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Research Article

Numerical nonlinear model solutions for the hepatitis C transmission between people and medical equipment using Jacobi wavelets method

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Abstract

In this work, we present a new mathematical model for the spread of hepatitis C disease in two populations: human population and medical equipment population. Then, we apply the Jacobi wavelets method combined with the decoupling and quasi-linearization technique to solve this set of nonlinear differential equations for numerical simulation.

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Keywords: Hepatitis C; Sterilization; Jacobi wavelets; Operational matrix of derivative; Simulation.

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1 Introduction

Viral hepatitis is a major health problem worldwide, comparable to that posed by other major communicable diseases, such as human immunodeficiency virus (HIV), tuberculosis, malaria, or, more recently, coronavirus disease 2019 (COVID-19). In this work, we are interested in viral hepatitis C (HCV).
Hepatitis C is an inflammation of the liver caused by the hepatitis C virus. The virus can cause both acute and chronic hepatitis. According to the fact sheet of the World Health Organization (WHO) updated on October 2017 for hepatitis C, 71 million people have been estimated for chronic hepatitis C infection in the whole world, and approximately 399,000 people die each year from hepatitis C [28]. Until today, researchers could not develop a vaccine or effective treatment that heals hepatitis C at 100% [25].

The hepatitis C virus is transmitted by exposure to contaminated blood resulting from bringing the blood of an infected person into contact with that of a person likely to be contaminated directly (transfusion) or indirectly (equipment of contaminated injection for example). In 2016, WHO introduced global targets, for the care and management of HCV, a 90% reduction in new cases of chronic hepatitis C, a 65% reduction in hepatitis C deaths, and treatment of 80% of eligible people with chronic hepatitis C infections [30]. In Algeria, the president of the "SOS hepatitis association", spoke in an interview about the need to draw up a national plan against viral hepatitis, which will aim to improve prevention, care, and the availability of drugs. He also mentioned that the prevention of viral hepatitis poses a problem in Algeria because there is no real prevention against these viral infections, especially at the dentist. It is obvious to know that the majority of contaminations by these viruses are done during dental care. Therefore, raising awareness against viral hepatitis "B" and "C" is very important to detect these diseases, especially since they are silent. Indeed, better prevention requires better knowledge of the modes of transmission and the populations at risk in order to improve education and teach the appropriate protective measures. The last century has seen the emergence and rapid development of mathematical modeling, which plays an important role in assessing and anticipate the impact of Public Health programs.

Over the last decade, a large number of mathematical models have been developed to simulate, analyze, and understand the dynamics of a population of hepatitis C. In a related research work, Martcheva and Castillo-Chavez [19] proposed a model to study the role of a chronic infectious stage on the dynamics of HCV over the long term. Incorporating the immune class in [10], in [32], the latency period was merged. In [4], the authors showed both the effect of processing and immigration. Another model describes the effect of isolating chronically infected people [15]. Several studies have been carried out in [11, 20, 23, 21, 22, 33] showing the impact of HCV treatment in drug users on the prevalence of the disease. The optimal control theory has been

used to understand the efforts made to prevent the spread of the disease by different measures and strategies [1, 31, 34].

Our aim in this article is to understand how hepatitis C disease can evolve, by highlighting the role of sterilization of infected material, modeled by ordinary differential equation (ODE), unlike the model of Miller et al. [24], which targets the population of drug users. Therefore a single mode of contamination which plays the role of a vector of the disease and a single host. Our new model SIR-MI consists of taking into account other causes of contamination, such as dental equipment, toilet equipment, needles, tattooing, and piercing equipment in interaction with a mixed human population, and then we resolve this model.

