Optimal control analysis for modeling HIV transmission

K. R. Cheneke

Abstract

In this study, a modified model of HIV with therapeutic and preventive controls is developed. Moreover, a simple evaluation of the optimal control problem is investigated. We construct the Hamiltonian function by way of integrating Pontryagin's maximal principle to achieve the point-wise optimal solution. The effects obtained from the version analysis strengthen public health education to a conscious population, PrEP for early activation of HIV infection prevention, and early treatment with artwork for safe life after HIV infection. Moreover, numerical simulations are done using the MATLAB platform to illustrate the qualitative conduct of the HIV infection. In the end, we receive that adhering to ART protective prone people, the usage of PrEP along with different prevention control is safer control measures.

1 Introduction

Human immunodeficiency virus (HIV), the cause of HIV infection, has no curative medication until now [2]. Moreover, the long-time existence of the virus in the body leads to a serious infection called acquired immunodeficiency syndrome (AIDS) disease [6]. However, optimal controls are the effective way to combat HIV transmission and progression in the community [2, 6]. Public health education, condom, and anti-retrovirus therapy are the

Kumama Regassa Cheneke

Department of Mathematics, Wollega University, Nekemte, Ethiopia. e-mail: kumamaregassa@gmail.com

major measures taken by both governmental and nongovernmental institutions to stop further progression and transmission of HIV in the populations [1, 3, 12, 4, 5, 7, 8, 9]. Moreover, effective pre-exposure prophylaxis (PrEP) is the drug used to prevent the survival of HIV in the human blood [14]. On the other hand, the abstinence of sexual practices through the activation of public health education reduces the fate of acquiring HIV from potentially infectious individuals. Mathematical models are very important tools to describe the behavior of biological events. Particularly, with the great contribution of Pontryagin's Maximum Principle (PMP) in the construction of optimal control problems, the nature of biological dynamics is studied intensively [13, 14, 15, 18, 19, 20, 10, 22, 23]. Based on the works done in [12], the motivation of this study is due to the significant contribution of public health education and prophylaxis in controlling the transmission of HIV infection among human individuals. Particularly, abstinence due to consolidated public health education builds positive awareness toward controlling oneself, whereas prophylaxis helps to prevent the progression of HIV in the human body. Mathematical models are important tools to control infections [24, 11, 27, 26, 16, 17]. In this study, we have included prophylaxis, antiretroviral therapy (ART), and prevention for controlling the transmission of HIV infection by modifying the model studied in [12].

2 Formulation of model

In this study, a mathematical model is formulated by classifying the total population into compartments of (i) Susceptible individuals (S), (ii) Individuals on Pre-exposure prophylaxis (E), (iii) HIV infected with primary stage (P), (iv) Not on treatment HIV infected individuals (J), (v) HIV Undetectable individuals (U), and (vi) On treatment HIV infected individuals (I).

Moreover, the subsequent assumptions are considered in the modeling of the infection (i) a new susceptible individuals becomes susceptible at recruitment rate of λ, (ii) individuals in S transfer to E due to taking PrEP at the rate of ρ; (iii) transmission rate of HIV infection from individuals in P to S is β_1 and transmission rate of HIV infection from I to S is β_2; (iv) individuals transfer from P to I at progression rate of ξ; (v) individuals transfer from P to J at transfer rate of η; (vi) individuals transfer from J to I at transfer rate of γ; (vii) individuals in the compartment J die due to infection at the rate ζ; (viii) individuals transfer from I to U due to adherence to ART at transferring rate of θ; (ix) individuals transfer from U to I due to default using of ART at the rate of ϕ; (x) natural induced death rate of all people is μ; (xi) AIDS induced death rate is δ; (xii) in this study, standard incidence rate is applied; (xiii) PrEP engagement effort is u_3; (xiv) Condom using effort is u_1; (xv) ART using effort is u_2.

The pictorial representation of the deterministic model with control measures is given in Figure 1.
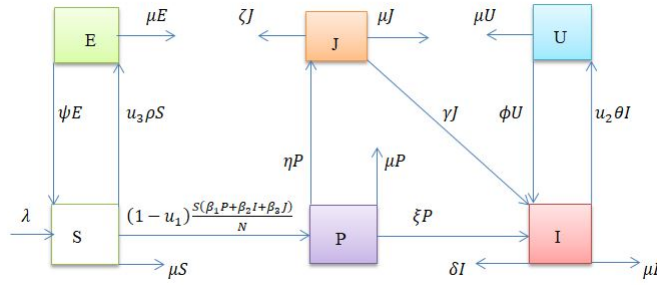
Figure 1: Schematic diagram of HIV transmission dynamics.

