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Research Article

Elzaki transform method for solving deterministic modeling of terrorism

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Abstract

A deterministic mathematical model of terrorism with government intervention was constructed from five compartments and subdivided into two core and non-core groups. A non-core group is a general group $G(t)$, while the core group is susceptible $S(t)$, moderate $I(t)$, terrorism $T(t)$, and recovered $R(t)$. The Elzaki transform method with differential transform to handle the nonlinear terms is employed to solve the model. The results show that government intervention on susceptible groups proved to be 90% effective in reducing terrorist threats since the group of susceptible in the population appears to be at risk of adopting the ideology through different means of contact. Also, due to the government intervention, the moderate group reduces gradually in time.

AMS subject classifications (2020): 93A30, 65C20, 39A39.

Keywords: Modeling; Terrorism; Elzaki transform; Differential transform and Intervention.

1 Introduction

Terrorism can be seen as extra-normal violence or a threat to intimidate innocent civilians through imposing their ideology, religiosity, and political objectives in a population. Global terrorism index 2020 defined terrorism as a threatened, or actual use of illegal force and violence by a non-state

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actor to attain political, economic, religious, or social goals through fear, oppression or intimidation. This definition deduces that terrorism is not only the physical act of an attack but also the mental blow it had on humanity for many years; see [18]. Government intervention strategies consist of law enforcement. When violent extremists are investigated, prosecuted, and imprisoned, rehabilitation involves psychological counseling and correcting of their extremist views through religious education followed by vocational training. The military strategy consists of when violent extremists are killed or captured. Counter-terrorism, however, agrees that these strategies cannot reduce the threat of terrorism solitarily in [26, 8].

Similarly, the authors of [24, 25] stated that governments intervention could use an additional set of strategies collectively known as countering violent extremism and classified it into three programs: 1) Prevention program which seeks to prevent the terrorist process from occurring, 2) disengagement program, which attempts to stop or control terrorist from occurring, and 3) terrorist recovery program, which attempts to change an individual extremist belief to re-integrate them into the society. These programs frequently aim to target convicted terrorist foot soldiers. Martcheva [22] discussed a mathematical model as a system using mathematical tools and language. Castillo-Chavez and Song [10] first developed transmission dynamics and spread of terrorism via mathematical modeling, in which they incorporated the spread of radical ideologies, fanaticism, recruitment, and terrorist activities. The models of terrorism were first developed in [10]. A mathematical model of the dynamic behavior of terrorism was developed in [1], and the authors controlled to check the recruitment pool. The model is developed to control the spread of terrorist ideologies in society. Furthermore, Santoprete [26] built a model of countering violent extremism (CVE) for prevention programs. It seeks to stop the radicalization process from occurring. According to [13], it was stated that Elzaki strictly connected with the Laplace transform has some advantages such as Elzaki over Laplace converges faster to the exact solution.

Moreover, Akinola, Akinpelu, and Oladejo [6] applied the Elzaki decomposition method for solving the epidemic model. The obtained result agreed with the Adomian decomposition method, homotopy perturbation method, variational iteration method, and differential transform method (DTM). Suleman et al. [31] used the Elzaki projected differential transform method to investigate the numerical solution of the fractional-order of the HIV model. According to [14], the Elzaki transform method (ETM) is a new integral transform, which is based on the Fourier series to handle only linear differential equations. Also, in [15, 23, 17, 16, 12, 9, 33], the Elzaki method transforms the fundamental properties and their applications to differential equations for the exact solution and analyzes boundary value problems via the Elzaki transform. The ETM was introduced to facilitate the process of solving ordinary and partial differential equations in the time domain. This method is incapable of handling nonlinear equations, because of the difficulties that are caused by the nonlinear terms in [15, 23].

Furthermore, Kalavathi, Kohila, and Upadhyaya [21] presented the degenerate Elzaki transform to examine some of its properties and relations. Also, they investigated a scale preserving theorem for the degenerate Elzaki transform and theoretical dual transform to the degenerate Laplace transform. Aggarwal, Mishra, and Kumar [3] showed a comparative study of two new integral transforms Mohand and Elzaki transforms, respectively. Also, Anjum et al. [7] applied the ETM on the numerical solution for nonlinear oscillators. Bhadane and Ghadle [9] solved a system of linear differential equations using the ETM. Although, in [27], it was proposed a numerical solution for a third-order ordinary differential equation, using the DTM and ETM, then compared the results to see which method converges quickly to the exact solution. Some new applications of the ETM were defined in [29] for the solution to linear Volterra type integral equations. The results obtained were compared with solutions of the Adomian decomposition method, variational iteration method, method of successive approximation, Galerkin method, and Laplace transform method. The authors developed dualities in [2, 4] between the Elzaki transform and some useful integral transforms and then developed equations with decay problems. Their results showed that the Elzaki transform is in complete agreement with some useful methods. Therefore, in [31, 20, 35, 28, 32], it was worked a new technique of the ETM for solving differential equations to an exact solution. Their obtained results showed that Elzaki converges to the exact solution faster than any other method. The Ezaki transform is a more powerful tool than the Laplace transform, which can still serve as an auxiliary method to the Laplace transform.

This study aims to modify the existing models in [1, 26, 24] by incorporating terrorist recovered groups and government intervention. Then we apply the ETM to solve the model equations together with DTM, to linearize the nonlinear term in the equations for numerical/exact solutions and compare the result of the methods on a graph.

2 Materials and method

Mathematical modeling of terrorism with government intervention is built, and then we apply numerical methods called ETM and DTM, respectively, to solve the model equations.