On the other hand, wavelet theory plays an important role in many areas of mathematics and applied sciences, for instance, signal analysis in medicine, image processing, signal processing, data compression, statistics, and numerical methods [7, 18]. In recent years, wavelets based on orthogonal polynomials have been used in many researches to solve different problems such as ODE, partial differential equations, fractional differential equations, optimal control, and variational calculus [2, 9, 8, 26, 27], and this is due to orthogonality property. We propose the Jacobi wavelets method with general indices (α, β) in this work in order to obtain computational solutions. This method generalized other methods like Legendre wavelets and Chebyshev wavelets. The Jacobi wavelets method reduces an ODE to a system of algebraic equations by using the operational matrix of the derivative of Jacobi wavelets. In our numerical simulations, we have found that using the operational matrix of derivative simplifies the implementation of the method compared to using the operational matrix of integration [3]. Then, we apply the decoupling and quasi-linearization technique (DQLT) combined with the Jacobi wavelets method to solve the underlying problem.

In this paper, we propose, in section 2, a mathematical model SIR-MI that describes the dynamics of a population of hepatitis C. Section 3 will concern the mathematical analysis of the proposed model. Section 4 is devoted to explaining the different steps that lead to the implementation of the Jacobi wavelet method combined with DQLT. In Section 5, we apply the proposed method to simulate the model SIR-MI. Finally, Section 6 presents our conclusions.

2 Model formulation

In order to understand the effect of sterilization of the material on the transmission and dynamics of hepatitis C, we propose a mathematical model SIR-MI developed by Miller et al. [24] with five compartments. That is, let N_H be the total population of humans, which is subdivided into three subclasses: S_H (susceptible), I_H (infected), R_H (recovered). The total population of N_M material is divided into two subclasses: M_U (uninfected), M_I (infected).

The graphical representation of the proposed model is shown in Figure 1.

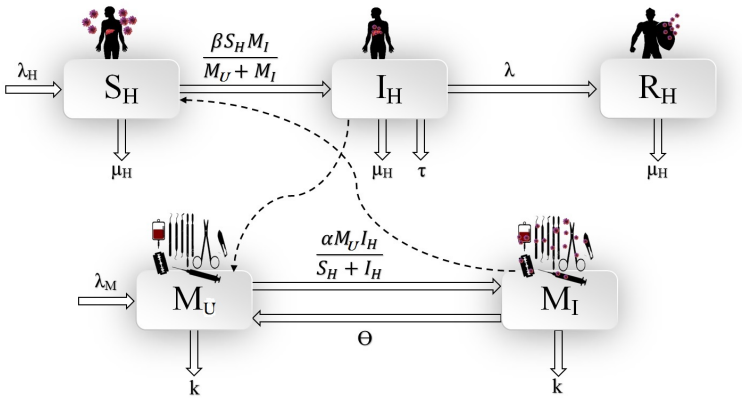


Figure 1: The compartmental model diagram.

The model SIR-MI is given by the following system of ODEs:

$$\begin{cases} \frac{dS_H}{dt}(t) = \lambda_H - \frac{\beta S_H(t) M_I(t)}{M_I(t) + M_U(t)} - \mu_H S_H(t), \\ \frac{dI_H}{dt}(t) = \frac{\beta S_H(t) M_I(t)}{M_I(t) + M_U(t)} - \lambda I_H(t) - (\tau + \mu_H) I_H(t), \\ \frac{dR_H}{dt}(t) = \lambda I_H(t) - \mu_H R_H(t), \\ \frac{dM_U}{dt}(t) = \lambda_M - \frac{\alpha M_U(t) I_H(t)}{S_H(t) + I_H(t)} + \theta M_I(t) - k M_U(t), \\ \frac{dM_I}{dt}(t) = \frac{\alpha M_U(t) I_H(t)}{S_H(t) + I_H(t)} - \theta M_I(t) - k M_I(t), \\ S_H(0) = S_{H_0}, I_H(0) = I_{H_0}, R_H(0) = R_{H_0}, M_U(0) = M_{S_0}, M_I(0) = M_{I_0}, \\ S_{H_0}, I_{H_0}, R_{H_0}, M_{U_0}, M_{I_0} > 0, \end{cases} \tag{1}$$

with

$$\lambda_M = k N_M \neq 0.$$

The parameters used in our model are defined in Table 1.

Table 1: Definitions of parameters used in model.