The deterministic model of population dynamics subject to HIV infection in the presence of control measures is given by

$$
\begin{aligned}
\frac{dS}{dt} &= \lambda - \frac{(1 - u_1)\, S\, (\beta_1 P + \beta_2 I + \beta_3 J)}{N} + \psi E - (u_3 \rho + \mu)\, S, \\
\frac{dE}{dt} &= u_3 \rho S - (\mu + \psi)\, E, \\
\frac{dP}{dt} &= \frac{(1 - u_1)\, S\, (\beta_1 P + \beta_2 I + \beta_3 J)}{N} - (\xi + \eta + \mu)\, P, \\
\frac{dJ}{dt} &= \eta P - (\gamma + \zeta + \mu) J, \\
\frac{dI}{dt} &= \xi P + \gamma J + \phi U - (u_2 \theta + \mu + \delta)\, I, \\
\frac{dU}{dt} &= u_2 \theta I - (\phi + \mu)\, U,
\end{aligned}
\tag{1}
$$

with initial conditions: $S(0) \geq 0, \; E(0) \geq 0, \; P(0) \geq 0, \; J(0) \geq 0, \; I(0) \geq 0, \; U(0) \geq 0, \; 0 \leq u_i \leq 1, \; i = 1, 2, 3, 4.$

3 Analysis of the model without control

3.1 Invariant region

Theorem 1. The solution of model (1) is invariant in the region Ω proper-subset of six-dimensional space over the set of nonnegative real numbers such that

$$
\Omega = \{(S, E, P, J, U, I) \in R_+^6 : N(0) \leq \frac{\lambda}{\mu}\}.
\tag{2}
$$

Proof. The equations of model (1) gives the subsequent equation:

$$\frac{dN}{dt} = \lambda - \mu N - \delta I - \zeta J,$$

which implies

$$\frac{dN}{dt} \leq \lambda - \mu N.$$

Applying mathematical procedures, the preceding inequality gives

$$N(t) \leq \frac{\lambda}{\mu} - \left(\frac{\lambda}{\mu} - N(0)\right) e^{-\mu t},$$

which implies, as time t varies, the total population size is bounded for all time t, with the given initial condition. $\square$

3.2 Nonnegative property

Theorem 2. All solution variables of model (1) without control are nonnegative in the stated invariant region of the solution.

Proof. Consider the first equation of model (1) without control. Then

$$\frac{dS}{dt} = \lambda - \frac{S\left(\beta_1 P + \beta_2 I + \beta_3 J\right)}{N} + \psi E - \mu S, \tag{3}$$

which implies

$$\frac{dS}{dt} \geq -\frac{S\left(\beta_1 P + \beta_2 I + \beta_3 J\right)}{N} - \mu S. \tag{4}$$

Solving the preceding inequality, we get

$$S(t) \geq S(0)e^{-\mu t - \int_0^t \frac{(\beta_1 P(\xi) + \beta_2 I(\xi) + \beta_3 J(\xi))}{N(\xi)} d\xi}. \tag{5}$$

Hence, based on the initial condition, the susceptible population size is nonnegative for all time t. $\square$

3.3 Basic reproduction number

The basic reproduction number R_0 of model (1) without control is the average number of infected individuals produced by typical infectious individuals in the susceptible population during the entire period of infection. Based on the techniques applied, we compute basic reproduction numbers from model (1) without control as follows. Let F and V be the Jacobian matrices obtained from model (1) as given below:

$$F = \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \xi + \eta + \mu & 0 & 0 & 0 \\ -\eta & \gamma + \zeta + \mu & 0 & 0 \\ -\xi & -\gamma & \delta + \mu & -\phi \\ 0 & 0 & 0 & \phi + \mu \end{pmatrix}.$$

The spectral radius $\rho\left(FV^{-1}\right)$ computed from next-generation matrix FV^{-1} of foregoing matrices is given by

$$\rho\left(FV^{-1}\right) = \frac{\beta_1}{\xi + \eta + \mu} + \frac{\beta_2 \eta}{(\xi + \eta + \mu)(\gamma + \zeta + \mu)} + \frac{\beta_3(\xi\mu + \xi\gamma + \xi\zeta + \eta\gamma)}{(\delta + \mu)(\xi + \eta + \mu)(\gamma + \zeta + \mu)}.$$

Therefore, by the definition, we obtain

$$R_0 = \frac{\beta_1}{\xi + \eta + \mu} + \frac{\beta_2 \eta}{(\xi + \eta + \mu)(\gamma + \zeta + \mu)} + \frac{\beta_3(\xi\mu + \xi\gamma + \xi\zeta + \eta\gamma)}{(\delta + \mu)(\xi + \eta + \mu)(\gamma + \zeta + \mu)}.$$

3.4 Global stability of disease-free equilibrium

Theorem 3. The global stability of a disease-free equilibrium point is described as a steady state where the trajectory of solution shows the tendency of moving toward it for all time t.