2.1 Basic functions of ETM

A set of function $f(t)$ of exponential order is defined by A as follows:

$$A = \left\{ f(t) : \exists M, k_1, k_2 > 0, |f(t)| < M \exp(\frac{|t|}{k_j}), \text{ if } t \in (-1)^j [0, \infty) \right\}, \quad (1)$$

where M is a real number and k_1 and k_2 are integer. Then the Elzaki transform denoted by the operator E as the new integral equation is defined by

$$E\left[f(t)\right] = T(u) = u \int\limits_0^\infty f(t) \exp\left(\frac{-t}{u}\right) dt, \qquad t \geq 0, k_1 \leq u \leq k_2, 0 \leq t \leq \infty. \quad (2)$$

Theorem 1. Let $T(u)$ be the Elzaki transform. Then $f(t)$ is a function taking the Elzaki $f(t)$ as

$E\left[f^{'}(t)\right] = \frac{T(u)}{u} - uf(0)$.

$E\left[f^{''}(t)\right] = \frac{T(u)}{u^2} - f(0) - uf'(0)$.

$E\left[f^{(n)}(t)\right] = \frac{T(u)}{u^n} - \sum\limits_{k=0}^{n-1} u^{2-n+k} f_{(0)}^{(k)}$

Proof. Suppose that $E\left[f(t)\right] = u \int\limits_0^\infty f'(t) \exp\left(-\frac{t}{u}\right) dt$. Using integration by part gives $E\left[f'(t)\right] = \frac{T(u)}{u} - uf(0)$ and $E\left[f''(t)\right] = \frac{T(u)}{u^2} - f(0) - uf'(0)$. The last expression of the theorem can be proved by mathematical induction. □

Therefore, Elzaki, Elzaki, and Elnour [17] stated the properties of Elzaki transform of a function below:

Let $T_1(u)$ and $T_2(u)$ be the Elzaki transform of $F_1(t)$ and $F_2(t)$, respectively. Then the Elzaki transform of $[aF_1(t) + bF_2(t)]$ is given by $[aT_1(u) + bT_2(u)]$, where a and b are arbitrary constants, and the inverse of Elzaki transform can be defined as $f(t) = E^{-1}(T(u))$.

The Elzaki transform of some functions is useful when applying on differential equations as follows:

(1) If $f(t) = 1$, then $E(1) = u \int\limits_0^\infty \exp\left(-\frac{t}{u}\right) dt = u^2$.

(2) If $f(t) = t$, then $E(t) = u \int\limits_0^\infty t \exp\left(-\frac{t}{u}\right) dt = u^3$.

(3) If $f(t) = t^2$, then $E(t^2) = u \int\limits_0^\infty t^2 \exp\left(-\frac{t}{u}\right) dt = 2!u^4$.

(4) If $f(t) = t^n, n \in \mathbb{N}$, then $E(t^n) = u \int\limits_0^\infty t^n \exp\left(-\frac{t}{u}\right) dt = n!u^{n+2}$.

(5) If $f(t) = \exp(at)$, then $E(\exp(at)) = u \int\limits_0^\infty \exp(at) \exp\left(-\frac{t}{u}\right) dt = \frac{u^2}{1-au}$.

(6) If $f(t) = \cos(at)$, then $E(\cos at) = u\int\limits_0^\infty \cos at \exp\left(-\frac{t}{u}\right) dt = \frac{u^2}{1-a^2u^2}$.

(7) If $f(t) = \sin(at)$, then $E(\sin at) = u\int\limits_0^\infty \sin at \exp\left(-\frac{t}{u}\right) dt = \frac{2au^4}{1+a^2u^2}$.

(8) If $f(t) = t\cos(at)$, then $E(t\cos at) = u\int\limits_0^\infty t\cos at \exp\left(-\frac{t}{u}\right) dt = \frac{u(1-a^2u^2)}{(1+a^2u^2)^2}$.

2.2 Advantages of Elzaki transform in comparison with Laplace transform

In this subsection, the advantages of ETM, in comparison with the Laplace transform method are as follows:

(a) The ETM has the capability of combining two powerful methods at the same time, compared to the Laplace transform method.

(b) The Ezaki transform method is a powerful tool compared to the Laplace transform, which can still serve as an auxiliary method to the Laplace transform method.

(c) The ETM converges to exact solutions faster than the Laplace transform method.

(d) The ETM is a new integral transform that does not require perturbation.

2.3 Differential transform method (DTM)

There are many methods for solving differential equations. One of them is the Taylor series. The new form of the Taylor series called DTM was proposed by [34], and they applied it to solve mathematical problems in an electrical circuit. The idea of DTM is to determine the root of the Taylor series of a function by solving the induced recursive equation. In [19], it was developed a mathematical model for the transmission dynamics of the Syphilis disease and employed the DTM to handle the nonlinear systems. The DTM is a semi-analytic technique that depends on the Taylor series. The Taylor polynomial of degree n is defined by

$$P_n(x) = \sum_{k=0}^{n} \frac{1}{k!}[f^{(k)}(c)(x-c)^k]. \tag{3}$$

Suppose that the function f has $(n+1)$ continuous derivative on the interval $(c-r, c+r)$, for all $r > o$, and that $\lim_{n\to\infty} R_n(x) = 0$, where $R_n(x)$ is the error between $P_n(x)$ and the function $f(x)$. Then the Taylor series expanded about $x = c$ converges to $f(x)$; that is,

$$f(x) = \sum_{k=0}^{\infty} \frac{1}{k!}[f^{(k)}(c)(x-c)^k], \qquad \text{for all} \quad x \in (c-r, c+r). \tag{4}$$

Differential transform of the function $f(x)$ for the kth derivative is given in [5, 19] as

$$F(k) = \frac{1}{k!}[\frac{d^k f(x)}{dx^k}]_{x=x_0}, \tag{5}$$

where $f(x)$ is the original function and $F(k)$ is the transformed function. Using the ideas of [5, 19], the inverse differential transform of $F(k)$ is defined as

$$f(x) = \sum_{k=0}^{\infty} (x-x_0)^k F(k). \tag{6}$$

Substituting (7) into (8) yields

$$f(x) = \sum_{k=0}^{n} (x-x_0)^k \frac{1}{k!} \left[\frac{d^k f(x)}{dx^k}\right]_{x=x_0}. \tag{7}$$

The basic idea of DTM is considered in Table 1.

