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Research Article

# Mathematical modeling and optimal control of customer's behavior toward e-commerce

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**Abstract**

The extensive influence of digital platforms has reshaped societal interactions and daily routines, integrating e-commerce into every aspect of modern life. This evolution not only redefines traditional business models

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but also fosters global connectivity and economic restructuring. However, despite its critical role in the global economy, e-commerce faces challenges, notably the hesitation of some consumers due to concerns about security and trust. To address this, we propose a novel mathematical model to examine customer behavior dynamics toward e-commerce, particularly the impact of the refusal behavior. Our study comprehensively examines the characteristics of our mathematical model, conducts a thorough stability analysis, and investigates the parameter sensitivity. Furthermore, control theory has been adopted to optimize the adoption of e-commerce using Pontryagin's maximum principle, with numerical simulations to evaluate the effectiveness of our proposed strategies.

**AMS subject classifications (2020):** Primary 93-10; Secondary 49J20, 34D20.

**Keywords:** Mathematical modeling, Stability analysis, E-commerce, Pontryagin Maximum principle, Sensitivity Analysis.

## 1 Introduction

The increasing dependence on digital platforms and the acceleration of technical progress have fundamentally transformed how people engage, converse, and go through their daily lives in modern society [7, 30]. Digital platforms have become an integral component in almost every aspect of contemporary life, ranging from interpersonal communication to career pursuits [16]. Global connectivity is fostered by social media, e-commerce, and online communication tools, which have become indispensable aspects of daily life. This significant change is redefining conventional corporate models and economic structures in addition to changing societal dynamics [36, 35]. The transformative potential of technology is clear as more people and organizations gravitate to digital solutions. This is because technology not only improves efficiency and convenience but also catalyzes paradigm shifts across varied industries, laying the groundwork for a future where everything is connected through the Internet. On the grounds of this, the major developments that e-commerce encountered made it an essential foundation for the growth of the nation's economy, as well as for the country's opening toward the worldwide market's accessibility of product presentation both domestically and abroad, and convenience of buying from any domestic or foreign market at any time or location [11, 39].

Nevertheless, in the contemporary landscape, e-commerce is one of the most important aspects of the global economy in the modern world. It has a significant impact on how businesses operate and how customers engage with goods and services [6]. In [29], the authors encountered the ever-expanding reach of digital platforms that have not only facilitated the seamless exchange of goods and services but has also blurred the traditional boundaries of market accessibility.

Online businesses or shops notice that their offered services can cease functioning due to some technical problems or more importantly, a decreasing number of customers, and sometimes it is difficult to control this behavior due to the lack of time or a misunderstanding of the customer's behavior toward online businesses or e-commerce in general. This lack of information can blur the future of the online business economy. Consequently, these situations serve as the inspiration behind the research question of this paper: How can we more significantly understand the customer's behavior toward e-commerce?

Regarding the enormous role e-commerce has in our lives and how it impacts people's behavior, scholars explored the interaction between people and e-commerce. In [8, 31], the authors analyzed the correlation between e-commerce and people's trust, forecasting the differences between a virtual and a conventional marketplace and how customers' trust affects their consumption and vision toward e-commerce. Moreover, some other researchers talked about and provided examples of the age, educational background, and income requirements of users on e-commerce platforms [26], and even the rumors dissemination impact on it [5]. Therefore, the significant attention that the public and governments accord e-commerce has created numerous opportunities for business owners. Thus, academics paid attention to this topic and studied the relationship between e-commerce and entrepreneurship [37, 15]. From another perspective, in [9], the authors investigated the impact of COVID-19 on e-commerce and its effects on consumer purchase behavior.

Following the pattern of modeling the customer's behavior, it is getting increasingly important to better understand the dynamic of customers toward these online businesses regarding the great evolution of digital platforms and the customer's needs and requirements. Consequently, various academics investigated the subject. In [41], the authors focused on the modeling of customer preferences using multi-network analysis, which gives a significant database to understand the customer's conduct. Furthermore, Park and Fader [32] developed a stochastic model of website visiting behavior to better understand what makes a customer change his conduct toward one web website to another. During the time customers spend on websites, they certainly leave behind a pattern trait that sums up their behavior. This process generates what we call big data information and web analytics that are significant in understanding customers' behavior when being on the website [13].

Perceiving the supreme role e-commerce took in people's lives and the correlation it has with our behavior and consumption aptitude, it is instinctive to better understand and analyze the process of how customers act toward e-commerce. Consequently, a proper and effective modeling approach must be used, such as the compartmental model approach, which was first introduced by Kermack and McKendrick [21] in 1927 in the Proceedings of the Royal Society of London, which is a seminal work that laid the groundwork for the mathematical modeling of infectious diseases, principally named the SIR models. Inspired by the epidemiological modeling approach, a population can be divided into three main compartments: susceptible, infected, and recovered, which make it convenient to use for customer's behavior modeling for a better and simpler understanding, interpreting, and controlling. In the literature, we can see that all the proposed mathematical modeling of customers' behavior toward e-commerce does not take into consideration the influence of customers with each other and the bad influence a refusal behavior can have on a group of customers toward these shopping websites. That is why a better modeling approach must take place. In our work, we formulate a new mathematical model to overview and understand the dynamic behavior of customers toward e-commerce and specifically to study the influence of customers' refusal to use e-commerce on others. To that end, we propose a compartmental model of four compartments (potential customers, refusers of e-commerce, users of e-commerce, and finally, those who temporally stop using e-commerce) that widely describe the interaction between customers regarding their behavior in the e-commerce environment. We study the main characteristics of the proposed model to discover the various aspects of the proposed modeling and to better understand the process of customers' behavior towards e-commerce. Nevertheless, the stability analysis of the proposed model is investigated in its local and global aspects for the equilibrium points subjected to this model. We also use control theory analysis and the Pontryagin maximum principle to seek our objective of maximization of users of e-commerce platforms and minimizing the refusers of e-commerce, which consequently will have a great impact on the global economy.

Control theory has been applied to various problems by a range of authors, such as the electoral behavior optimal control strategies regarding the political appurtenance [3]. In addition, in [18], the authors applied mathematical modeling and optimal control strategies to prevent the spread of the coronavirus. Further research on control theory can be found in the following works: [25, 22, 10, 2].

In our research, we formulate a new mathematical model to study the impact of the behavior of customers who refuse to use e-commerce on e-commerce. We study the basic characteristics of the proposed model. We will also apply control theory to this model to increase the number of customers using e-commerce.

The research is organized as follows: In Section 2, a mathematical model is presented that analyzes customers' behavior toward e-commerce. Then, in Section 3, a stability analysis is performed around the equilibrium points, in both the locally and globally aspects. Furthermore, a sensitivity analysis of the reproduction number parameters is also performed to find out the most effective parameters for the optimal control problem of the model. Following that, Section 4 presents the optimal control problem, where the existence and characterization of the proposed controls are proven. Section 5 showcases the numerical simulations of the model's different compartments to evaluate the effectiveness of the proposed strategies. Finally, the paper is concluded in Section 6.

## 2 The mathematical model formulation

In this research, the problem studied is that of customers who refuse to use electronic commerce because they are afraid of electronic crime or fraud, or because they are afraid that their personal data will be stolen. To study this problem mathematically, we have proposed a deterministic, non-linear model that describes the problem in all its aspects and its relationship with the rest of the parties that could be affected by the refusal to use electronic commerce. The proposed model has four compartments.

### 2.1 Description of the model

**Compartment $P$:** It represents potential customers for electronic commerce who have never made any purchases through electronic stores located on the Internet, knowing that these people are connected to the Internet. The behavior of this compartment is due to a group of reasons, including lack of knowledge of electronic commerce, fear of electronic crimes, fear of fraud regarding the quality of products, fear of leakage of personal data, or living in places far from means of transportation for transporting goods. The number of members of this compartment increases with the number $\Lambda$, which represents the rate of new people connected to the Internet. The number of individuals in this compartment is reduced by the number $\beta$, which represents the percentage of people affected by the refusal to use e-commerce, the number $\alpha$, which represents the percentage of people who decided to use e-commerce as a result of positive thoughts about it, and the number $\mu$, which represents the percentage of natural deaths of customers.

**Compartment $A$:** It represents customers who refuse to use e-commerce because of negative thoughts about it, knowing that the members of this compartment have never purchased through e-commerce stores. The number of members of this compartment increases with the number $\beta$, which represents the percentage of people affected by the refusal to use e-commerce, while the number of members of this compartment decreases with the numbers $\gamma$ and $\mu$, which, respectively, represent the percentage of people who refused to use e-commerce but decided to use it as a result of positive thoughts about it and the percentage of natural deaths of customers.

**Compartment $E$:** This category represents customers who use e-commerce, and they are people who purchase from electronic stores on the Internet. The number of members of this compartment increases by three numbers $\alpha$, $\gamma$, and $\theta$, which, respectively, represent the percentage of people who decided to use e-commerce as a result of positive thoughts about it, the percentage of people who refused to use e-commerce but decided to use it as a result of positive thoughts about it, and the rate of people who returned to using it. The number of people in this compartment is reduced by the number $\mu$, which represents the percentage of natural deaths of customers, and the number $\delta$, which represents the percentage of people who stopped using e-commerce as a result of a problem.