Proof. To show the global stability of disease-free equilibrium, we incorporate the method applied in the works of [21]. Next, from the computed matrices for construction of next-generation, we obtain

$$F = \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} \xi + \eta + \mu & 0 & 0 & 0 \\ -\eta & \gamma + \zeta + \mu & 0 & 0 \\ -\xi & -\gamma & \delta + \mu & -\phi \\ 0 & 0 & 0 & \phi + \mu \end{pmatrix}.$$

Moreover, the rate of change of variables (P, J, I, U) at disease-free equilibrium can be written as

$$\begin{pmatrix} \frac{dP}{dt} \\ \frac{dJ}{dt} \\ \frac{dI}{dt} \\ \frac{dU}{dt} \end{pmatrix} \leq (F - V) \begin{pmatrix} P \\ J \\ I \\ U \end{pmatrix}.$$

Therefore, by the comparison method applied in [21], we justify that model (1) without control has a globally asymptotically stable disease-free equilibrium. $\qquad\square$

4 Extension to control problem

The deterministic model of population dynamics subject to HIV infection, in the presence of control measures, is given by

$$
\begin{aligned}
\frac{dS}{dt} &= \lambda - \frac{(1-u_1)\,S\,(\beta_1 P + \beta_2 I + \beta_3 J)}{N} + \psi E - (u_3 \rho + \mu)\,S, \\
\frac{dE}{dt} &= u_3 \rho S - (\mu + \psi)\,E, \\
\frac{dP}{dt} &= \frac{(1-u_1)\,S\,(\beta_1 P + \beta_2 I + \beta_3 J)}{N} - (\xi + \eta + \mu)\,P, \\
\frac{dJ}{dt} &= \eta P - (\gamma + \zeta + \mu)J, \\
\frac{dI}{dt} &= \xi P + \gamma J + \phi U - (u_2 \theta + \delta + \mu)\,I, \\
\frac{dU}{dt} &= u_2 \theta I - (\phi + \mu)\,U,
\end{aligned}
\tag{6}
$$

with initial conditions $S(0) \geq 0$, $E(0) \geq 0$, $P(0) \geq 0$, $J(0) \geq 0$, $I(0) \geq 0$, $U(0) \geq 0$, $0 \leq u_i \leq 1$, $i = 1, 2, 3$.
To study the optimal levels of the controls, we define the Lebesgue measurable control set U as

$$
U = \{(u_1, u_2, u_3) : 0 \leq u_1 \leq 1, 0 \leq u_2 \leq 1, 0 \leq u_3 \leq 1, 0 \leq t \leq t_f\}. \tag{7}
$$

Our goal is to find the optimal controls that minimize objective functional J given by

$$
J = \min_{(u_1, u_2, u_3)} \int_0^{t_f} c_1 P + c_2 I + c_3 J + \frac{1}{2}\left(w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2\right), \tag{8}
$$

where c_j, $j = 1, 2, 3$ and w_i, $i = 1, 2, 3$ are constants. The expressions $0.5 w_i u_i^2$, $i = 1, 2, 3$ are costs associated with controls. The form of cost is quadratic because we assumed it to be nonlinear in nature [24]. Also, for four optimal controls u_1^*, u_2^*, u_3^*, we have

$$
J(u_1^*, u_2^*, u_3^*) = \min\{J(u_1, u_2, u_3) : u_1, u_2, u_3 \in U\},
$$

where $U = \{(u_1, u_2, u_3) : 0 \leq u_1 \leq 1, 0 \leq u_2 \leq 1, 0 \leq u_3 \leq 1\}$. Furthermore, $u_1, u_2 and u_3$ are measurable controls.

4.1 Existence of optimal control solution

Theorem 4. The optimal control solution of a control problem exists if the following Fleming's and Rishel's conditions are satisfied:

(i) The set of all solutions to optimal control problem and objective functional must be nonempty.

(ii) The state system is a linear function of controls with coefficients dependent on state variables and time.

(iii) The integrand in objective functional is convex and bounded above by $d_1(|u_1|^2+|u_2|^2+|u_3|^2)^d-d_2 \leq c_1P+c_2I+c_3J+\frac{1}{2}\left(w_1u_1^2 + w_2u_2^2 + w_3u_3^2\right), d_1 > 0$ and $d > 1$.