Table 1: The fundamental operations of DTM

Original Function	**Transform Function**
$y(x) = u(x) \pm m(x)$	$Y(k) = U(k) \pm M(k)$
$y(x) = \alpha m(x)$	$Y(k) = \alpha M(k)$
$y(x) = \frac{du(x)}{dx}$	$Y(k) = (k+1)U(k+1)$
$y(x) = \frac{d^2u(x)}{dx^2}$	$Y(k) = (k+1)(k+2)U(k+2)$
$y(x) = \frac{d^n u(x)}{dx^n}$	$Y(k) = (k+1)(k+2)k(k+n)U(k+n)$
$y(x) = 1$	$Y(k) = \delta(k)$
$y(x) = x$	$Y(k) = \delta(k-l)$
$y(x) = x^m$	$Y(k) = \delta(k-m)$
$y(x) = g(x)h(x)$	$Y(k) = \sum_{m=0}^{k} H(m)G(k-m)$
$y(x) = \exp(\lambda x)$	$Y(k) = \frac{\lambda^k}{k!}$
$y(x) = (1+x)^m$	$Y(k) = \frac{m(m-1)\cdots(m-k+1)}{k!}$

3 Formulation of the model

The flow diagram of terrorist network within a host community can be deduced in Figure 1.

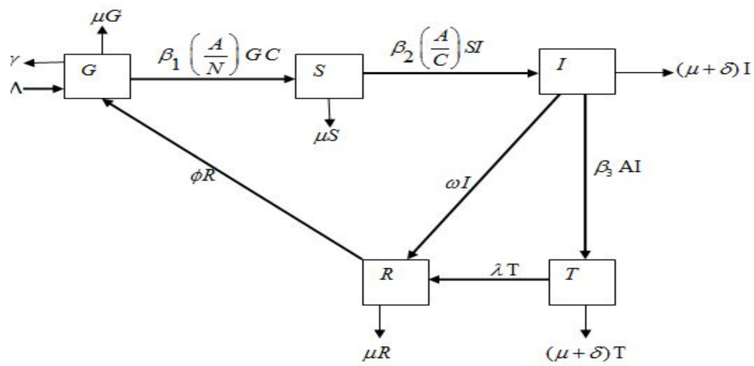

Figure 1: Flow diagram of terrorist with government intervention

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The model of terrorism with government intervention corresponding to the flow diagram in Figure 1 is given by the deterministic system of nonlinear differential equations as

$$\left.\begin{aligned}\frac{dG}{dt} &= \Lambda - \gamma + \phi R(t) - \mu G(t) - \beta_1\left(\frac{A}{N}\right)G(t)C(t)\\ \frac{dS}{dt} &= \beta_1\left(\frac{A}{N(t)}\right)G(t)C(t) - \mu S(t) - \beta_2\left(\frac{A}{C(t)}\right)S(t)I(t)\\ \frac{dI}{dt} &= \beta_2\left(\frac{A}{C(t)}\right)S(t)I(t) - (\omega+\mu+\delta)I(t) - \beta_3(A)I(t)\\ \frac{dT}{dt} &= \beta_3 AI(t) - (\mu+\delta)T(t) - \lambda T(t)\\ \frac{dR}{dt} &= \lambda T(t) + \omega I(t) - \mu R(t) - \phi R(t)\end{aligned}\right\}. \tag{8}$$

The model focuses on a population of interest $T(t)$, which is divided into noncore group $G(t)$ and core group $C(t)$ in [11]. The noncore group $G(t)$ is the general population, which the individuals in the group are at risk of adopting the ideology. The noncore group is usually the source of the recruitment pool, where the entire core group builds their members from. The core group $C(t)$ consists of susceptible group $S(t)$, the moderate group $I(t)$, Terrorist group $T(t)$, and recovered group $R(t)$. The susceptible group $S(t)$ is individuals who have not yet been converted into the ideology but have begun to walk around and fall into terrorist beliefs. Moderate group $I(t)$ includes individuals who have been transformed with beliefs to reinforce their terrorist views (Training stage). The terrorist groups $T(t)$ are individuals who have completely converted and have received training to become either leaders or foot soldiers of the terrorist respectively. The recovered group $R(t)$ is individuals who are neutralized due to counter-terrorist/NGOs activities applying to each group.

Table 2: Description of parameters of terrorism model equation (10)

Parameters	**Description**
Λ	Recruitment per-capital (birth/migration rate)
δ	Death due to suicide attack or arrest by counter-terror activities
β_1	Rate at which terrorists recruit their members from general to moderate group
β_2	Contact rate between susceptible and moderate groups
μ	Natural death rates
β_3	Contact rate between moderate and terrorist groups
A	Probability the government measure intervention on each groups
γ	Immigration rate
λ	Rate at which individuals recover from the ideology
ω	Probability that the moderate terrorists move to $R(t)$ due to government intervention
ϕ	Rate at which terrorist individuals progress from $R(t)$ to $G(t)$ after being certified by the counter-terrorist/NGO's

Table 3: Description of variables of terrorism model equation (10)

Variables	**Description**
$G(t)$	Number of general group at time t
$S(t)$	Number of partial-terrorist at time t
$I(t)$	Number of moderate terrorist (indoctrinization) at time t
$T(t)$	Number of terrorist at time t
$R(t)$	Number of terrorist recovered at time t

We have

$$C(t) = S(t) + I(t) + T(t) + R(t). \tag{9}$$

The total population is

$$N(t) = G(t) + C(t). \tag{10}$$

Then $\beta_1, \beta_2, \beta_3$ are parameters, which define the recruitment pool of each subgroup with each other. Similarly, $\beta_1 \left(\frac{A}{N(t)}\right) G(t)C(t)$ is the transition rate from $G(t)$-$S(t)$. The moderate group has more chances of recruiting, since the group builds its members from $S(t)$ and transfers them into the terrorist group individuals. There will accept their duties and execute the mission given to them (to become complete terrorist at $T(t)$), in [11]. The dynamics between $S(t)$-$I(t)$ and the transfer rate from $I(t)$-$T(t)$ within the core are $\beta_2 \left(\frac{A}{C(t)}\right) S(t)I(t)$ and $\beta_3 AI(t)$, respectively.