Parameters	Description
λ_H	birth rate of susceptible
μ_H	natural mortality rate of the human population
β	rate of interaction between susceptible humans and infected material
τ	mortality rate due to the disease
λ	disease cure rate
λ_M	birth rate of uninfected material
α	interaction rate between infected humans and uninfected material
k	rejection rate of infected or non-infected material
θ	sterilization rate of infected material

3 Mathematical Analysis of the Model

In this section and in the first moment, we will apply the theorem of Cauchy–Lipschitz to demonstrate the existence and the uniqueness of the solution of the system (1). Then we will study the behavior of this solution by going through the calculation of the points of equilibrium as well as the stability of these points. However, first, we note that the population of material $N_M(t)$ is constant.
Indeed,

$$N_M(t) = M_I(t) + M_U(t) \Longleftrightarrow \frac{dN_M}{dt}(t) = \frac{dM_U}{dt}(t) + \frac{dM_I}{dt}(t).$$

So

$$\begin{aligned} \frac{dM_U}{dt}(t) + \frac{dM_I}{dt}(t) &= \lambda_M - k\left(M_I(t) + M_U(t)\right), \\ \frac{dN_M}{dt}(t) &= 0. \end{aligned}$$

We show that the human population is not constant. So we have

$$N_H(t) = S_H(t) + I_H(t) + R_H(t) \Longleftrightarrow \frac{dN_H}{dt}(t) = \frac{dS_H}{dt}(t) + \frac{dI_H}{dt}(t) + \frac{dR_H}{dt}(t).$$

Then

$$\begin{aligned} \frac{dS_H}{dt}(t) + \frac{dI_H}{dt}(t) + \frac{dR_H}{dt}(t) &= \lambda_H - \mu_H\left(S_H(t) + I_H(t) + R_H(t)\right) - \tau I_H(t), \\ \frac{dN_H}{dt}(t) &= -\tau I_H(t). \end{aligned}$$

3.1 Existence and uniqueness of a positive solution

To study the existence and uniqueness of the solution of problem (1), we need to apply the Cauchy–Lipschitz theorem.
Our model (1) is a system of nonlinear, autonomous first-order differential equations that can be written as the following Cauchy problem:

$$\begin{cases} X'(t) = F(X(t)), t \in [0,T], \\ X(0) = X_0, \end{cases} \tag{2}$$

with

$$X(t) = \begin{pmatrix} S_H(t) \\ I_H(t) \\ R_H(t) \\ M_U(t) \\ M_I(t) \end{pmatrix} \text{ and } F(X(t)) = \begin{pmatrix} f_1(X(t)) \\ f_2(X(t)) \\ f_3(X(t)) \\ f_4(X(t)) \\ f_5(X(t)) \end{pmatrix},$$

where

$$f_1(X(t)) = \lambda_H - \frac{\beta S_H(t) M_I(t)}{N_M} - \mu_H S_H(t), \tag{3}$$

$$f_2(X(t)) = \frac{\beta S_H(t) M_I(t)}{N_M} - \lambda I_H(t) - (\tau + \mu_H) I_H(t), \tag{4}$$

$$f_3(X(t)) = \lambda I_H(t) - \mu_H R_H(t), \tag{5}$$

$$f_4(X(t)) = \lambda_M - \frac{\alpha M_U(t) I_H(t)}{S_H(t) + I_H(t)} + \theta M_I(t) - k(t), \tag{6}$$

$$f_5(X(t)) = \frac{\alpha M_U(t) I_H(t)}{S_H(t) + I_H(t)} - \theta M_I(t) - k M_I(t). \tag{7}$$

We recall that the norm $Norm(\cdot)$ in the space of continuous functions from I to $\mathbb{R}^5$ (denoted by $C(I, \mathbb{R}^5)$) is defined by

$$Norm(F) = \max_{t \in I} \|F(t)\|_2,$$

with $\|\cdot\|_2$ is the usual Euclidean norm in $\mathbb{R}^5$.

We are now able to state the following result.