Proof. We employ the method from to demonstrate the existence of optimal control. The condition (i) is satisfied if the state system has bounded coefficients. Additionally, the state system operates in accordance with controls, satisfying requirement (ii). The integrand in the objective functional is used to demonstrate condition (iii). Moreover, $c_1P + c_2I + c_3J + \frac{1}{2}\left(w_1u_1^2 + w_2u_2^2 + w_3u_3^2\right)$ is convex on U as any constant, linear and quadratic are convex. Furthermore, assume that there are $d_1, d_2 > 0$, and $d > 1$ satisfying $d_1(|u_1|^2+|u_2|^2+|u_3|^2)^d-d_2 \leq c_1P+c_2I+c_3J+\frac{1}{2}\left(w_1u_1^2 + w_2u_2^2 + w_3u_3^2\right), d_1 = \min\{w_i, i = 1, 2, 3\}$, $d = 2$, and d_2 is the half of coefficient of control functions. Therefore, the optimal solution exists. $\square$

4.2 The Hamiltonian and optimality system

The PMP stated the necessary conditions that are satisfied optimal pair. Hence, by this principle, we obtain the Hamiltonian function (H) defined as [24]

$$
\begin{aligned}
H(x, u, t) =& c_1P + c_2I + c_3J + \frac{1}{2}\left(w_1u_1^2 + w_2u_2^2 + w_3u_3^2\right) \\
&+ \lambda_1\frac{dS}{dt} + \lambda_2\frac{dE}{dt} + \lambda_3\frac{dP}{dt} + \lambda_4\frac{dJ}{dt} + \lambda_5\frac{dI}{dt} + \lambda_6\frac{dU}{dt},
\end{aligned}
$$

where $\lambda_i, i = 1, 2, 3, 4, 5, 6$ are the adjoint variable corresponding to state variables S, E, P, J, I, and U, respectively, and to be determined using the PMP for the existence of optimal pairs.

Theorem 5. Let S, E, P, J, I, U be optimal state variables and let optimal control $u_i, i = 1, 2, 3$ be the optimal controls. Then there exist costate variables $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, and\lambda_6$ that satisfy

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial S}, \qquad \frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial E},$$
$$\frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial P}, \qquad \frac{d\lambda_4}{dt} = -\frac{\partial H}{\partial J},$$
$$\frac{d\lambda_5}{dt} = -\frac{\partial H}{\partial I}, \qquad \frac{d\lambda_6}{dt} = -\frac{\partial H}{\partial U},$$

with transversality or final time conditions $\lambda_1(t_f) = \lambda_2(t_f) = \lambda_3(t_f) = \lambda_4(t_f) = \lambda_5(t_f) = \lambda_6(t_f) = 0$, where H is the Hamiltonian function. Moreover, the optimal controls u_1^*, u_2^*, and u_3^* are $u_1^* = \min\{0, \max\{\frac{\beta S I(\lambda_2 - \lambda_1)}{(w_1 N)}, 1\}\}$ and $u_2^* = \min\{0, \max\{\frac{\alpha I(\lambda_4 - \lambda_2)}{(w_2 N)}, 1\}\}$, over the constraints $0 \leq u_1 \leq 1, 0 \leq u_2 \leq 1$.

Proof. The PMP gives the standard form of adjoint equation with transversality conditions. Now, differentiating the Hamiltonian function with respect to state variables, we have

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial S} = (1 - u_1)\frac{(\beta_1 P + \beta_2 I + \beta_3 J) N - S(\beta_1 P + \beta_2 I + \beta_3 J)}{N^2}(\lambda_1 - \lambda_3)$$
$$+ u_3\rho(\lambda_1 - \lambda_2) + \mu\lambda_1,$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial E} = \psi(\lambda_2 - \lambda_1) + \mu\lambda_2,$$

$$\frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial P} = -c_1 + \frac{(1 - u_1)(\beta_1 SN - S(\beta_1 P + \beta_2 I + \beta_3 J))}{N^2}(\lambda_1 - \lambda_3)$$
$$+ \xi(\lambda_3 - \lambda_5) + \eta(\lambda_3 - \lambda_4) + \mu\lambda_3,$$

$$\frac{d\lambda_4}{dt} = -\frac{\partial H}{\partial J} = \frac{(1 - u_1)(\beta_3 SN - S(\beta_1 P + \beta_2 I + \beta_3 J))}{N^2}(\lambda_1 - \lambda_3)$$
$$+ \gamma(\lambda_4 - \lambda_5) + (\zeta + \mu)\lambda_4,$$

$$\frac{d\lambda_5}{dt} = -\frac{\partial H}{\partial I} = -c_2 + \frac{(1 - u_1)(\beta_2 SN - S(\beta_1 P + \beta_2 I + \beta_3 J))}{N^2}(\lambda_1 - \lambda_3)$$
$$+ u_2\theta(\lambda_5 - \lambda_6) + (\delta + \mu)\lambda_5,$$

$$\frac{d\lambda_6}{dt} = -\frac{\partial H}{\partial U} = \phi(\lambda_6 - \lambda_5) + \mu\lambda_6.$$