The intervention rate A is bounded in an interval $0 \leq A < 1$, which is the measure of intervention on each compartment. The intervention rate A is the military/NGO's activities, which aimed at reducing the recruitment pool/threats, respectively. Due to the intervention, some individuals will move to a recovered group. After some time, individuals who received either jail time or certified repentance and others will fall back to $G(t)$. It is assumed that recovery from terrorism is not permanent, since it deals with ideology, marginalization and poverty. Moreover, Λ is the birth/migration rate, γ is the immigration rate, μ is the natural death from each compartment, δ is terrorist induced death rate due to military intervention or by suicide, while λ is the rate at which individuals recover from the ideology due to intervention and ϕ is the rate at which terrorist individuals progress from $R(t)$-$G(t)$ after being certified by the counter-terrorist/NGO's.

3.1 Assumptions of the model

From the model formulated, it is assumed that A is the intervention rate, while $C(t)$ is equal to 1, $(C(t) = 1)$. Similarly, it is assumed that $N(t) = 1$. The recovered rate of individual terrorists in this model is not permanent since their behavior is dynamics. This means individual terrorists can change after being integrated into society can be re-engaged. Also, A is assumed to be a probability that not all individuals terrorists the intervention will capture (some will hide). The rate at which an individual leaves the terrorist group to enter the recovered group is λ, where ϕ is the fraction at which terrorists successfully move to the general group due to the intervention by the government. Susceptible, moderates and terrorists are assumed to have frequent contacts with the general group.

Finally, the age limit of an individual ranges from five years and above while everybody in the population has the same average natural death rate μ.

3.2 Solution of the model using ETM and DTM

Consider the nonlinear differential equation corresponding to the model diagram as

$$\frac{dG}{dt} = \Lambda - \gamma + \phi R - (\mu + \beta_1 A)G, \tag{11}$$

$$\frac{dS}{dt} = \beta_1 AG - \mu S - \beta_2(A)SI, \tag{12}$$

$$\frac{dI}{dt} = \beta_2(A)SI - (\omega + \mu + \delta + \beta_3 A)I, \tag{13}$$

$$\frac{dT}{dt} = \beta_3 AI - (\mu + \delta + \lambda)T, \tag{14}$$

$$\frac{dR}{dt} = \lambda T + \omega I - (\mu + \phi)R. \tag{15}$$

So, using (14)-(15) gives the following equations:

$$\frac{dG}{dt} = L_1 + \phi R - L_2 G, \tag{16}$$

$$\frac{dS}{dt} = L_3 G - \mu S - L_4 IS, \tag{17}$$

$$\frac{dI}{dt} = L_4 IS - L_5 I, \tag{18}$$

$$\frac{dT}{dt} = L_6 I - L_7 T, \tag{19}$$

$$\frac{dR}{dt} = \lambda T + \omega I - L_8 R, \tag{20}$$

where

$$\begin{gathered} L_1 = \Lambda - \gamma, \\ L_2 = \mu + \beta_1 A, \\ L_3 = \beta_1 A, \\ L_4 = \beta_2 A, \\ L_5 = \omega + \mu + \delta + \beta_3 A, \\ L_6 = \beta_3 A, \\ L_7 = \mu + \delta + \lambda, \\ L_8 = \mu + \phi. \end{gathered}$$

Applying the ETM on both sides of equations (16)–(20) gives

$$E\left(\frac{dG}{dt}\right) = E\{L_1 + \phi R - L_2 G\}, \tag{21}$$

$$E\left(\frac{dS}{dt}\right) = E\{L_3 G - \mu S - L_4 IS\}, \tag{22}$$

$$E\left(\frac{dI}{dt}\right) = E\{L_4 IS - L_5 I\}, \tag{23}$$

$$E\left(\frac{dT}{dt}\right) = E\{L_6 I - L_7 T\}, \tag{24}$$

$$E\left(\frac{dR}{dt}\right) = E\{\lambda T + \omega I - L_8 R\}. \tag{25}$$

Suppose that $E\left[G(t)\right] = T_1(u)$, that $E\left[S(t)\right] = T_2(u)$, that $E\left[I(t)\right] = T_3(u)$, that $E\left[T(t)\right] = T_4(u)$, that $E\left[R(t)\right] = T_5(u)$ and that L_1 in (21) is a constant. Then using the ETM properties of constant on L_1 yields $E(L_1) = u\int_0^\infty L_1 \exp(\frac{-t}{u})dt = u\left\{\lim_{t\to\infty}\left(L_1\exp(\frac{-t}{u}) + uL_1\right)\right\} = u^2 L_1$ as $E(L_1) = u^2 L_1$.

Applying the DTM on the nonlinear part in (22) and (23) implies

$$E\left(\frac{dS}{dt}\right) = E\{L_3 G - \mu S - L_4 A_n\}, \tag{26}$$

$$E\left(\frac{dI}{dt}\right) = E\{L_4 A_n - L_5 I\}, \tag{27}$$

where $A_o = \sum_{m=0}^{k} S(m)I(k-m)$, which is defined by the DTM and the polynomial as

$$\left.\begin{array}{l} A_0 = S_0 I_0, \\ A_1 = S_0 I_1 + S_1 I_0, \\ A_2 = S_0 I_2 + S_1 I_1 + S_2 I_0, \\ A_3 = S_0 I_3 + S_1 I_2 + S_2 I_0, \\ A_4 = S_0 I_4 + S_1 I_3 + S_2 I_2 + S_3 I_2 + S_4 I_0, \end{array}\right\} \tag{28}$$