**Compartment $D$:** It represents customers who have temporarily stopped using e-commerce, and they are people who have temporarily stopped using e-commerce as a result of a group of factors, including exposure to fraud, traveling to places where the Internet is not available, or exposure to electronic crime. The number of members of this compartment increases with the number $\delta$, which represents the percentage of people who stopped using e-commerce as a result of a problem. The number of people in this compartment is reduced by the number $\mu$, which represents the percentage of natural deaths of customers, and by the number $\theta$, which represents the percentage of people who returned to using e-commerce after solving their previous problem.

The schematic diagram in Figure 1 showcases the trends of customer flows between compartments. We will fill these trends with directional arrows to further clarify these flows. The parameters used in this model are summarized in Table 1.

The model outlines a dynamic flow of customers through four distinct compartments, each representing different stages of engagement with electronic commerce (e-commerce). Initially, individuals enter the system as potential customers (Compartment P), influenced by factors such as their familiarity with e-commerce, concerns about security, and geographical accessibility. From there, they may transition to become active customers

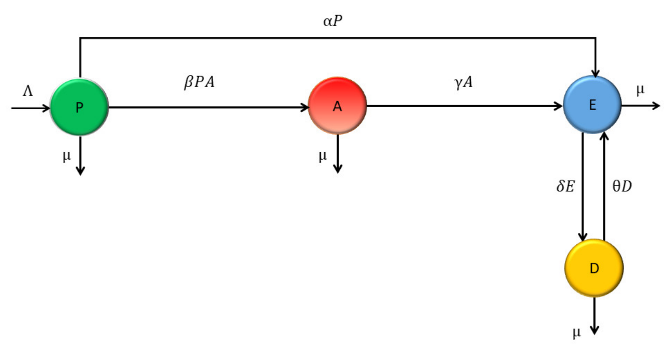


Figure 1: Compartments model diagram.

(Compartment E) through positive experiences or resolution of initial apprehensions. Alternatively, some individuals may initially refuse to engage with e-commerce (Compartment A) due to negative perceptions but may later be persuaded to join the active customer base through positive interventions. However, the model also accounts for fluctuations in customer behavior, as evidenced by the existence of temporarily inactive customers (Compartment D), who may pause their e-commerce activities due to encountered problems but could return to active engagement following issue resolution. Throughout this process, the flow of individuals between compartments is governed by rates of adoption, refusal, retention, and attrition, reflecting the complex interplay of factors shaping e-commerce participation. Following the hypothesis and description above, we present the mathematical model for customer’s behavior, who refuses to use e-commerce when purchasing their daily needs. The model is governed by the following system of differential equations:

$$\begin{cases} \frac{dP(t)}{dt} = \Lambda - \beta P(t)A(t) - \alpha P(t) - \mu P(t), \\ \frac{dA(t)}{dt} = \beta P(t)A(t) - \gamma A(t) - \mu A(t), \\ \frac{dE(t)}{dt} = \alpha P(t) + \gamma A(t) + \theta D(t) - \delta E(t) - \mu E(t), \\ \frac{dD(t)}{dt} = \delta E(t) - \theta D(t) - \mu D(t), \end{cases} \tag{1}$$

Table 1: Model's parameter description

| Parameter | Description |
|---|---|
| $\Lambda$ | The rate of new people connected to the Internet. |
| $\beta$ | The Rate of people affected by refusal to use e-commerce. |
| $\alpha$ | The rate of people who decided to use e-commerce as a result of positive thoughts about it. |
| $\gamma$ | The rate of people who refused to use e-commerce but decided to use it as a result of positive thoughts about it. |
| $\delta$ | The rate of people who stopped using e-commerce as a result of a problem. |
| $\theta$ | The rate of people who returned to using e-commerce after solving their previous problem. |
| $\mu$ | Natural mortality rate of clients. |

with the initial conditions $P(0) \geq 0$, $A(0) \geq 0$, $E(0) \geq 0$, and $D(0) \geq 0$. The total number of people who can buy from online stores at time $t$ is $N(t)$, so we have $N(t) = P(t) + A(t) + E(t) + D(t)$.

## 2.2 Model's fundamental characteristics

Considering that the system model we proposed describes the dynamic of a human population, it is primordial to demonstrate that the system's solutions will remain positive for any initial positive condition and for all time $t > 0$. For the system model (1) and for $t > 0$, we have the following properties:

$$\begin{aligned} \left.\frac{dP}{dt}\right|_{P=0} &= \Lambda \geq 0, \\ \left.\frac{dA(t)}{dt}\right|_{A=0} &= 0, \\ \left.\frac{dE(t)}{dt}\right|_{E=0} &= \alpha P(t) + \gamma A(t) + \theta D(t) \geq 0, \\ \left.\frac{dD(t)}{dt}\right|_{D=0} &= \delta E(t) \geq 0. \end{aligned}$$

Thus, noting that all rates are nonnegative at the boundary of the nonnegative cone in $\mathbb{R}^4_+$ and considering the inward directions of the vector fields on these boundaries, starting the system from any interior point within this cone ensures that the solutions will remain inside the cone indefinitely.

### 2.2.1 Invariant region

In this section, we came to the conclusion that all possible solutions of the system described by model (1) are bounded.

**Theorem 1.** Let the region $\Omega$ be defined by
$\Omega = \left\{ (P(t),\ A(t),\ E(t),\ D(t)) \in \mathbb{R}^4_+,\ 0 \leq P(t) + A(t) + E(t) + D(t) \leq \frac{\Lambda}{\mu} \right\}.$
If the initial conditions $P(0)$, $A(0)$, $E(0)$, and $D(0)$ are positive, then the region $\Omega$ is positively invariant for the model (1).

*Proof.* To demonstrate the positivity invariance of $\Omega$ in the context of Theorem 1, let $t \geq 0$.

We have

$$\begin{aligned}\frac{dN(t)}{dt} &= \Lambda - \mu P(t) - \mu A(t) - \mu E(t) - \mu D(t),\\ &= \Lambda - \mu N(t).\end{aligned}$$

Hence

$$N(t) = N(0)\exp(-\mu t) + \left(\frac{\Lambda}{-\mu}\right)(\exp(-\mu t) - 1)\,.$$

This implies

$$\lim_{t\to+\infty} \sup N(t) = \frac{\Lambda}{\mu}.$$

Therefore

$$N(t) \leq \frac{\Lambda}{\mu}, \tag{2}$$

where $N(t) = P(t) + A(t) + E(t) + D(t)$. This inequality ensures that trajectories starting within the region defined by

$$\Omega = \left\{ (P(t), A(t), E(t), D(t)) \in \mathbb{R}^4_+ \mid 0 \leq P(t) + A(t) + E(t) + D(t) \leq \frac{\Lambda}{\mu} \right\}$$

remain within $\Omega$ over time. Specifically, $N(t)$ cannot exceed $\frac{\Lambda}{\mu}$ due to the upper limit condition, thereby establishing the positivity invariance of $\Omega$ under the dynamics of model (1) [14]. This property allows us to focus our analysis exclusively on the behaviors within $\Omega$. □

### 2.2.2 Existence of solutions

In this subsection, we delve into the fundamental question of the existence of solutions within the context of our proposed compartment model. To this end, we present and prove the following theorem.

**Theorem 2.** System (1) that fulfills the initial conditions $P(0) \geq 0$, $A(0) \geq 0$, $E(0) \geq 0$, and $D(0) \geq 0$ has a unique solution.

*Proof.* Let

$$X = \begin{pmatrix} P(t) \\ A(t) \\ E(t) \\ D(t) \end{pmatrix} \quad \text{and} \quad \Psi(X) = \begin{pmatrix} \frac{dP(t)}{dt} \\ \frac{dA(t)}{dt} \\ \frac{dE(t)}{dt} \\ \frac{dD(t)}{dt} \end{pmatrix}.$$

Then

$$\Psi(X) = SX + Q(X),$$

in which

$$S = \begin{bmatrix} -\alpha-\mu & 0 & 0 & 0 \\ 0 & -\gamma-\mu & 0 & 0 \\ \alpha & \gamma & -\delta-\mu & \theta \\ 0 & 0 & \delta & -\theta-\mu \end{bmatrix} \quad \text{and} \quad Q(X) = \begin{bmatrix} \Lambda - \beta P(t)A(t) \\ \beta P(t)A(t) \\ 0 \\ 0 \end{bmatrix},$$

$$\begin{aligned} \|Q(X_1) - Q(X_2)\| &\leq 2 \left| \beta P_1 A_1 - \beta P_2 A_2 \right|, \\ &\leq 2\beta \left| P_1 \left(A_1 - A_2\right) + A_2 \left(P_1 - P_2\right) \right|, \\ &\leq 2\beta \left| P_1 \left|A_1 - A_2\right| + A_2 \left|P_1 - P_2\right| \right|, \\ &\leq \tfrac{2\beta\Lambda}{\mu} \left| \left|A_1 - A_2\right| + \left|P_1 - P_2\right| \right|, \\ &\leq L \left\|X_1 - X_2\right\|, \end{aligned}$$

with

$$L = \frac{2\beta\Lambda}{\mu}.$$

Hence

$$\|\Psi(X_1) - \Psi(X_2)\| \leq K \left\|X_1 - X_2\right\|,$$

where $K = \max\left(L, \|S\|\right) < \infty$. Thus, it follows that the function $\Psi$ is a Lipschitz continuous function, resulting at the system (1) admits a unique solution [23]. □

## 3 Stability analysis and model's parameters sensitivity

As we dig into the dynamics of our proposed compartment model, we now turn our analysis to two important aspects: stability analysis and sensitivity analysis of considered parameters in the model. These considerations are crucial in understanding the reliability of the model dynamics, putting light on its behavior under various conditions for model optimization.
This section will investigate the considered system (1) comportment at the rejection-free equilibrium point (RFEP) and rejection equilibrium point (REP).