Theorem 1. The differential problem (1) admits a unique solution $(S_H(t), I_H(t), R_H(t), M_U(t), M_I(t))^T \in \mathbb{R}^5$ for all $t \in [0,T]$.

Proof. To demonstrate that the Cauchy problem (1) admits a unique solution, it suffices to show that the vector function F of the equivalent problem (2), is Liptschizian.

Let $t \in [0,T]$, $X_1, X_2 \in \mathbb{R}^5$. Then

$$\|F(X_1(t)) - F(X_2(t))\| = \max \begin{cases} |f_1(X_1(t)) - f_1(X_2(t))|, \\ |f_2(X_1(t)) - f_2(X_2(t))|, \\ |f_3(X_1(t)) - f_3(X_2(t))|, \\ |f_4(X_1(t)) - f_4(X_2(t))|, \\ |f_5(X_1(t)) - f_5(X_2(t))|. \end{cases}$$

We assume that at any instant $t \in [0,T]$, the human population $N_H(t) = S_H(t) + I_H(t)$ is between two real numbers strictly positive $N_{\min}$ and $N_{\max}$.

We will examine each of the components $|f_i(X_1(t)) - f_i(X_2(t))|$, $i = 1,\ldots,5$. Therefore

$$\begin{aligned} &|f_1(X_1(t)) - f_1(X_2(t))| \\ &= \left| -\frac{\beta S_{H_1}(t) M_{I_1}(t)}{N_M(t)} - \mu_H S_{H_1}(t) + \frac{\beta S_{H_2}(t) M_{I_2}(t)}{N_M(t)} + \mu_H S_{H_2}(t) \right| \\ &\leq \frac{\beta}{N_M(t)} |-S_{H_1}(t) M_{I_1}(t) + S_{H_2}(t) M_{I_2}(t)| + \mu_H |-S_{H_1}(t) + S_{H_2}(t)|. \end{aligned}$$

By adding and subtracting the term $S_{H_1} M_{I_2}$, we have

$$\begin{aligned} &|f_1(X_1(t)) - f_1(X_2(t))| \\ &\quad \leq \frac{\beta}{N_M(t)} S_{H_1}(t) |-M_{I_1}(t) + M_{I_2}(t)| + \beta \frac{M_{I_2}(t)}{N_M(t)} |-S_{H_1}(t) + S_{H_2}(t)| \\ &\qquad + \mu_H |-S_{H_1}(t) + S_{H_2}(t)|. \end{aligned}$$

Since $S_{H_1} \leq N_{\max}$ and $\dfrac{M_{I_2}}{N_M} \leq 1$, then

$$|f_1(X_1(t)) - f_1(X_2(t))| \leq \left(\frac{\beta}{N_M(t)} N_{\max} + \beta + \mu_H \right) \|X_1(t) - X_2(t)\|.$$

For (4) and following the same reasoning, we find

$$\begin{aligned} &|f_2(X_1(t)) - f_2(X_2(t))| \\ &\leq \left(\frac{\beta}{N_M(t)} N_{\max} + \beta + \lambda + \tau + \mu_H \right) \|X_1(t) - X_2(t)\|. \end{aligned}$$

The linearity of terms in (5) leads to

$$|f_3(X_1(t)) - f_3(X_2(t))| \leq (\lambda + \mu_H) \|X_1(t) - X_2(t)\|.$$

From (6), we have

$$|f_4(X_1(t)) - f_4(X_2(t))| \leq \frac{\alpha}{N_H(t)} |-M_{U_1}(t)I_{H_1}(t) + M_{U_2}(t)I_{H_2}(t)| + \theta |M_{I_1}(t) - M_{I_2}(t)| + k |-M_{U_1}(t) + M_{U_2}(t)|.$$