Subsequently

$$\left[\frac{T_1(u)}{u} - uG_0\right] = L_1[u^2] + \phi[T_5(u)] - L_2[T_1(u)], \tag{29}$$

$$\left[\frac{T_2(u)}{u} - uS_0\right] = L_3[T_1(u)] - \mu[T_2(u)] - E[L_4 A_n], \tag{30}$$

$$\left[\frac{T_3(u)}{u} - uI_0(0)\right] = E[L_4 A_n] - L_5[T_3(u)], \tag{31}$$

$$\left[\frac{T_4(u)}{u} - uT_0(0)\right] = L_6[T_3(u)] - L_7[T_4(u)], \tag{32}$$

$$\left[\frac{T_5(u)}{u} - uR_0(0)\right] = \lambda[T_4(u)] + \omega[T_3(u)] - L_8[T_5(u)], \tag{33}$$

with the initial conditions

$$\left.\begin{array}{l} G(0) = G_0(t) = G_0 = 100, \\ S(0) = S_0(t) = S_0 = 50, \\ I(0) = I_0(t) = I_0 = 15, \\ T(0) = T_0(t) = T_0 = 25, \\ R(0) = R_0(t) = R_0 = 10, \end{array}\right\} \text{for } t \geq 0. \tag{34}$$

By simplifying and collecting the same terms of (29)–(33), we have

$$\left[\frac{1}{u}+L_2\right]T_1(u)-\phi T_5(u)=u^2L_1+uG_0, \tag{35}$$

$$\left[\frac{1}{u}+\mu\right]T_2(u)-L_3T_1(u)=E[L_4A_n]+uS_0, \tag{36}$$

$$\left[\frac{1}{u}+L_5\right]T_3(u)=E[L_4A_n]+uI_0, \tag{37}$$

$$\left[\frac{1}{u}+L_7\right]T_4(u)-L_6T_3(u)=uT_0, \tag{38}$$

$$\left[\frac{1}{u}+L_8\right]T_5(u)-\omega T_3(u)-\lambda T_4(u)=uR_0. \tag{39}$$

Solve equations (35)–(39), for $T_1(u)$, $T_2(u)$, $T_3(u)$, $T_4(u)$, and $T_5(u)$. Then truncate A_n at S_0, I_0, using (28) and applying the ETM. Subsequently, the DTM is employed to handle the nonlinear terms in (22) and (23), and we solved the system with the help of Maple 17 software (Intel (R) Celeron (R) CPU@ 1.10GH$_Z$, (RAM) 4.00GB), which gives the following results:

$$\begin{aligned}T_1(u)=&\frac{1}{(1+uL_8)(uL_2+1)(1+uL_7)(uL_5+1)}(((u\lambda L_6+u\omega L_7)+\omega\phi EuL_4(A_n)\\&+u^2L_8L_1L_5L_7+(((L_1+R_0\phi+G_0L_8)L_5+L_1L_8+\phi\omega I_0)L_7\\&+(\phi\lambda T_0+L_1L_8)L_5+\phi\lambda L_6I_0)u^3\\&+((R_0\phi+L_5G_0+G_0L_8+L_1)L_7+(L_1+R_0\phi+G_0L_8)L_5\\&+(\lambda T_0+\omega I_0)\phi+L_1L_8)u^2\\&+(L_1+G_0L_7+R_0\phi+L_5G_0+G_0L_8)u+G_0)u^2)\end{aligned} \tag{40}$$

$$\begin{aligned}T_2(u)=&(((1+((L_3\phi\omega+L_2L_8L_5)L_7+L_3\phi\lambda L_6)u^4+(((L_8+L_2)L_5+L_2L_8)L_7\\&+L_3\phi\omega+L_2L_8L_5)u^3+((L_8+L_5+L_2)L_7+(L_8+L_2)L_5+L_2L_8)u^2\\&+(L_5+L_2+L_7+L_8)u)EL_4(A_n)\\&u(u^5L_3L_8L_1L_5L_7+(((L_1+R_0\phi+G_0L_8)L_3+L_8L_2S_0)L_5\\&+L_3(L_1L_8+\phi\omega I_0))L_7\\&+L_3((\phi\lambda T_0+L_1L_8)L_5+\phi\lambda L_6I_0))u^4+(((L_3G_0+S_0(L_8+L_2))L_5\\&+(L_1+R_0\phi+G_0L_8)L_3+L_8L_2S_0)L_7\\&+((L_1+R_0\phi+G_0L_8)L_3+L_8L_2S_0)L_5+L_3(L_1L_8+(\lambda T_0+\omega I_0)\phi))u^3\\&+((S_0L_5+L_3G_0+S_0(L_8+L_2))L_7\\&+(L_3G_0+S_0(L_8+L_2))L_5+(L_1+R_0\phi+G_0L_8)L_3+L_8L_2S_0)u^2\\&+(S_0L_7+S_0L_5+L_3G_0+S_0(L_8+L_2))u+S_0))u)/\end{aligned}$$

$$((1+uL_8)(uL_2+1)(1+uL_7)(uL_5+1)) \tag{41}$$

$$T_3(u) = \frac{(EL_4(A_n)+uI_0)u}{uL_5+1} \tag{42}$$

$$T_4(u) = \frac{u^2(EL_4(A_n)L_6+(L_5T_0+L_6I_0)u+T_0)}{(1+uL_7)(uL_5+1)} \tag{43}$$

$$\begin{aligned} T_5(u) =& \frac{1}{(1+uL_7)(uL_5+1)(1+uL_8)}(u^2(\lambda L_6+\omega L_7)u+\omega)EL_4(A_n) \\ &+((L_5R_0+\omega I_0)L_7+\lambda(L_5T_0+L_6I_0))u^2 \\ &+(R_0L_7+\lambda T_0+L_5R_0+\omega I_0)u+R_0)). \end{aligned} \tag{44}$$

Thus the initial condition and parameters are
$G(0) = 100$, $S(0) = 50$, $I(0) = 15$, $T(0) = 25$, $R(0) = 10$, $\Lambda = 600$, $\phi = 0{,}0022$, $\omega = 0.015$, $\lambda = 0.008$, $A = 0.7$, $\beta_1 = 0.0035$, $\beta_2 = 0.000005$, $\beta_3 = 0.00045$, $\delta = 0.0036$, $\mu = 0.000034247$ and $\gamma = 0.7$.