Furthermore, the characterization of optimal controls u_1^*, u_2^* and u_3^* shows that

$$\frac{\partial H}{\partial u_1} = \frac{\partial H}{\partial u_2} = \frac{\partial H}{\partial u_3} = 0.$$

Hence, optimal controls over $0 \leq u_1 \leq 1, 0 \leq u_2 \leq 1, 0 \leq u_3 \leq 1$ are given by

$$u_1^* = u_1 = \frac{S(\beta_1 P + \beta_2 I + \beta_3 J)(\lambda_3 - \lambda_1)}{w_1 N},$$

$$u_2^* = u_2 = \frac{\theta I (\lambda_5 - \lambda_6)}{w_2},$$

$$u_3^* = u_3 = \frac{\rho S (\lambda_1 - \lambda_2)}{w_3}.$$

Therefore, the bounds of the optimal control variables are given by

$$u_1^* = \begin{cases} \frac{S(\beta_1 P+\beta_2 I+\beta_3 J)(\lambda_3-\lambda_1)}{w_1 N} & \text{if } 0 < \frac{S(\beta_1 P+\beta_2 I+\beta_3 J)(\lambda_3-\lambda_1)}{w_1 N} < 1, \\ 0 & \text{if } \frac{S(\beta_1 P+\beta_2 I+\beta_3 J)(\lambda_3-\lambda_1)}{w_1 N} \leq 0, \\ 1 & \text{if } 1 \leq \frac{S(\beta_1 P+\beta_2 I+\beta_3 J)(\lambda_3-\lambda_1)}{w_1 N}, \end{cases}$$

$$u_2^* = \begin{cases} \frac{\theta I(\lambda_5-\lambda_6)}{w_2} & \text{if } 0 < \frac{\theta I(\lambda_5-\lambda_6)}{w_2} < 1, \\ 0 & \text{if } \frac{\theta I(\lambda_5-\lambda_6)}{w_2} \leq 0, \\ 1 & \text{if } 1 \leq \frac{\theta I(\lambda_5-\lambda_6)}{w_2}, \end{cases}$$

$$u_3^* = \begin{cases} \frac{\rho S(\lambda_1-\lambda_2)}{w_3} & \text{if } 0 < \frac{\rho S(\lambda_1-\lambda_2)}{w_3} < 1, \\ 0 & \text{if } \frac{\rho S(\lambda_1-\lambda_2)}{w_3} \leq 0, \\ 1 & \text{if } 1 \leq \frac{\rho S(\lambda_1-\lambda_2)}{w_3}. \end{cases}$$

In a compact form, the optimal controls can be written as

$$u_1^* = \min\{0, \max\{\tfrac{S(\beta_1 P+\beta_2 I+\beta_3 J)(\lambda_3-\lambda_1)}{w_1 N}, 1\}\},$$
$$u_2^* = \min\{0, \max\{\tfrac{\theta I(\lambda_5-\lambda_6)}{w_2}, 1\}\},$$
$$u_3^* = \min\{0, \max\{\tfrac{\rho S(\lambda_1-\lambda_2)}{w_3}, 1\}\}. \qquad \Box$$

Moreover, the optimality system of the optimal control problem can be written as

$$\frac{dS}{dt} = \lambda - \frac{(1-u_1) S (\beta_1 P + \beta_2 I + \beta_3 J)}{N} + \psi E - (u_3\rho + \mu) S,$$

$$\frac{dE}{dt} = u_3\rho S - (\mu + \psi) E,$$

$$\frac{dP}{dt} = \frac{(1-u_1) S (\beta_1 P + \beta_2 I + \beta_3 J)}{N} - (\xi + \eta + \mu) P,$$

$$\frac{dJ}{dt} = \eta P - (\gamma + \zeta + \mu) J,$$

$$\frac{dI}{dt} = \xi P + \gamma J + \phi U - (u_2\theta + \delta + \mu) I,$$

$$\frac{dU}{dt} = u_2\theta I - (\phi + \mu) U,$$

$$\frac{d\lambda_1}{dt} = (1-u_1)\frac{(\beta_1 P + \beta_2 I + \beta_3 J) N - S (\beta_1 P + \beta_2 I + \beta_3 J)}{N^2} (\lambda_1 - \lambda_3)$$
$$\qquad + u_3\rho (\lambda_1 - \lambda_2) + \mu\lambda_1,$$