1. The rejection-free equilibrium is given by $X_0 = (P_0, A_0, E_0, D_0)$, where
$$P_0 = \frac{\Lambda}{\alpha + \mu}, \quad A_0 = 0,$$
with
$$E_0 = \alpha \frac{\Lambda}{\mu} \frac{\theta + \mu}{(\alpha + \mu)(\theta + \delta + \mu)}$$
and
$$D_0 = \alpha \Lambda \frac{\delta}{\mu (\alpha + \mu)(\theta + \delta + \mu)}.$$
This equilibrium aligns with a scenario where customers exhibit a consistent acceptance and utilization of e-commerce platforms. In essence, it signifies a state where there is a harmonious balance between the demand for e-commerce services and the willingness of customers to engage with such platforms. This equilibrium suggests that customers are not inclined to reject or resist the adoption of e-commerce, indicating a stable and favorable environment for online transactions and interactions.

2. The rejection equilibrium corresponds to the situation in which the refusal to use electronic commerce is able to spread among the potential customers of electronic commerce. This equilibrium is given by $X_* = (P_*, A_*, E_*, D_*)$ with
$$P_* = \frac{1}{\beta} (\gamma + \mu), \quad A_* = \frac{1}{\beta} (R_0 - 1)(\alpha + \mu),$$

$$E_* = \frac{1}{\beta\mu}\frac{\theta+\mu}{\theta+\mu+\delta}\left(\alpha\mu+\alpha\gamma R_0+\gamma\mu\left(R_0-1\right)\right),$$

and

$$D_* = \frac{1}{\beta\mu}\frac{\theta+\mu}{\theta+\mu+\delta}\left(\alpha\mu+\alpha\gamma R_0+\gamma\mu\left(R_0-1\right)\right).$$

Here, the $R_0$ is the basic reproduction number corresponding to the number of secondary infections caused by an infected person during the course of their infection in a susceptible population. It is a measure widely used in epidemiology because an epidemic can only occur if $R_0 > 1$, and if $R_0 < 1$, the disease cannot invade and no outbreak is expected.

In our research, the basic reproduction number corresponds to the number of people who reject e-commerce, which was the result of a person who refuses to use e-commerce in a sensitive population.

We relied on the approach presented by P.Van Driessche and Watmough [33], in order to calculate $R_0$.

Letting $X = (A, E, P)$, then system (1) can be written as $X' = \Gamma(X) - \Upsilon(X)$.

We have

$$\Gamma(X) = \begin{pmatrix}\beta PA\\ 0\\ 0\end{pmatrix} \quad \text{and} \quad \Upsilon(X) = \begin{bmatrix}\gamma A+\mu A\\ -\alpha P-\gamma A-\theta D+\delta E+\mu E\\ -\Lambda+\beta PA+\alpha P+\mu P\end{bmatrix}.$$

Next, we can compute the Jacobian matrix of $\Gamma(X)$.

We have

$$J(\Gamma(X)) = \begin{bmatrix}\beta P & 0 & \beta A\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \quad \text{so} \quad J(\Gamma(X_0)) = \begin{bmatrix}\beta P_0 & 0 & \beta A_0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}. \tag{3}$$

We can then evaluate this Jacobian matrix at the equilibrium point $X_0$.

We have

$$J(\Gamma(X_0)) = \begin{bmatrix}\beta\frac{\Lambda}{\alpha+\mu} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}.$$

Next, we consider the function $\Upsilon(X)$.

We have

$$J(\Upsilon(X_0)) = \begin{bmatrix} \gamma+\mu & 0 & 0 \\ -\gamma & \delta+\mu & -\alpha \\ \beta P_0 & 0 & \beta A_0+\alpha+\mu \end{bmatrix}, \tag{4}$$

so that

$$J(\Upsilon(X_0)) = \begin{bmatrix} \gamma+\mu & 0 & 0 \\ -\gamma & \delta+\mu & -\alpha \\ \beta\frac{\Lambda}{\alpha+\mu} & 0 & \alpha+\mu \end{bmatrix}.$$

We can now define the matrices $V$ and $F$ as follows:

$$V = \begin{bmatrix} \gamma+\mu & 0 \\ -\gamma & \delta+\mu \end{bmatrix}$$

and

$$F = \begin{bmatrix} \beta\frac{\Lambda}{\alpha+\mu} & 0 \\ 0 & 0 \end{bmatrix}.$$

Finally, we compute the basic reproduction number $R_0$.

At the end, we get

$$R_0 = \rho\left(F \times V^{-1}\right) = \frac{\Lambda\beta}{(\alpha+\mu)(\gamma+\mu)}, \tag{5}$$

where $\rho\left(F \times V^{-1}\right)$ is the spectrum of the matrix $F \times V^{-1}(X_0)$.

## 3.1 The local stability study

In this subsection, we will delve into the local stability analysis, first of the RFEP. Consequently, we present the following theorem that characterizes local stability of the $RFEP$.

**Theorem 3.** The $RFEP$ is locally asymptotically stable if $R_0 < 1$.

*Proof.* We have the Jacobian matrix of $J_0$ in $X_0$:

$$J_0 = \begin{bmatrix} -\alpha-\mu & -\beta\frac{\Lambda}{\alpha+\mu} & 0 & 0 \\ 0 & \beta\frac{\Lambda}{\alpha+\mu}-\gamma-\mu & 0 & 0 \\ \alpha & \gamma & -\delta-\mu & \theta \\ 0 & 0 & \delta & -\theta-\mu \end{bmatrix}.$$

The Eigen values of the characteristic equation of $J_0$ are

$V_1 = -\alpha - \mu < 0$,

$V_2 = -\mu < 0$,

$V_3 = -\theta - \mu - \delta < 0$,

$V_4 = -\frac{1}{\alpha+\mu}\left(1 - R_0\right)\left(\alpha+\mu\right)\left(\gamma+\mu\right) < 0$.

Therefore, $X_0$ is locally asymptotically stable if $R_0 < 1$. □

Thereafter, let us now present the following theorem that concerns $REP$ characterization.

**Theorem 4.** The $REP$ is locally asymptotically stable if $R_0 > 1$.

*Proof.* We have the Jacobian matrix $J_*$ evaluated at $X_*$:

$$J_* = \begin{bmatrix} -R_0(\alpha+\mu) & -\gamma-\mu & 0 & 0 \\ (R_0-1)(\alpha+\mu) & 0 & 0 & 0 \\ \alpha & \gamma & -\mu-\delta & \theta \\ 0 & 0 & \delta & -\theta-\mu \end{bmatrix}.$$

The characteristic polynomial of $J_*$ is given by

$$P(X) = X^4 + a_3X^3 + a_2X^2 + a_1X + a_0,$$

where

$$\begin{aligned} a_3 &= \theta + 2\mu + \delta + R_0(\alpha+\mu), \\ a_2 &= \theta\mu + (R_0-1)(\alpha\gamma+\alpha\mu+\gamma\mu) + \mu\delta + R_0(3\mu^2+\theta\alpha+\theta\mu+2\alpha\mu \\ &\quad + \alpha\delta+\mu\delta), \\ a_1 &= (\alpha+\mu)\left[(R_0-1)(\gamma+\mu)(\theta+2\mu+\delta) + \mu\delta R_0 + \mu^2R_0 + \theta\mu R_0\right], \\ a_0 &= \mu(R_0-1)(\alpha+\mu)(\gamma+\mu)(\theta+\mu+\delta). \end{aligned}$$

According to the Routh–Hurwitz criterion, for the system (1) to be locally asymptotically stable, all the coefficients $a_i$ must be positive, and the following condition must hold:

$$a_1(a_2a_3 - a_1^2) > a_0a_3^2.$$

Let us verify these conditions step-by-step:

1. Positivity of Coefficients:

- $a_3 = \theta + 2\mu + \delta + R_0(\alpha + \mu) > 0$ since all parameters are positive and $R_0 > 1$.
- $a_2 = \theta\mu + (R_0 - 1)(\alpha\gamma + \alpha\mu + \gamma\mu) + \mu\delta + R_0(3\mu^2 + \theta\alpha + \theta\mu + 2\alpha\mu + \alpha\delta + \mu\delta) > 0$ given $R_0 > 1$ and all parameters are positive.
- $a_1 = (\alpha + \mu)[(R_0 - 1)(\gamma + \mu)(\theta + 2\mu + \delta) + \mu\delta R_0 + \mu^2 R_0 + \theta\mu R_0] > 0$ given $R_0 > 1$ and all parameters are positive.
- $a_0 = \mu(R_0 - 1)(\alpha + \mu)(\gamma + \mu)(\theta + \mu + \delta) > 0$ since $R_0 > 1$ and all parameters are positive.