Applying the conditions above on (40)–(44) gives

$$\begin{aligned} T_1(u) =& 100u^2+599.0035754u^3-1.90652029u^4+0.00605328738u^5 \\ &-0.0000189756893u^6+\cdots, \\ T_2(u) =& 50u^2+0.4708656101u^3+1.886410919u^4-0.00606738403u^5 \\ &+0.0000192353168u^6+\cdots, \\ T_3(u) =& 14u^2-0.2800526593u^3+0.005228652322u^4 \\ &-0.00009762023033u^5+0.000001822593812u^6+\cdots, \\ T_4(u) =& 25u^2-0.2847812124u^3+0.003197551982u^4 \\ &-0.00003499897343u^5+3.65259688510^{-7}u^6+\cdots, \\ T_5(u) =& 10u^2+0.3886565195u^3-0.007974537753u^4 \\ &+0.0001361224536u^5-0.000002327546774u^6+\cdots. \end{aligned}$$

Take the inverse of the Elzaki transform to solve the induced recursive equation of $G(t) = E^{-1}(T_1(u)), S(t) = E^{-1}(T_2(u)), I(t) = E^{-1}(T_3(u)), T(t) = E^{-1}(T_4(u))$, and $R(t) = E^{-1}(T_5(u))$.
Then using Maple 17 software to solve the above expression, the closed form of the solution when $k = 4$ can be written as

$$\begin{aligned} G(t) =& 100+599.0035t-0.95326t^2+0.001008t^3+7.9065\times10^{-7}t^4+\cdots, \\ S(t) =& 50+0.02628t+0.0839t^2-0.000009706t^3+5.667\times10^{-10}t^4+\cdots, \\ I(t) =& 15-0.28005t+0.00261t^2-0.00001626t^3+7.594\times10^{-8}t^4+\cdots, \\ T(t) =& 25-0.2847t+0.00159t^2-0.0000058t^3+1.5219\times10^{-8}t^4+\cdots, \end{aligned}$$

$$R(t) = 10 + 0.3886t - 0.00398t^2 + 0.0000226t^3 - 9.698 \times 10^{-8}t^4 + \cdots,$$

Furthermore, the DTM is also employed to solve (16)–(20) numerically and compare the result on the graph with that of Elzaki transform. The initial condition and the parameter values are
$G(0) = 100$, $S(0) = 50$, $I(0) = 15$, $T(0) = 25$, $R(0) = 10$, $\Lambda = 600$, $\phi = 0{,}0022$, $\omega = 0.015$, $\lambda = 0.008$, $A = 0.7$, $\beta_1 = 0.0035$, $\beta_2 = 0.000005$, $\beta_3 = 0.00045$, $\delta = 0.0036$, $\mu = 0.000034247$ and $\gamma = 0.7$.
Also, using the transformation in Table 1, for (16)–(20), gives

$$G(k+1) = \frac{L_1(k) + \phi R(k) - L_2 G(k)}{k+1}, \tag{45}$$

$$S(k+1) = \frac{L_4 \sum\limits_{m=0}^{k} S(m) I(k-m) + L_3 G(k) - \mu S(k)}{k+1}, \tag{46}$$

$$I(k+1) = \frac{L_4 \sum\limits_{m=0}^{k} S(m) I(k-m) - L_5 I(k)}{k+1}, \tag{47}$$

$$T(k+1) = \frac{L_6 I(k) - L_7 T(k)}{k+1}, \tag{48}$$

$$R(k+1) = \frac{\lambda T(k) + \omega I(k) - L_8 R(k)}{k+1}. \tag{49}$$

Applying the initial condition in (45) to (49) and using Maple 17 software, we have

$$G(t) = \sum_{m=0}^{4} G(k) t^k$$
$$= 100 + 599.2905t - 0.09373t^2 + 0.0000069105t^3 + 1.1743 \times 10^{-8}t^4 + \cdots,$$
$$S(t) = \sum_{m=0}^{4} S(k) t^k$$
$$= 50 + 0.02658t + 0.08389t^2 - 0.000009522t^3 - 1.8694 \times 10^{-9}t^4 + \cdots,$$
$$I(t) = \sum_{m=0}^{4} I(k) t^k$$
$$= 15 - 0.27975t + 0.002608t^2 - 0.00001605t^3 + 7.24869 \times 10^{-8}t^4 + \cdots,$$
$$T(t) = \sum_{m=0}^{4} T(k) t^k$$
$$= 25 - 0.29031t + 0.0016837t^2 - 0.000006498t^3 + 1.87567 \times 10^{-8}t^4 + \cdots,$$
$$R(t) = \sum_{m=0}^{4} R(k) t^k$$
$$= 10 + 0.3886t - 0.003965t^2 + 0.000022338t^3 - 9.3485 \times 10^{-8}t^4 + \cdots.$$

4 Numerical simulation

The simulations were carried out using the following variables and parameters for initial conditions. The final time was $t = 5$ years. Computations were run in Maple 17 software (Intel (R) Celeron (R) CPU@ 1.10GH$_Z$, (RAM) 4.00GB) for analyzing. Therefore, the variable and parameter values of source are
$G(t) = 100$, $S(t) = 50$, $I(t) = 15$, $T(t) = 25$, $R(t) = 10$, $\Lambda = 600$ [26], $\delta = 0.0036$ [24], $\beta_1 = 0.0035$ [24], $\beta_2 = 0.000005$ [24], $\beta_3 = 0.00045$ [24], $\mu = 0.000034247$ [24], $\lambda = 0.008$ [26], $A = 0.1$, 0.7,0.9 varies, $\gamma = 0.7$ [26], $\omega = 0.015$ and $\phi = 0.0022$.