$$\frac{d\lambda_2}{dt} = \psi(\lambda_2 - \lambda_1) + \mu\lambda_2,$$

$$\frac{d\lambda_3}{dt} = -c_1 + \frac{(1-u_1)(\beta_1 SN - S(\beta_1 P + \beta_2 I + \beta_3 J))}{N^2}(\lambda_1 - \lambda_3)$$
$$+ \xi(\lambda_3 - \lambda_5) + \eta(\lambda_3 - \lambda_4) + \mu\lambda_3,$$

$$\frac{d\lambda_4}{dt} = \frac{(1-u_1)(\beta_3 SN - S(\beta_1 P + \beta_2 I + \beta_3 J))}{N^2}(\lambda_1 - \lambda_3)$$
$$+ \gamma(\lambda_4 - \lambda_5) + (\zeta + \mu)\lambda_4,$$

$$\frac{d\lambda_5}{dt} = -c_2 + \frac{(1-u_1)(\beta_2 SN - S(\beta_1 P + \beta_2 I + \beta_3 J))}{N^2}(\lambda_1 - \lambda_3)$$
$$+ u_2\theta(\lambda_5 - \lambda_6) + (\delta + \mu)\lambda_5,$$

$$\frac{d\lambda_6}{dt} = \phi(\lambda_6 - \lambda_5) + \mu\lambda_6,$$

with $\lambda_1(t_f) = \lambda_2(t_f) = \lambda_3(t_f) = \lambda_4(t_f) = \lambda_5(t_f) = \lambda_6(t_f) = 0, S(0) = S_0, P(0) = P_0, J(0) = J_0, I(0) = I_0, U(0) = U_0$.

4.3 Numerical simulations and discussion

4.3.1 Analysis using the numerical methods

In this study, the numerical methods are involved in simulating the general results of the analytical findings that give real meaning to both the mathematical and biological communities. Furthermore, the parameter values used in the simulation are either taken from the literature or assumed, as given in Table 1. Also, $w_1 = 50, w_2 = 20, w_3 = 30, c_1 = 5, c_2 = 25, T = 20, S(0) = 1000, H(0) = 0, W(0) = 300, I(0) = 500, U(0) = 0, A(0) = 0$.
Moreover, MATLAB software is applied in the simulation process. Fractional derivatives and stochastic findings are widely applied as reviewed in this paper. Hence, we incorporate both forward and backward sweep methods of fourth-order Runge–Kutta method to simulate the results. The applied control strategies are as follows:
Strategy 1: Using together control measures u_1 and u_2.
Strategy 2: Using together control measures u_1 and u_3.
Strategy 3: Using together control measures u_2 and u_3.
Strategy 4: Using together control measures u_1, u_2, and u_3.
Moreover, we have used the parameters given in Table 1 to simulate subsequent numerical solutions.

Table 1: Parameter/constants value.

Parameter/constants	Value
λ	200
β_1	0.9915
β_2	0.75
β_3	0.9815
ξ	0.5
μ	0.02
η	0.5
ζ	0.1
ϕ	0.09
θ	0.5
δ	1
ρ	0 .1
γ	0.1
ψ	0.001

Based on the aforementioned control strategies, the following numerical simulations are performed.
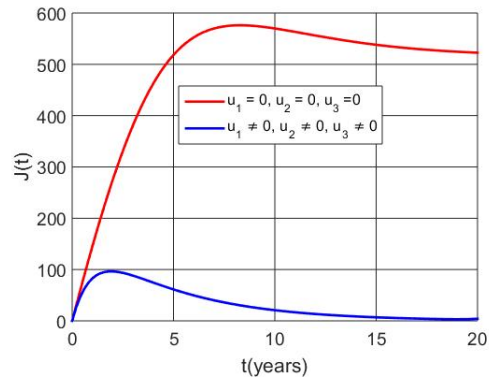

Figure 2: Primary HIV infected population.

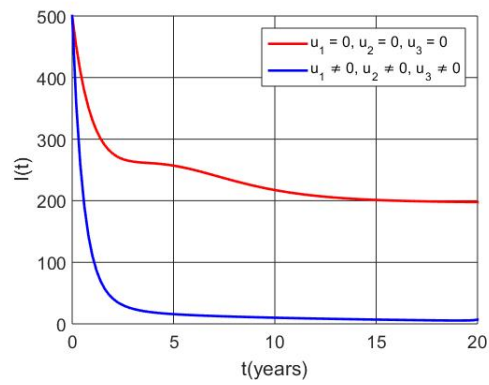Figure 3: HIV not tested population.

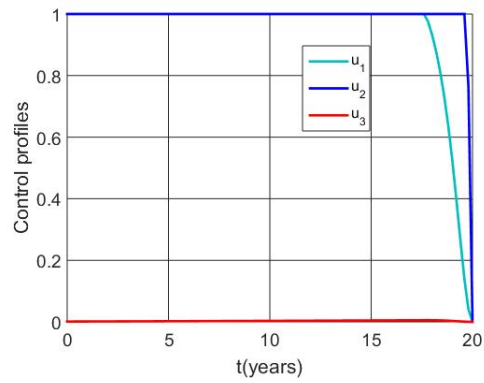Figure 4: HIV infected and on treatment individuals.