2. Routh–Hurwitz Condition:

$$a_1(a_2a_3 - a_1^2) > a_0a_3^2.$$

By substituting the expressions for $a_1$, $a_2$, $a_3$, and $a_0$ and verifying algebraically, we confirm this inequality holds when $R_0 > 1$.

Thus, from the Routh–Hurwitz criterion, the system is locally asymptotically stable if $R_0 > 1$. □

## 3.2 The global stability study

Let us now investigate the global stability of the concerned states, the $RFEP$ and $REP$. In the following, we present the theorem showcasing the global stability characterization of $RFEP$.

**Theorem 5.** The RFEP is globally asymptotically stable if $R_0 \leq 1$.

*Proof.* Consider a Lyapunov function $V : \Omega \to \mathbb{R}$ given by

$$V(P, A) = \frac{1}{2}\left((P - P_0) + A\right)^2 + \frac{(\alpha + \gamma + 2\mu)}{\beta}A. \tag{6}$$

After deriving $V$, we get

$$V^{'}(P,A) = ((P-P_0)+A)\left(P^{'}+A^{'}\right)+\frac{(\alpha+\gamma+2\mu)}{\beta}A^{'}.$$

Then

$$\begin{aligned}V^{'}(P,A) =& -(\alpha+\mu)\left(P-\frac{\Lambda}{\alpha+\mu}\right)^2-(\gamma+\mu)A^2\\&-(\alpha+\gamma+2\mu)\times\frac{(\mu+\gamma)}{\beta}(1-\Lambda\frac{\beta}{(\alpha+\mu)(\gamma+\mu)})\times A\\=&(\alpha+\mu)\left(P-\frac{\Lambda}{\alpha+\mu}\right)^2-(\gamma+\mu)A^2\\&-(\alpha+\gamma+2\mu)\times\frac{(\mu+\gamma)}{\beta}(1-R_0)\times A\leq 0.\end{aligned}$$

Also, we obtain $V(P,A)=0 \Longleftrightarrow P=P_0$ and $A=0$.

Therefore, the previous theorem has been proved. Moreover, by La Salle's invariance principle [17], the RFEP is globally asymptotically stable on $\Omega$. □

In the following, we get to the global stability of the REP, which is given by the upcoming theorem

**Theorem 6.** The REP is globally asymptotically stable if $R_0>1$.

*Proof.* Consider Lyapunov function $V:\Omega\to\mathbb{R}$ given by

$$V(P,A)=(P-P_*)-P_*\ln\left(\frac{P}{P_*}\right)+(A-A_*)-A_*\ln\left(\frac{A}{A_*}\right). \tag{7}$$

After deriving $V$, we get

$$\begin{aligned}V^{'}(P,A) =&P^{'}+A^{'}-P_*\times\frac{P^{'}}{P}-A_*\times\frac{A^{'}}{A}.\\V^{'}(P,A) =&-\frac{1}{P}\frac{\Lambda}{\beta(\gamma+\mu)}(\gamma+\mu-P\beta)^2\\=&-\frac{1}{P}\frac{\Lambda}{P_*}(P-P_*)^2\leq 0.\end{aligned}$$

Also, we obtain

$V'(P,A)=0 \Longleftrightarrow P=P_*$.

Therefore, by La Salle's invariance principle [17], the $REP$ is globally asymptotically stable on $\Omega$.

□

**Remark 1.** After thorough examination of local and global stability, particularly within our proposed model concerning customer rejection behavior, significant insights have emerged. Local stability analysis delves into the dynamics surrounding equilibrium points, revealing how the system behaves under small perturbations. Meanwhile, global stability analysis extends beyond local considerations, providing insights into the system's overall behavior across its entire phase space. This combined analysis offers a comprehensive understanding vision of the interaction between customer rejection behavior and e-commerce dynamics, aiding in strategic decision-making.

### 3.3 Model's parameters sensitivity analysis

The concept of sensitivity investigation and analysis is used to measure the prevailing parameters in the model that significantly influence the prevalence of people who refuse to use e-commerce. We know that the prevalence of people who refuse to use e-commerce in the rest of society is linked to the basic reproduction number $R_0$.

In this section, we study the effect of each parameter entered into the system (1) on the value of $R_0$. In order to identify the parameters that contribute most to the spread of people who refuse to use e-commerce within society, a sensitivity analysis is carried out in the sense of [28, 1]. We compute the partial derivatives of the value of $R_0$ with respect to the values of the system parameters (1) using the sensitivity index.

**Definition 1.** The sensitivity index $Z_q^{R_0}$ of the basic reproduction number $R_0$, varying with parameter $q$, is defined as

$$Z_q^{R_0} = \frac{\partial R_0}{\partial q} \cdot \frac{q}{R_0},$$

where $\frac{\partial R_0}{\partial q}$ denotes the partial derivative of $R_0$ with respect to $q$.

We have

$$\begin{cases} Z_{\Lambda}^{R_0} = \frac{\partial R_0}{\partial \Lambda} \cdot \frac{\Lambda}{R_0} = 1, \\ Z_{\beta}^{R_0} = \frac{\partial R_0}{\partial \beta} \cdot \frac{\beta}{R_0} = 1, \\ Z_{\alpha}^{R_0} = \frac{\partial R_0}{\partial \alpha} \cdot \frac{\alpha}{R_0} = \frac{-\alpha}{(\alpha+\mu)}, \\ Z_{\gamma}^{R_0} = \frac{\partial R_0}{\partial \gamma} \cdot \frac{\gamma}{R_0} = \frac{-\gamma}{(\gamma+\mu)}, \\ Z_{\mu}^{R_0} = \frac{\partial R_0}{\partial \mu} \cdot \frac{\mu}{R_0} = \frac{-\mu(\alpha+\gamma+2\mu)}{(\alpha+\mu)(\gamma+\mu)}. \end{cases} \tag{8}$$

The sign of the sensitivity index of the basic reproduction figure $R_0$ for the model parameters indicates that an increase (or decrease) in the value of each of the parameters, in this case, will lead to an increase (or decrease) in the value of the basic reproduction figure for people who refuse to use e-commerce. For example, $Z_{\beta}^{R_0} = 1$ means that increasing (or decreasing) the effective contact rate $\beta$ by 10% will increase (or decrease) the basic reproduction number $R_0$ by 10%. On the other hand, the negative sign of the sensitivity index, for the basic reproduction number of the model parameters, means that an increase (or decrease) in the value of each of the parameters, in this case, leads to a corresponding decrease (or increase) in the value of the basic reproduction number for people who refuse to use e-commerce. For example, $Z_{\alpha}^{R_0} =$ -0.63 means that increasing (or decreasing) the parameter $\alpha$ by 10% decreases (or increases) $R_0$ by 6.3%. Figure 2 below shows the sensitivity indices of all the model parameters to $R_0$. The parameters are ordered from most to least sensitive. Through sensitivity analysis, it is therefore possible to obtain a comprehensive view of appropriate strategies to increase the number of people using e-commerce.

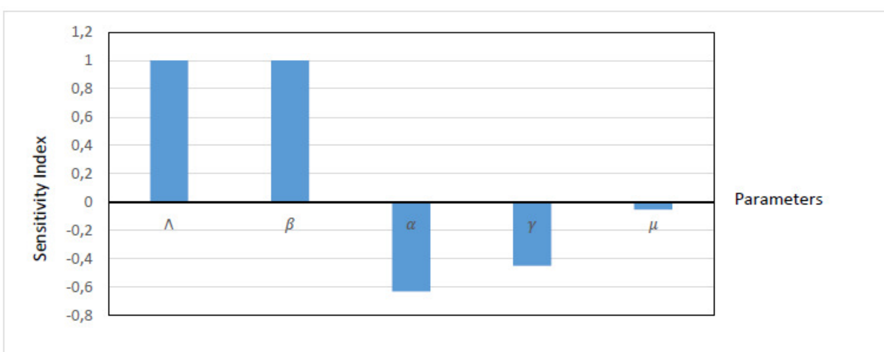


Figure 2: Sensitivity analysis of $R_0$.