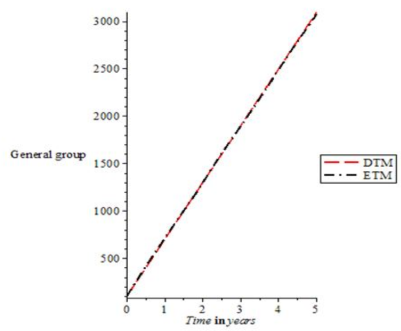

Figure 2: Comparison graph between ETM and DTM on general group in time.

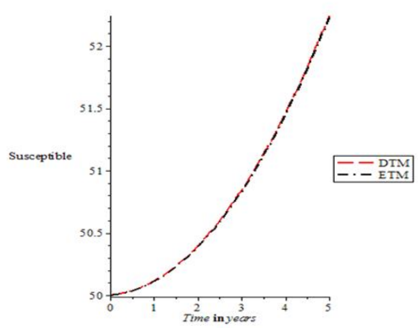

Figure 3: Comparison graph between ETM and DTM on susceptible in time.

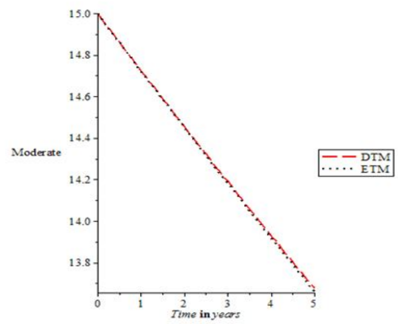

Figure 4: Comparison graph between ETM and DTM on moderate group in time.

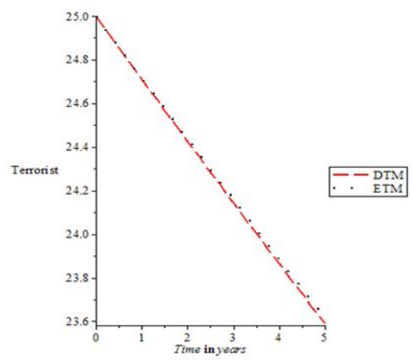

Figure 5: Comparison graph between ETM and DTM on terrorist group in time.

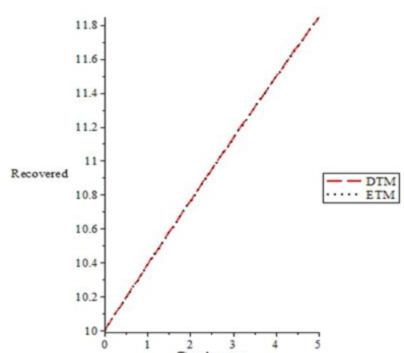

Figure 6: Comparison graph between ETM and DTM on recovered group in time.

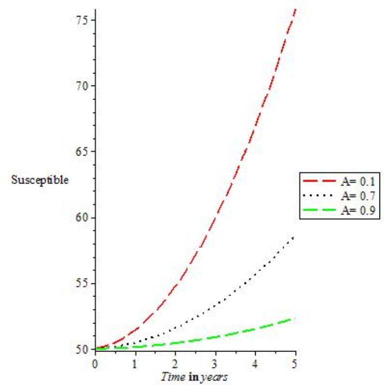

Figure 7: Graph of susceptible group in time with different rate of government intervention A.

5 Discussion of results

The ETM was introduced to determine the approximate numerical solution of a system of ordinary differential equations. Maple 17 software was used for simulations. Government intervention on susceptible groups proved to be effective in reducing terrorist threats since the susceptible group in this research is seen to be at risk of adopting the ideology through contact. similarly, this can be deduced from Figure 3 with different rates of intervention A.

The graph in Figure 2 for five years, showed that ETM and DTM agreed with each other without any significant difference in comparison. Similarly, this can be deduced from Figure 3 where the two methods also agreed on the comparison. Figure 4 shows the comparison graph between ETM and DTM on the moderate group against time, which is gradually reduced due to government intervention. From the graph, this can be seen when time duration continues after 5 years, the methods for analytic solution may not agree with each other due to control on moderate terrorist since it is dynamics. Furthermore, a similar scenario can be deduced from Figure 5 on the comparison between ETM and DTM. This can also be observed from the graph in Figure 6, where the effect of the comparison showed no significant difference between ETM and DTM on the recovered group. Finally, Figures 2–3 showed that both methods are more effective, more powerful, and accurate for solving a system of differential equations for an approximate solution, but Elzaki transform converges faster than DTM.

6 Conclusion

A mathematical model of terrorism with government intervention was developed to reduce the recruitment pools within the five compartments. The ETM and DTM were tested and solved the governing differential equations compared to the results on the graph. Government intervention on susceptible groups proved to be 90% effective as well as a moderate group. The susceptible and moderate group in this study appears to be exposed to adopting the ideology through contact. The ETM was employed to solve the model with Maple 17 software and applied DTM to decompose the nonlinear term since some differential equations have difficulty finding analytic/exact solutions. Finally, a plan to incorporate other issues would be addressed in the future study.

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References

1. Adamu, G. and Ibrahim, M.O. *Mathematical modelling of dynamics behaviour of terrorism and control*, Caspian. J. Math. Sci., 9(1) (2020), 68–85.

2. Aggarwal, S., Bhatnagar, K. and Dua, A. *Dualities between Elzaki transform and some useful integral transforms.* Int. J. Inno. Tech. Expl. Eng., 8(12)(2019), 4312–4318.

3. Aggarwal, S., Mishra, R. and Kumar, A. *A comparative study of Mohand and Elzaki Transforms*, Global. J. Eng. Sci. Res., 6(2) (2019), 203–213.

4. Aggarwal, S., Singh, D.P., Asthana, N. and Gupta, A.R. *Application of Elzaki transform for solving population growth and decay problems*, J. Emer. Tech. Inno Res., 5(9) (2018), 281–284.

5. Ahmad, M.Z., Alsarayreh, D., Alsarayreh, A. and Qaralleh, I. *Differential transformation method (DTM) for solving SIS and SI epidemic models*, Sains. Malay., 46(10) (2017), 2007–2017.

