1. \end{cases} \tag{33}$$

The exact solution of this equation is $u(t) = \mathrm{e}^t$. We construct the following homotopy

$$U'(t) - u_0(t) + p\left[u_0(t) - \mathrm{e}^{pt} + \mathrm{e}^{3pt} - 2\mathrm{e}^{2pt}U(t) + \sum_{n=0}^{\infty} \mathrm{e}^{pt}U^2(t)\right] = 0 \quad (34)$$

Applying Laplace transform on both sides of (34), we have

$$\mathrm{L}\left\{U'(t) - u_0(t) + p\left[u_0(t) - \mathrm{e}^{pt} + \mathrm{e}^{3pt} - 2\mathrm{e}^{2pt}U(t) + \sum_{n=0}^{\infty} \mathrm{e}^{pt}U^2(t)\right]\right\} = 0 \quad (35)$$

Using the differential property of Laplace transform we have

$$\begin{aligned} s\mathrm{L}\{U(t)\} - U(0) =& \mathrm{L}\Big\{u_0(t) - p\Big[u_0(t) - \mathrm{e}^{pt} \\ & + \mathrm{e}^{3pt} - 2\mathrm{e}^{2pt}U(t) + \sum_{n=0}^{\infty} \mathrm{e}^{pt}U^2(t)\Big]\Big\} \end{aligned} \quad (36)$$

or

$$\begin{aligned} \mathrm{L}\{U(t)\} =& \frac{1}{s}\Big\{U(0) + \mathrm{L}\Big\{u_0(t) \\ & - p\Big[u_0(t) - \mathrm{e}^{pt} + \mathrm{e}^{3pt} - 2\mathrm{e}^{2pt}U(t) + \sum_{n=0}^{\infty} \mathrm{e}^{pt}U^2(t)\Big]\Big\}\Big\} \end{aligned} \quad (37)$$

By applying inverse Laplace transform on both sides of (37) and using the Taylor series of $e^{\alpha t} = \sum_{n=0}^{\infty} \frac{(\alpha t)^n}{n!}$, we have

$$\begin{aligned} U(t) =& \mathrm{L}^{-1}\Big\{\frac{1}{s}\Big(U(0) + \mathrm{L}\Big\{u_0(t) - p\Big[u_0(t) - \sum_{n=0}^{\infty} p^n \frac{t^n}{n!} \\ & + \sum_{n=0}^{\infty} p^n \frac{(3t)^n}{n!} - 2\sum_{n=0}^{\infty} p^n \frac{(2t)^n}{n!}U(t) + \sum_{n=0}^{\infty} p^n \frac{t^n}{n!}U^2(t)\Big]\Big\}\Big)\Big\} \end{aligned} \quad (38)$$

Suppose the solution of U can be expanded into infinite series as $U(t) = \sum_{n=0}^{\infty} p^n U_n(t)$. Substituting $U(t)$ into equation (38), collecting the same powers of p , and equating each coefficient of p to zero, results in

$$\begin{array}{l}
p^0: \ U_0(t) = \mathrm{L}^{-1}\left\{\frac{1}{s}\left(U(0) + \mathrm{L}\left\{u_0(t)\right\}\right)\right\} \\
p^1: \ U_1(x) = \mathrm{L}^{-1}\left\{-\frac{1}{s}\left(\mathrm{L}\left\{u_0(t) - 2U_0 + U_0^2\right\}\right)\right\} \\
p^2: \ U_2(x) = \mathrm{L}^{-1}\left\{-\frac{1}{s}\left(\mathrm{L}\left\{2t - 4tU_0 - 2U_1 + tU_0^2 + 2U_0U_1\right\}\right)\right\} \\
p^3: \ U_3(x) = \mathrm{L}^{-1}\{-\frac{1}{s}(\mathrm{L}\{4t^2 - 4tU_1 - 2U_2 - 4t^2U_0 + 2tU_0U_1 \\
\qquad\qquad + U_1^2 + 2U_0U_2 + \frac{1}{2}t^2U_0^2\})\} \\
\vdots
\end{array} \tag{39}$$

Assuming $u_0(t) = U(0) = 1$, and solving the above equation for $U_j(t)$, $j = 0, 1, \cdots$ leads to the result

$$\begin{array}{l}
U_0(t) = 1 + t, \\
U_1(t) = -\frac{t^3}{3}, \\
U_2(t) = -\frac{t^2}{12}\left(-6 - 8t + t^2\right), \\
U_3(t) = -\frac{t^3}{5040}\left(840 - 4620t + 504t^2 - 840t^3 - 240t^4 + 35t^5\right) \\
\vdots
\end{array} \tag{40}$$

Therefore we gain the solution of equation (33) as

$$\begin{array}{l}
u(t) = U_0(t) + U_1(t) + U_3(t) + \cdots \\
\quad = 1 + t + \frac{1}{2!}t^2 + \frac{1}{3!}t^3 + \cdots \\
\quad = \sum_{n=0}^{\infty} \frac{t^n}{n!} = e^t,
\end{array} \tag{41}$$

which is exact solution.