### 3.4 Model's numerical simulation

This section presents some numerical solutions to the system (1) for different values of the parameters. We use different initial values for each state variable and use the following parameters:
$\Lambda$= 880750, $\mu = 1.3\times10^{-2}$, $\alpha = 1.7\times10^{-2}$, $\gamma = 8\times10^{-3}$, $\delta = 7\times10^{-4}$, $\theta = 3\times 10^{-4}$, and $\beta = 6\times10^{-11}$. We have the equilibrium point without e-commerce refusal $X_0 = ($ $2.9358\times10^7$, 0, $3.6472\times10^7$, $1.9196\times10^6$) and $R_0 = 0.0839 < 1$. In this case, we observe that the number of people potentially refusing to use e-commerce approaches $X_0$ over time; that is, all state variables converge to the equilibrium point for four different initial values of each of the state variables studied. We also observe that the percentage of those who refuse to use e-commerce is close to zero. See Figure 3.

Also, for different initial values of each state variable, we have the following parameters: $\Lambda$= 880750, $\mu = 1.3\times10^{-2}$, $\alpha = 1.7\times10^{-2}$, $\gamma = 8\times10^{-3}$, $\delta = 7\times10^{-4}$, $\theta = 3\times10^{-4}$ and $\beta = 2\times10^{-9}$. Our equilibrium point, which corresponds to the situation where the refusal to use e-commerce is able to spread among potential e-commerce customers, is $X_* = ($ $1.0500\times10^7$, $2.6940\times10^7$, $2.8794\times10^7$, $1.5155\times10^6$) and $R_0 = 2.7960 > 1$, and the state variables converge to the equilibrium point $X_*$. In this case, we note that the number of people $P$ and $E$ converge over time to the same value for each of them. See Figure 4.

## 4 The optimal control problem

### 4.1 Problem synopsis

To increase the number of people using e-commerce for their daily needs and reduce the number of those who refuse to use it, we recommend two techniques.

Control $v_1$ involves encouraging customers to use e-commerce. The importance of e-commerce is introduced to customers, highlighting its advantages such as time-saving, reduced traffic accidents, and the convenience of shop-

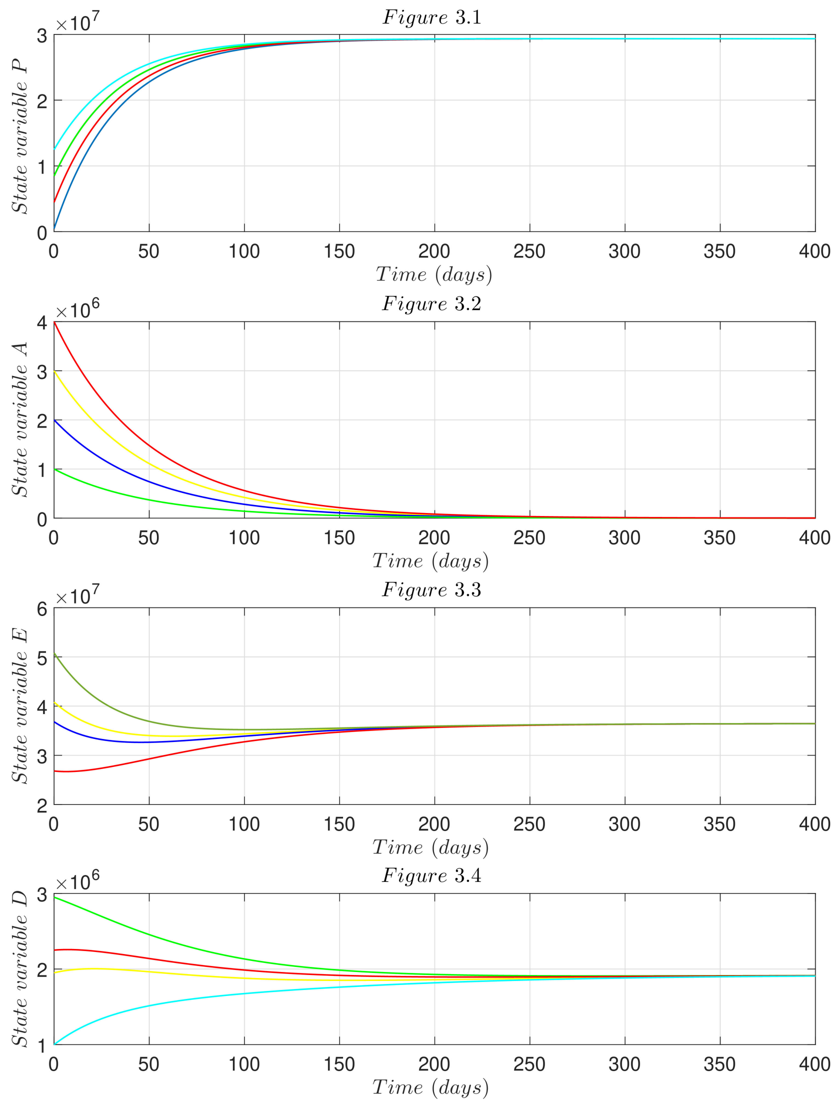


Figure 3: Rejection-free equilibrium model for e-commerce.

ping from anywhere at any time. Advertising is carried out through various means, such as audio and visual media, social media, and newspapers.

Control $v_2$ is to provide insurance and guarantees to individuals who refuse to use e-commerce for fear of cybercrime or online fraud. This will

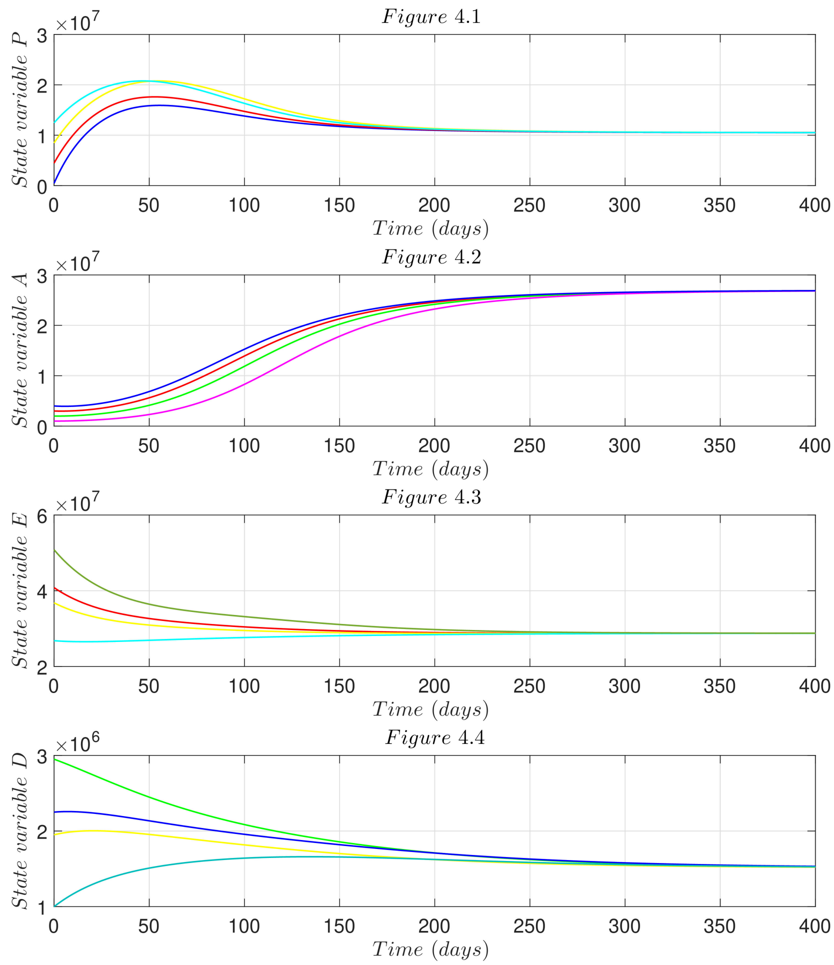


Figure 4: Equilibrium model with rejection of e-commerce.

protect them if their money is stolen, they fall victim to fraud, or their personal information is stolen.

By adding the two controls $v_1(t)$ and $v_2(t)$ to the first system (1), we obtain the following control model:

$$\begin{cases} \frac{dP(t)}{dt} = \Lambda - \beta P(t)A(t) - \alpha P(t) - \mu P(t) - v_1(t)P(t), \\ \frac{dA(t)}{dt} = \beta P(t)A(t) - \gamma A(t) - \mu A(t) - v_2(t)A(t), \\ \frac{dE(t)}{dt} = \alpha P(t) + \gamma A(t) + \theta D(t) \\ \qquad -\delta E(t) - \mu E(t) - \mu E(t) + v_1(t)P(t) + v_2(t)A(t), \\ \frac{dD(t)}{dt} = \delta E(t) - \theta D(t) - \mu D(t), \end{cases} \tag{9}$$

with the initial conditions $P(0) \geq 0$, $A(0) \geq 0$, $E(0) \geq 0$, and $D(0) \geq 0$. Here, our goal is to reduce the total number of customers who refuse to use e-commerce using the control variables $v_1(t)$ and $v_2(t)$ in the model (1). We define the objective function as follows:

$$J(v_1, v_2) = A(t_f) - E(t_f) + \int_{t_0}^{t_f} \left[ A(t) - E(t) + \frac{C_1}{2}(v_1(t))^2 + \frac{C_2}{2}(v_2(t))^2 \right] dt, \tag{10}$$

where $C_1 > 0$ and $C_2 > 0$ are the cost coefficients. They are selected to weigh the relative importance of $v_1(t)$ and $v_2(t)$ at time $t$, and $t_f$ is the final time.