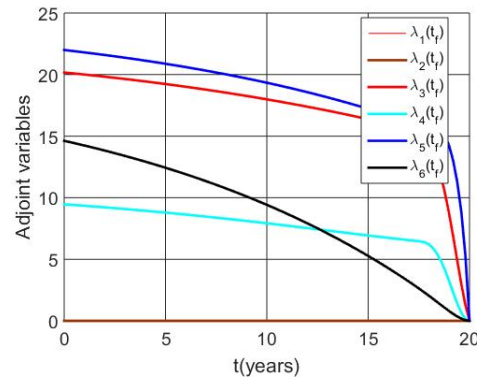Figure 5: Control functions effect illustration.

Figure 6: Adjoint variables condition descriptions.

4.3.2 Numerical results and discussion

This study develops and analyzes a mathematical model of HIV with the best possible control measures. The conceptual diagram for population dynamics is shown in Figure 1. The inclusion of the intervention with control serves to emphasize the significance of control measures in minimizing the effects of HIV infection. The numerical simulation results are shown in Figure 2 and show how a successful combination of control methods lowers the number of newly infected people. The numerical results in Figure 3 show that a reduction in the number of people who have not begun ART is shown when an intervention with control functions is present. Figure 4 shows a simulation of the number of HIV-positive people who are now receiving treatment. The results show that the intervention with three control groups dramatically lowers the number of people infected with HIV. When applied correctly, u_1 and u_2 are effective from the beginning to the end of initiation, as seen in Figure 5, where the applied control functions are simulated. Control u_3, on the other hand, makes a smaller contribution to regulating HIV infection dynamics because of its limited availability. In Figure 6, the adjoint variable is simulated to show that the transversal requirement has been satisfied.

5 Conclusion

According to the results of analytical and numerical simulations, adopting the best control measures to stop the further progression and transmission of HIV dynamics is more successful if done before the HIV infection even begins to spread. Additionally, maintaining ART and protecting those who are susceptible are considered the most crucial ways to lessen the effects of

HIV infection. Intervention with pre-exposure prophylaxis contributes less to lowering the risk of HIV infection since it is less affordable and accessible.

Acknowledgements

References

[1] Ali Biswas, H. *On the evolution of AIDS/HIV treatment: an optimal control approach,* Curr. HIV Res. 12(1), (2014), 1–12.

[2] Akudibillah G., Pandey A., and Medlock J. *Optimal control for HIV treatment.* Math. Biosci. Eng. 16(1), (2018), 373–396.

[3] Arenas, A.J., González-Parra, G., Naranjo, J.J., Cogollo, M. and De La Espriella, N. *Mathematical analysis and numerical solution of a model of HIV with a discrete time delay,* Mathematics, 9(3), (2021), 257.

[4] Bakare, E.A. and Hoskova-Mayerova S.*Optimal control analysis of cholera dynamics in the presence of asymptotic transmission, Axioms* 10 (2), (2021), 60.

[5] Bórquez, A., Guanira, J.V., Revill, P., Caballero, P., Silva-Santisteban, A., Kelly, S., Salazar, X., Bracamonte, P., Minaya, P., Hallett, T.B. and Cáceres, C.F. *The impact and cost-effectiveness of combined HIV prevention scenarios among transgender women sex-workers in Lima, Peru: a mathematical modelling study,* The Lancet Public Health, 4(3), (2019), e127–e136.

[6] Boukhouima, A., Lotfi, E.M., Mahrouf, M., Rosa, S., Torres, D.F. and Yousfi, N. *Stability analysis and optimal control of a fractional HIV-AIDS epidemic model with memory and general incidence rate,* Eur. Phys. J. Plus, 136(1), (2021), 1–20.

[7] Camlin, C.S., Koss, C.A., Getahun, M., Owino, L., Itiakorit, H., Akatukwasa, C., Maeri, I., Bakanoma, R., Onyango, A., Atwine, F. and Ayieko, J. *Understanding demand for PrEP and early experiences of PrEP use among young adults in rural Kenya and Uganda: a qualitative study,* AIDS Behav. 24, (2020), 2149–2162.

[8] Campos, C., Silva, C.J. and Torres, D.F. *Numerical optimal control of HIV transmission in Octave/MATLAB,* Math. Comput. Appl. 25(1) (2020), Paper No. 1, 20 pp.