6. Akinola, E.I., Akinpelu, F.O. and Oladejo, A.A.J. *Elzaki decomposition method for solving epidemic model*, Int. j. Chem. Math. Phys., 1(1) (2017), 68–72.

7. Anjum, N., Suleman, M., Lu, D., He, J.H. and Ramzan, M. *Numerical iteration for nonlinear oscillators by Elzaki transform*, J. Freq. Nois. Vibr. Cont., 0(0) (2019), 1–6.

8. Basit, A. *Countering violent extremism: evaluating Pakistan's counter-radicalization and de-radicalization initiatives*, Islamabad. Pol. Res. Inst. J., 15(2) (2015), 44–68.

9. Bhadane, P. and Ghadle, K. *Application of Elzaki transform to system of linear differential equations*, Int. J. Res. Pub., 6(2) (2016), 36–43.

10. Castillo-Chavez, C. and Song, B. *Models for the transmission dynamics of fanatic behaviours*, In Bioterrorism: Math. Modell. Appl. Homel. Secur. Soci. Indust. Appl. Math., 28 (2003), 155–172.

11. Cherif, A., Yoshioka, H., Ni, W. and Bose, P. *Terrorism mechanism of radicalization process, control of contagion and counter-terrorist measures*, Santa Fe. Insti., 910 (2010), 1–14.

12. Duz, M. and Elzaki, T. M. *Solution Of constant coefficients partial derivative equations with Elzaki transform method*, TWMS J. Appl. Eng. Mathe., 9(3) (2019), 563–570.

13. Elzaki. T.M. *On the connections between Laplace and Elzaki transforms*, Adv. Theory.Appl. Math., 6(1) (2011), 1–11.

14. Elzaki, T.M. *The new integral transform Elzaki transform*, Global J. Pure Appl. Math., 7(1) (2011), 57–64.

15. Elzaki, T.M. *Solution of nonlinear differential equations using mixture of Elzaki transform and differential transform method*, In Int. Math. Forum., 7(13) (2012), 631–638.

16. Elzaki, T.M., Alamri, B.A.S. *Combination of transform methods for solving linear and nonlinear differential equations with proportional delay*, Int. Math. Forum., 10(10) (2015), 467–476.

17. Elzaki, T.M., Elzaki, S.M. and Elnour, E.A. *On the new integral transform "ELzaki Transform" fundamental properties investigations and applications*, Global J. Math. Sci. Theory Pract., 4(1) (2012), 1–13.

18. Global Terrorism Index 2020. *Measuring the impact of terrorism, institute for economics and peace*, Sydney, November 2020.

19. Ifeyinwa, M.H. *Mathematical modeling of the transmission dynamics of syphilis disease using differential transformation method*, Math. Model. Appl., 5(2) (2020), 47–54.

20. Ige, O.E. and Oderinu, R.A. *Adomian polynomial and Elzaki transform method for solving Sine-Gordon equations*, IAENG Int. J. Appl. Math., 49(3) (2019), 1–7.

21. Kalavathi, A., Kohila, T. and Upadhyaya, L.M. *On the degenerate Elzaki transform*, Bull. Pure Appl. Sci. Sect. E Math. Stat. 40E (2021). 99–107.

22. Martcheva, M. *An introduction to mathematical epidemiology*, New York, Springer, 2015.

23. Mohamed, M.Z. and Elzaki, T.M. *Applications of new integral transform for linear and nonlinear fractional partial differential equations*, J. King Saud Uni. Sci., 32(1) (2020), 544–549.

24. Nathan, O., Lawi, G. and Nthiiri, J. *Modelling the dynamics of radicalization with government intervention*, Neural Parallel. Sci. Comput., 26 (2018), 211–224.

25. Orakzai, S.B. *Pakistan's approach to countering violent extremism (CVE), reframing the policy framework for peace building and development strategies*. Study. Conf. Terro., 42(8) (2019), 755–770.

26. Santoprete, M. *Countering violent extremism: A mathematical model*, Appl. Math. Comput., 358 (2019), 314–329.

27. Saqlain, M., Irshad, W., Bashir, U. and Murtaza, G.*Numerical solutions for third order ordinary differential equation by differential transform method & Elzaki transform method*, Sci. Inquiry Review, 2(4) (2018), 01–12.

28. Shah, N.A., Dassios, I. and Chung, J.D. *A decomposition method for a fractional-order multi-dimensional telegraph equation via the Elzaki transform*, Symt., 13(1) (2021), 1–8.

29. Sharjeel, S. and Barakzai, M.A.K. *Some new applications of Elzaki transform for solution of linear Volterra type integral equations*, J. Appl. Math. Phys., 7(8) (2019), 1877–1892.

30. Suleman, M., Elzaki, T., Wu, Q., Anjum, N. and Rahman, J.U. *New application of Elzaki projected differential transform method*, J. Comput. Theoretical Nanosci., 14(1) (2017), 631–639.

31. Suleman, M., Lu, D., He, J.H., Farooq, U., Hui, Y.S. and Rahman, J.U. *Numerical investigation of fractional HIV model using Elzaki projected differential transform method*, Fract, 26(5) (2018), 1850062.

32. Verma, D. and Alam, A. *Analysis of simultaneous differential equations By Elzaki transform approach*, Sci. Tech. Devel., 9(1) (2020), 364–367.

33. Verma, D. and Gupta, R. *Analyzing Boundary Value Problem in Physical Sciences Via Elzaki Transform*, ASIO J. Chem. Phys. Math. Appl. Sci., 4(1) (2020), 17–20.

34. Zhou, J.K., *Differential transformation and its applications for electrical circuits*, Huazhong University Press, Wuhan, China, 1986.

35. Ziane, D., Elzaki, T.M. and Cherif, M.H. *Elzaki transform combined with variational iteration method for partial differential equations of fractional order*, Fundamental J. Math. Appl., 1(1) (2018), 102–108.

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