In other words, we seek the optimal controls $v_1^*$ and $v_2^*$ such that

$$J(v_1^*, v_2^*) = \min_{(v_1,v_2)\in E_d} J(v_1, v_2), \tag{11}$$

where $E_d$ is the set of admissible controls defined by

$$E_d = \{(v_1(t), v_2(t)) : 0 \leq v_1(t) \leq 1 \text{ and } 0 \leq v_2(t) \leq 1 \; for \; t \in [t_0, t_f]\}. \tag{12}$$

## 4.2 The existence result of the optimal control

In this part, we present a theorem, which proves the existence of an optimal control ($v_1^*$,$v_2^*$) minimizing the cost function $J$.

**Theorem 7.** There exists an optimal control $(v_1^*, v_2^*) \in E_d$ such that

$$J(v_1^*, v_2^*) = \min_{(v_1,v_2)\in E_d} J(v_1, v_2).$$

*Proof.* To use the existence result in [12], we must check the following properties:

$(Q_1)$: The set of controls and the corresponding state variables is nonempty.

$(Q_2)$: The control set $E_d$ is convex and closed.

$(Q_3)$: The right-hand side of the state system is bounded by a linear function in the state and control variables.
$(Q_4)$: The integrand $L(A,E,v_1,v_2)$ in the objective functional is convex on $E_d$, and there exist constants $\chi_1 > 0$, $\chi_2 > 0$, and $\sigma > 1$ such that

$$L(A,E,v_1,v_2) = A(t) - E(t) + \frac{C_1}{2}\left(v_1(t)\right)^2 + \frac{C_2}{2}\left(v_2(t)\right)^2 \\ \geq -\chi_1 + \chi_2\left(|v_1|^2 + |v_2|^2\right)^{\frac{\sigma}{2}}.$$

The first condition $(Q_1)$ is verified using the result in [27]. The set $E_d$ is convex and closed by definition, thus the condition $(Q_2)$ is verified. Our state system is linear in $v_1$ and $v_2$. Moreover the solutions of the system are bounded as proved in model (1) so the condition $(Q_3)$. Also, we have the last condition needed,

$$L(A,E,v_1,v_2) \geq -\chi_1 + \chi_2\left(|v_1|^2 + |v_2|^2\right)^{\frac{\sigma}{2}},$$

where

$$\chi_1 = 2\sup_{t\in[t_0,t_f]}(A(t),E(t)),\ \chi_2 = \inf(\frac{C_1}{2},\frac{C_2}{2}) \text{ and } \sigma = 2 \text{ since } C_1 > 0.$$

We conclude that there exists an optimal control $(v_1^*, v_2^*) \in E_d$ such that

$$J(v_1^*, v_2^*) = \min_{(v_1,v_2)\in E_d} J(v_1,v_2).$$

□

### 4.3 Characterization of the optimal controls

Pontryagin's maximum principle [24, 34, 4] transforms (9)–(11) and (12) into a problem of minimizing a Hamiltonian, $H$, point-wise with respect to $v_1$ and $v_2$:

$$H(t) = A(t) - E(t) + \frac{C_1}{2}\left(v_1(t)\right)^2 + \frac{C_2}{2}\left(v_2(t)\right)^2 + \sum_{i=1}^{4}\lambda_i g_i, \tag{13}$$

where $g_i$ is the right side of the differential equations system (9).

In the following theorem, we give the necessary conditions for an optimal control to exist.

**Theorem 8.** Given the optimal controls $(v_1^*, v_2^*)$ and solutions $P^*$, $A^*$, $E^*$, and $D^*$ of the corresponding state system (9), there exists adjoint variables $\lambda_1,\ \lambda_2,\ \lambda_3$, and $\lambda_4$ that satisfy the following equations:

$$\begin{aligned}
\lambda_1' &= -\frac{dH}{dP} = -\lambda_1[-\beta A(t) - \alpha - \mu - v_1(t)] - \lambda_2[\beta A(t)] - \lambda_3[\alpha + v_1(t)],\\
\lambda_2' &= -\frac{dH}{dA} = -1 - \lambda_1\left(-\beta P(t)\right) - \lambda_2[\beta P(t) - \gamma - \mu - v_2\left(t\right)]\lambda_3[\gamma + v_2(t)],\\
\lambda_3' &= -\frac{dH}{dE} = 1 - \lambda_3[-\delta - \mu] - \lambda_4\left(\delta\right),\\
\lambda_4' &= -\frac{dH}{dD} = -\lambda_3\left(\theta\right) - \lambda_4\left(-\theta - \mu\right),
\end{aligned}$$

such that the transversality conditions at time $t_f$ are: $\lambda_1(t_f) = 0$, $\lambda_2(t_f) = 1$, $\lambda_3(t_f) = -1$ and $\lambda_4(t_f) = 0$. Additionally we have, for $t \in [t_0, t_f]$, the optimal controls $v_1^*(t)$ and $v_2^*(t)$ are given by:

$$\begin{cases}
v_1^*(t) = \min\left(1, \max\left(0, \frac{P(t)}{C_1}\left(\lambda_1 - \lambda_3\right)\right)\right),\\
\\
v_2^*(t) = \min\left(1, \max\left(0, \frac{A(t)}{C_2}\left(\lambda_2 - \lambda_3\right)\right)\right).
\end{cases} \tag{14}$$

*Proof.* The Hamiltonian $H$ is defined in the following way:

$$H(t) = A(t) - E(t) + \frac{C_1}{2}\left(v_1(t)\right)^2 + \frac{C_2}{2}\left(v_2(t)\right)^2 + \sum_{i=1}^{4}\lambda_i g_i,$$

where

$$\begin{cases}
g_1 = \Lambda - \beta P(t)A(t) - \alpha P(t) - \mu P(t) - v_1(t)P(t),\\
g_2 = \beta P(t)A(t) - \gamma A(t) - \mu A(t) - v_2(t)A(t),\\
g_3 = \alpha P(t) + \gamma A(t) + \theta D(t) - \delta E(t) - \mu E(t)\\
\qquad -\mu E(t) + v_1(t)P(t) + v_2(t)A(t),\\
g_4 = \delta E(t) - \theta D(t) - \mu D(t).
\end{cases}$$

For $t \in [t_0, t_f]$, the adjoint equations and transversality conditions can be obtained by using Pontryagin's maximum principle [24, 20] such that

$$\lambda_1' = -\frac{dH}{dP} = -\lambda_1[-\beta A(t) - \alpha - \mu - v_1(t)] - \lambda_2[\beta A(t)] - \lambda_3[\alpha + v_1(t)],$$

$$\lambda_2' = -\frac{dH}{dA} = -1 - \lambda_1\left(-\beta P(t)\right) - \lambda_2[\beta P(t) - \gamma - \mu - v_2\left(t\right)] - \lambda_3[\gamma + v_2(t)],$$
$$\lambda_3' = -\frac{dH}{dE} = 1 - \lambda_3[-\delta - \mu] - \lambda_4\left(\delta\right),$$
$$\lambda_4' = -\frac{dH}{dD} = -\lambda_3\left(\theta\right) - \lambda_4\left(-\theta - \mu\right).$$

For $t \in [t_0, t_f]$, the optimal controls $v_1^*$ and $v_2^*$ can be solved from the optimal conditions,

$$\frac{dH}{dv_1} = 0,$$
$$\frac{dH}{dv_2} = 0.$$

That is,

$$\frac{dH}{dv_1} = C_1 v_1(t) - \lambda_1 P(t) + \lambda_3 P(t) = 0,$$
$$\frac{dH}{dv_2} = C_2 v_2(t) + \lambda_2\left(-A\left(t\right)\right) + \lambda_3\left(A(t)\right) = 0.$$

We have

$$v_1 = \frac{P(t)}{C_1}\left(\lambda_1 - \lambda_3\right),$$
$$v_2 = \frac{A(t)}{C_2}\left(\lambda_2 - \lambda_3\right).$$

Thus, by the bounds in $E_d$, the set of admissible controls, we can smoothly obtain $v_1^*$ and $v_2^*$ in the form of (14). □

## 5 Numerical simulations and discussion

In our research, a set of preliminary data was collected for the country of France to serve as initial conditions for the system (1). Thus, in 2021, the population of France available on the Internet reached 86.1% [38] of the total population of this country. In the same year, the percentage of the French population using e-commerce was 69.1% [40] of the total French population. Below, we present a set of data for the application of numerical simulation in MATLAB:

Conditions for system (1) are as follows: $P(0) = 8467320$, $A(0) = 3000000$, $E(0) = 46815250$, $D(0) = 50000$, $\Lambda = 880750$, $\mu = 9 \times 10^{-3}$, $\alpha = 1.7 \times 10^{-2}$, $\gamma = 8 \times 10^{-3}$, $\delta = 7 \times 10^{-4}$, $\theta = 3 \times 10^{-4}$ and $\beta = 10^{-9}$.