Example 4. Consider the following quadratic Riccati differential equation taken from [13, 7, 14, 6, 8]

$$\begin{cases} u'(t) = -u(t) + u^2(t), \\ u(0) = \frac{1}{2} \end{cases} \tag{42}$$

The exact solution of above equation was found to be of the form

$$u(t) = \frac{\mathrm{e}^{-t}}{1 + \mathrm{e}^{-t}} \tag{43}$$

The Taylor expansion of $u(t)$ about $t = 0$ gives

$$u(t) = \frac{1}{2} - \frac{1}{4}t + \frac{1}{48}t^3 - \frac{1}{480}t^5 + \frac{17}{80640}t^7 - \frac{31}{1451520}t^9 + \frac{691}{319334400}t^{11} - \cdots . \tag{44}$$

To solve equation (43) by the LTNHPM, we construct the following homotopy:

$$U'(t) = u_0(t) - p\left[u_0(t) + U(t) - U^2(t)\right] \tag{45}$$

By applying Laplace transform we get

$$\mathrm{L}\{U(t)\} = \frac{1}{s}\left\{U(0) + \mathrm{L}\left\{u_0(t) - p\left[u_0(t) + U(t) - U^2(t)\right]\right\}\right\} \tag{46}$$

Using inverse Laplace transform on both sides of (46), we have

$$U(t) = \mathrm{L}^{-1}\left\{\frac{1}{s}\left(U(0) + \mathrm{L}\left\{u_0(t) - p\left[u_0(t) + U(t) - U^2(t)\right]\right\}\right)\right\} \tag{47}$$

Suppose the solution of equation (47) to have the following form

$$U(t) = U_0(t) + pU_1(t) + p^2U_2(t) + \cdots , \tag{48}$$

Substituting equation (48) into equation (47), collecting the same powers of p , and equating each coefficient of p to zero, results in

$$\begin{aligned}
&p^0 : U_0(t) = \mathrm{L}^{-1}\left\{\tfrac{1}{s}\left(U(0) + \mathrm{L}\left\{u_0(t)\right\}\right)\right\}\\
&p^1 : U_1(x) = \mathrm{L}^{-1}\left\{-\tfrac{1}{s}\left(\mathrm{L}\left\{u_0(t) + U_0(t) - U_0^2(t)\right\}\right)\right\}\\
&p^j : U_j(x) = \mathrm{L}^{-1}\left\{\tfrac{1}{s}\left(\mathrm{L}\left\{-U_{j-1}(t) + \textstyle\sum_{k=0}^{j-1} U_k(t)U_{j-k-1}(t)\right\}\right)\right\},\ j = 2, 3, \ldots
\end{aligned} \tag{49}$$

Assuming $u_0(t) = U(0) = \frac{1}{2}$, and solving the above equation for $U_j(t)$, $j = 0, 1, \cdots$ leads to the result

$$\begin{aligned}
&U_0(t) = \tfrac{1}{2}(1 + t),\\
&U_1(t) = \tfrac{t}{12}\left(-9 + t^2\right),\\
&U_2(t) = \tfrac{t^3}{60}\left(-15 + t^2\right),\\
&U_3(t) = \tfrac{t^3}{5040}\left(945 - 387t^2 + 17t^4\right),\\
&U_4(t) = \tfrac{t^5}{45360}\left(5103 - 918t^2 + 31t^4\right),\\
&U_5(t) = \tfrac{t^5}{4989600}\left(-280605 + 227205t^2 - 25575t^4 + 691t^6\right),\\
&\vdots
\end{aligned} \tag{50}$$

Therefore we gain the solution of equation (42) as

$$\begin{aligned}
u(t) &= U_0(t) + U_1(t) + U_3(t) + \cdots\\
&= \tfrac{1}{2} - \tfrac{1}{4}t + \tfrac{1}{48}t^3 - \tfrac{1}{480}t^5 + \tfrac{17}{80640}t^7 - \tfrac{31}{1451520}t^9 + \tfrac{691}{319334400}t^{11} - \cdots ,
\end{aligned} \tag{51}$$

this in the limit of infinitely many terms, yields the exact solution of (42).

4 Conclusion

In the present work, we proposed a combination of Laplace transform method and homotopy perturbation method to solve nonlinear Riccati differential equation. The new method developed in the current paper was tested on several examples. The obtained results show that these approaches can solve the problem effectively. Unlike the previous approach implemented by the present authors, so-called NHPM, the present technique, does not need the initial approximation to be defined as a power series. In the NHPM we reach to a set of recurrent differential equations which must be solved consecutively to give the approximate solution of the problem. Sometimes we have to do many computations in order to reach to the higher orders of approximation with acceptable accuracy. Also as an advantage of the LTNHPM over decomposition procedure of Adomian, the former method provides the solution of the problem without calculating Adomian's polynomials. The advantage of the LTNHPM over numerical methods (finite difference, finite element, . . .) and ADM is that it solves the problem without any need to discretization of the variables. The Computations finally lead to a set of nonlinear equations with one unspecified value in each equation. The results show that the LTNHPM is an effective mathematical tool which can play a very important role in nonlinear sciences.

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