[9] Campos, N.G., Lince-Deroche, N., Chibwesha, C.J., Firnhaber, C., Smith, J.S., Michelow, P., Meyer-Rath, G., Jamieson, L., Jordaan, S., Sharma, M. and Regan, C. *Cost-effectiveness of cervical cancer screening in women living with HIV in South Africa: a mathematical modeling study.* Acquir. Immune Defic. Syndr. (1999), 79(2) (2018), 195.

[10] Cheneke, K.R. *Optimal Control and Bifurcation Analysis of HIV Model,* Comput. Math. Methods Med. (2023).

[11] Cheneke, K.R., Rao, K.P. and Edessa, G.K. *Application of a new generalized fractional derivative and rank of control measures on cholera transmission dynamics*, International Journal of Mathematics and Mathematical Sciences, 2021, (2021), 1–9.

[12] Cheneke, K.R., Rao, K.P. and Edessa, G.K. *Bifurcation and stabillity analysis of HIV transmission model with optimal control,* J. Math. 2021, (2021), 1–14.

[13] Choi, H., Suh, J., Lee, W., Kim, J.H., Kim, J.H., Seong, H., Ahn, J.Y., Jeong, S.J., Ku, N.S., Park, Y.S. and Yeom, J.S. *Cost-effectiveness analysis of pre-exposure prophylaxis for the prevention of HIV in men who have sex with men in South Korea: a mathematical modelling study,* Sci. Rep. 10(1), (2020), 14609.

[14] Đorđević, J. and Rognlien Dahl, K., 2022. *Stochastic optimal control of pre-exposure prophylaxis for HIV infection,* Math, Med. Biol. 39(3), (2022), 197–225.

[15] Ghosh, I., Tiwari, P.K., Samanta, S., Elmojtaba, I.M., Al-Salti, N. and Chattopadhyay, J. *A simple SI-type model for HIV/AIDS with media and self-imposed psychological fear,* Math. Biosci. 306 (2018), 160–169.

[16] Hattaf, K. and Yousfi, N. *Two optimal treatments of HIV infection model,* World J. Model. Simul. 8(1), (2012), 27–36.

[17] Hattaf, K. and Yousfi, N. *Optimal control of a delayed HIV infection model with immune response using an efficient numerical method,* Int. Sch. Res. Notices, (2012).

[18] Hidayat, N., R. B. E. Wibowo, et al., Marsudi, Hidayat, N. and Wibowo, R. B. E. *Optimal control and cost-effectiveness analysis of HIV model with educational campaigns and therapy,* Matematika (Johor) 35 (2019), Special issue, 123–138.

[19] Hove-Musekwa, S.D., Nyabadza, F., Mambili-Mamboundou, H., Chiyaka, C. and Mukandavire, Z. *Cost-effectiveness analysis of hospitalization and home-based care strategies for people living with HIV/AIDS: the case of Zimbabwe,* Int. Sch. Res. Notices, (2014).

[20] Huo, H.-F., Chen, R. and Wang, X.-Y. *Modelling and stability of HIV/AIDS epidemic model with treatment,* Appl. Math. Model. 40(13-14) (2016), 6550–6559.

[21] Huo, H.-F. and Li-Xiang, F. *Global stability for an HIV/AIDS epidemic model with different latent stages and treatment.* Applied Mathematical Modelling, 37(3) (2013), 1480–1489.

[22] Khajanchi, S., Bera, S. and Roy, T.K. *Mathematical analysis of the global dynamics of a HTLV-I infection model, considering the role of cytotoxic T-lymphocytes,* Math. Comput. Simul. 180 (2021), 354–378.

[23] Marsudi, M., Hidayat, N. and Wibowo, R.B.E. *Application of Optimal Control Strategies for the Spread of HIV in a Population,* J. Life Sci. Res. 4(1) (2017), 1–9.

[24] Marsudi, T., Suryanto, A. and Darti, I. *Global stability and optimal control of an HIV/AIDS epidemic model with behavioral change and treatment.* Eng. Lett. 29(2) (2021).

[25] Naik, P.A., Yavuz, M., Qureshi, S., Zu, J. and Townley, S. *Modeling and analysis of COVID-19 epidemics with treatment in fractional derivatives using real data from Pakistan,* Eur. Phys. J. Plus, 135 (2020), 1–42.

[26] Olaniyi, S., Obabiyi, O.S., Okosun, K.O., Oladipo, A.T. and Adewale, S.O. *Mathematical modelling and optimal cost-effective control of COVID-19 transmission dynamics,* Eur. Phys. J. Plus, 135(11) (2020), 938.

[27] Silva, C.J. and Torres, D.F. *Modeling and optimal control of HIV/AIDS prevention through PrEP.* arXiv preprint arXiv:1703.06446 (2017).

How to cite this article