## 5.1 Strategy $v_1$: Promotion of the importance of e-commerce through advertising

■ Figure 5.1: Before the implementation of the control element $v_1$, the number of people who refused to use e-commerce was 3 million. After applying the first strategy, from day 37, the number of people who refused to use e-commerce started to decrease, so that the percentage of this decrease reached about 6.92% on day 60. The analysis shows that after 100 days of implementing the first strategy, the number of merchants who refused to use e-commerce decreased by 35.20%.

■ Figure 5.2: Before applying the second strategy, the number of individuals using e-commerce was approximately 46815250 individuals. After applying the control element $v_1$, this number started to increase from day 17, so that the percentage of this increase reached about 6.33% on day 60. It is also clear that the number of people using electronic commerce increased by 18% after 100 days of application First strategy.

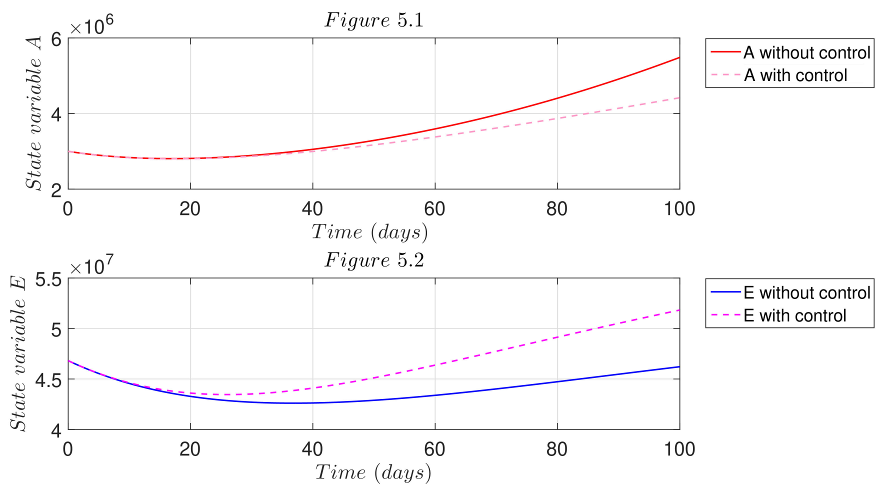


Figure 5: Dynamics with control $v_1$.

## 5.2 Strategy $v_2$ : Cybercrime insurance

■ Figure 6.1: Before implementing the $v_2$ control, the number of people who refused to use e-commerce was 3 million. After applying the second strategy, this number started to decrease from the seventh day. The percentage of this decrease reached 90.19% on the 60th day after the application of this strategy. From this analysis, we can conclude that the number of people who refused to use e-commerce decreased by 162.41% after 100 days of applying the second strategy.

■ Figure 6.2: Before applying the second strategy, the number of individuals using e-commerce was approximately 46815250 individuals. After applying the $v_2$ control, this number started to increase from day 7. The percentage of this increase reached 3.11% after 60 days of applying this strategy. According to this analysis, the number of people using e-commerce increased by 3.68% after 100 days of applying the second strategy.

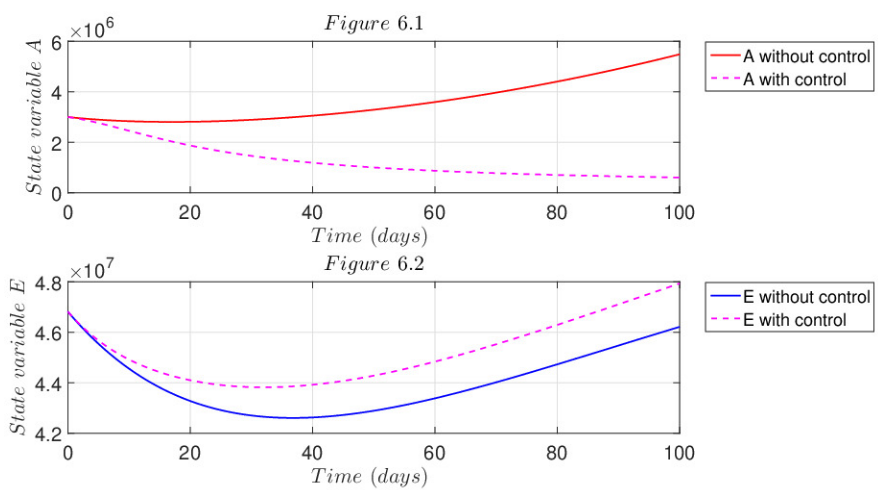


Figure 6: Dynamics with control $v_2$.

### 5.3 Strategy $v_1$ and $v_2$: Advertising and cybercrime insurance

■ Figure 7.1: Before applying controls $v_1$ and $v_2$, the number of people who refused to use e-commerce was 3 million. After applying the third strategy, this number started to decrease from the fourth day. The percentage of this decrease reached 91.56% on the 60th day after applying this strategy. From this analysis, we can conclude that the number of merchants who refused to use e-commerce decreased by 165.33% after 100 days of implementing the third strategy.

■ Figure 7.2: Before implementing the third strategy, the number of individuals using e-commerce was approximately 46815250 individuals. After applying the control elements $v_1$ and $v_2$, this number started to increase from the 4th day. The percentage of this increase reached 9.31% after 60 days of applying this strategy. From this analysis, we can conclude that the number of people using e-commerce increased by 14.61% after 100 days of implementing the second strategy.

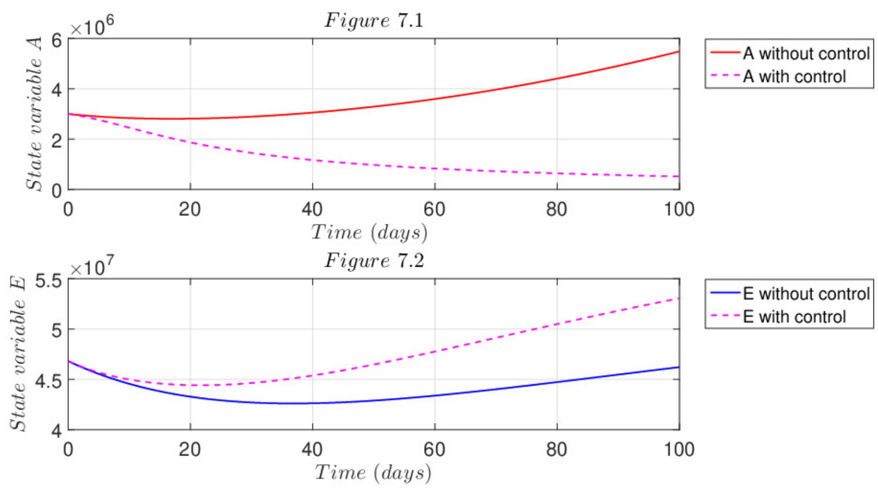


Figure 7: Dynamics with control $v_1$ and $v_2$.

In conclusion, the analysis of the three proposed strategies provides valuable insights into their effectiveness in addressing the issue of customer rejection behavior in e-commerce. Strategy $v_1$, focusing on promoting the importance of e-commerce through advertising, demonstrates promising results. It led to a significant decrease in the number of individuals rejecting e-commerce, with a reduction of 35.20% observed after 100 days of implementation. Moreover, there was a notable increase in the number of people utilizing e-commerce, indicating a positive impact on adoption rates. Strategy $v_2$, centered around cybercrime insurance, proved to be highly effective in mitigating customer rejection of e-commerce. The strategy resulted in a substantial decrease of 162.41% in the number of individuals rejecting e-commerce after 100 days. Additionally, there was a moderate increase in e-commerce adoption rates, further reinforcing the efficacy of this approach. Furthermore, combining both strategies, as in Strategy $v_1$ and $v_2$, yielded even more promising outcomes. The simultaneous implementation of advertising and cybercrime insurance led to a remarkable decrease of 165.33% in e-commerce rejection rates after 100 days. Furthermore, there was a notable increase in e-commerce adoption, highlighting the synergistic effects of employing multiple strategies.

Overall, these findings underscore the importance of tailored interventions in addressing customer rejection behavior in e-commerce. By strategically leveraging advertising and cybercrime insurance, policymakers and businesses can foster a more conducive environment for e-commerce adoption, ultimately driving growth and innovation in the digital marketplace.

## 6 Conclusion

In this research, we formulated a new mathematical model that describes the dynamics of people who refuse to use e-commerce and people who use e-commerce. We also computed the basic reproduction number $R_0$ and studied the parameters involved the sensitivity analysis to determine which parameters have the greatest influence on the $R_0$. We also analyzed the local and global stability of the behavior of those who refuse to use e-commerce. We also performed numerical simulations to verify the theoretical analysis using MATLAB. Finally, we proposed three different control strategies. This was done using Pontryagin's maximum principle to describe optimal controls. After analysis, it was found that the third strategy was the most effective in increasing the number of people using e-commerce and reducing the number of people rejecting it.

## Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## References


[1] Aarabate, O., Belhdid, S. and Balatif, O., *Stability analysis and optimal strategies for controlling a boycotting behavior of a commercial product,* Iranian Journal of Numerical Analysis and Optimization 14 (3) (2024), 708–735.

[2] Balatif, O., Khajji, B. and Rachik, M., *Mathematical modeling, analysis, and optimal control of abstinence behavior of registration on the electoral lists,* Discrete Dyn. Nature Soc. 2020 (2020), 9738934.

[3] Balatif, O., Labzai, A. and Rachik, M., *A discrete mathematical modeling and optimal control of the electoral behavior with regard to a political party,* Discrete Dyn. Nature Soc. 2018, (2018) 1–14.

[4] Benfatah, Y., Khaloufi, I., Boutayeb, H., Rachik, M. and Laarabi, H., *Optimal control for a discrete time epidemic model with zones evolution,* Commun. Math. Biol. Neurosci., 2022 (2022) 51.

[5] Belhdid, S., Balatif, O. and Khajji, B., *Optimal control and cost-effectiveness analysis of scam rumor propagation over social networks,* Results Control Optim. (2024) 100441.

[6] Behnam, M., Hollebeek, L.D., Clark, M.K. and Farabi, R., *Exploring customer engagement in the product vs. service context,* J. Retail. Consum. Serv. 60 (2021) 102456.

[7] Bonina, C., Koskinen, K., Eaton, B. and Gawer, A., *Digital platforms for development: Foundations and research agenda,* Inf. Syst. J. 31(6) (2021) 869–902.

[8] Corbitt, B.J., Thanasankit, T. and Yi, H., *Trust and e-commerce: A study of consumer perceptions,* Electron. Commer. Res. Appl.. 2(3) (2003) 203–215.

[9] Dionysiou, G., Fouskas, K. and Karamitros, D., *The impact of Covid-19 in E-commerce. effects on consumer purchase behavior,* In Strategic Innovative Marketing and Tourism in the COVID19 Era: 9th ICSIMAT Conference 2020 (pp. 199–210). Springer International Publishing, 2021.

[10] El Bhih, A., Benfatah, Y., Kouidere, A. and Rachik, M., *A discrete mathematical modeling of transmission of COVID-19 pandemic using optimal control,* Commun. Math. Biol. Neurosci. 2020 (2020) Article-ID.

[11] Fedorko, M.R., *The current status and importance of e-commerce in a global perspective,* Online J. 4509 (2014) 1339–4509.

[12] Fleming, W.H. and Rishel, R.W., *Deterministic and stochastic optimal control* (Vol. 1). Springer Science and Business Media, 2012.

[13] Gour, A., Aggarwal, S. and Erdem, M., *Reading between the lines: analyzing online reviews by using a multi-method Web-analytics approach,* Int. J. Contemp. Hosp. Manag. 33(2) (2021) 490–512.

[14] Hethcote, H.W. and Thieme, H.R., *Stability of the endemic equilibrium in epidemic models with subpopulations,* Math. Biosci. 75(2), (1985) 205–227.

[15] Huang, L., Huang, Y., Huang, R., Xie, G. and Cai, W., *Factors influencing returning migrants' entrepreneurship intentions for rural e-commerce: An empirical investigation in China,* Sustainability, 14(6) (2022) 3682

[16] Jones, L.S. and Samples, T.R., *On the systemic importance of digital platforms.* U. Pa. J. Bus. L., 25 (2023) 141.

[17] LaSalle, J.P. *The stability of dynamical systems,* Regional Conference Series in Applied Mathematics, Vol. 25, SIAM, Philadelphia, PA, USA, 1976.

[18] Khajji, B., Boujallal, L., Balatif, O. and Rachik, M., *Mathematical modelling and optimal control strategies of a multistrain COVID-19 spread,* J. Appl. Math. 2022 (1) (2022): 9071890.

[19] Khajji, B., Boujallal, L., Elhia, M., Balatif, O. and Rachik, M., *A fractional-order model for drinking alcohol behaviour leading to road accidents and violence,* Math. Model. Comput. 9, (2022), 501–518.

[20] Khaloufi, I., Benfatah, Y., Laarabi, H. and Rachik, M., *A scenario to fight monkeypox using a mathematical model,* Commun. Math. Biol. Neurosci., 2022 (2022) 99.

[21] Kermack, W.O. and McKendrick, A.G., *A contribution to the mathematical theory of epidemics. Proceedings of the royal society of London,* Proc. R. soc. Lond. Ser. A-Contain. Pap. Math. Phys. Character. 115(772) (1927), 700–721.

[22] Kouidere, A., Labzai, A., Khajji, B., Ferjouchia, H., Balatif, O., Boutayeb, A. and Rachik, M., *Optimal control strategy with multi-delay in state and control variables of a discrete mathematical modeling for the dynamics of diabetic population,* Commun. Math. Biol. Neurosci. 2020 (2020) 14.

[23] Kouidere, A., Balatif, O. and Rachik, M., *Cost-effectiveness of a mathematical modeling with optimal control approach of spread of COVID-19 pandemic: A case study in Peru.* Chaos, Solitons Fractals. X, 10 (2023) 100090.

[24] Kouidere, A., Youssoufi, L.E., Ferjouchia, H., Balatif, O. and Rachik, M., *Optimal control of mathematical modeling of the spread of the COVID-19 pandemic with highlighting the negative impact of quarantine on diabetics people with cost-effectiveness,* Chaos, Solitons Fractals. 145 (2021) 110777.

[25] Labzai, A., Kouidere, A., Khajji, B., Balatif, O. and Rachik, M., *Mathematical modeling and optimal control strategy for a discrete time drug consumption model,* Discrete Dyn. Nature Soc. 2020 (2020) 1–10.

[26] Lightner, N.J., *What users want in e-commerce design: effects of age, education and income,* Ergonomics, 46(1-3), (2003) 153–168.

[27] Lukes, D.L., *Differential equations: classical to controlled,* Academic Press, New York, 1982.

[28] Makinde, O.D. and Okosun, K.O., *Impact of chemo-therapy on optimal control of malaria disease with infected immigrants,* Biosyst. 104(1), (2011) 32–41.

[29] Moorman, C., van Heerde, H.J., Moreau, C.P. and Palmatier, R.W., *Challenging the boundaries of marketing,* J. Mark. 83(5) (2019) 1–4.

[30] Nelson, J.L. and Lei, R.F., *The effect of digital platforms on news audience behavior,* Digit. Journal. 6(5), (2018) 619–633.

[31] Olson, J.S. and Olson, G.M., *I2i trust in e-commerce.* Commun. ACM, 43(12) (2000), 41–44.

[32] Park, Y.H. and Fader, P.S., *Modeling browsing behavior at multiple websites,* Mark. Sci. 23(3), (2004) 280–303.

[33] Driessche, P.V.and Watmough, J. *Reproduction numbers and subthreshold endemic equilibria for compartmental models of disease transmission,* Math.l Biosci. 180 (2002) 29–48.

[34] Pontryagin, L.S., *The mathematical theory of optimal processes and differential games,* Trudy Mat. Inst. Steklov, 169, (1985) 119–158.

[35] Rajala, T., Huhtamäki, J., Johanson, J.E. and Vakkuri, J., *Social impacts of digital platforms,* Public Innovation and Digital Transformation, (2022) 32–55.

[36] Salminen, J., Gach, N. and Kaartemo, V., *Platform as a social contract: An analytical framework for studying social dynamics in online platforms,* Collaborative value co-creation in the platform economy, (2018) 41–64.

[37] Shemi, A.P. and Procter, C., *E-commerce and entrepreneurship in SMEs: case of myBot,* Journal of Small Business and Enterprise Development, 25(3) (2018) 501–520.

[38] Statista.https://fr.statista.com/statistiques/472073/utilisateurs-internet-france/

[39] Svatosova, V., *The importance of online shopping behavior in the strategic management of e-commerce competitiveness* J. Compet. 12(4) (2020), 143.

[40] The National Institute of Statistics and Economic Studies collects, analyses and disseminates information on the French economy and society. https://www.insee.fr/en/accueil

[41] Wang, M., Chen, W., Huang, Y., Contractor, N.S. and Fu, Y., *Modeling customer preferences using multidimensional network analysis in engineering design,* Des. Sci. 2 (2016) e11.