By adding and subtracting the term $M_{U_1}I_{H_2}$, we have

$$|f_4(X_1(t)) - f_4(X_2(t))| \leq \frac{\alpha}{N_H(t)} M_{U_1}(t) |-I_{H_1}(t) + I_{H_2}(t)| + \frac{\alpha}{N_H(t)} I_{H_2}(t) |-M_{U_1}(t) + M_{U_2}(t)| + \theta |M_{I_1}(t) - M_{I_2}(t)| + k |-M_{U_1}(t) + M_{U_2}(t)|.$$

Knowing $M_{U_1} \leq N_M$, $N_H(t) \geq N_{\min}$ and $\frac{I_{H_2}}{N_H(t)} \leq 1$, we arrive at

$$|f_4(X_1(t)) - f_4(X_2(t))| \leq \left(\frac{\alpha N_M(t)}{N_{\min}} + \alpha + \theta + k\right) \|X_1(t) - X_2(t)\|.$$

Finally, from (7) and following the previous steps, we have

$$|f_5(X_1(t)) - f_5(X_2(t))| \leq \left(\frac{\alpha N_M(t)}{N_{\min}} + \alpha + \theta + k\right) \|X_1(t) - X_2(t)\|.$$

Therefore, we have

$$\|F(X_1(t)) - F(X_2(t))\| \leq C \|X_1(t) - X_2(t)\|,$$

with

$$C = \max\left(\frac{\beta}{N_M(t)} N_{\max} + \beta + \mu_H,\ \frac{\beta}{N_M(t)} N_{\max} + \beta + \lambda + \tau + \mu_H,\ \lambda + \mu_H,\ \frac{\alpha N_M(t)}{N_{\min}} + \alpha + \theta + k\right).$$

□

3.2 Equilibrium points

In this subsection, we will look for points of equilibrium E_0 and E_1 (Theorem 2) and study their stabilities. We limit ourselves to the stability of the point E_0. The stability of the point E_1 will be made numerically.

The basic reproduction rate R_0

Understanding how an epidemic develops once it has appeared is crucial if we are to hope to control it. To do this, various models have been developed, which highlight the crucial role played by the R_0 parameter, describing the average number of new infections due to a sick individual. As one can imagine, if this number is less than 1, then the epidemic will tend to die out, while it may persist or even spread to the entire population if $R_0 > 1$ ([12]).

We recall, for a given matrix A, that $Sp(A)$ represents the spectrum of A and that the spectral radius of the matrix A, denoted $\rho(A)$, is defined by

$$\rho(A) = \max\left\{|\lambda|, \lambda \in Sp(A)\right\}.$$

The disease-free point is

$$(S_H = \frac{\lambda_H}{\mu_H}, I_H = 0, R_H = 0, M_U = N_M, M_I = 0).$$

We consider different infected populations of the model. That is,

$$\frac{dI_H}{dt}(t) = \frac{\beta S_H(t) M_I(t)}{N_M} - \lambda I_H(t) - (\tau + \mu_H) I_H(t)$$

and

$$\frac{dM_I}{dt}(t) = \frac{\alpha M_U(t) I_H(t)}{S_H(t) + I_H(t)} - \theta M_I(t) - k M_I(t).$$

To be able to calculate R_0, we use two matrices F and V, where the matrix F represents the appearance of new infected; that is, what comes from other compartments and which enters the infected compartment following an infection,

$$F(I_H, M_I) = \begin{pmatrix} 0 & \frac{\beta S_H}{N_M} \\ \frac{\alpha M_U}{N_H} & 0 \end{pmatrix}.$$

The matrix V represents all those who leave the compartments of the infected and those who come there for any other reason,

$$V(I_H, M_I) = \begin{pmatrix} -\lambda - (\tau + \mu_H) & 0 \\ 0 & -\theta - k \end{pmatrix}.$$

We have

$$-FV^{-1} = \begin{pmatrix} 0 & \dfrac{\beta\lambda_H}{N_M\,(k+\theta)\,\mu_H} \\ \dfrac{\alpha N_M \mu_H}{(\lambda+\tau+\mu_H)\,\lambda_H} & 0 \end{pmatrix}.$$

The matrix $-FV^{-1}$ represents the next generation matrix. The basic reproduction rate is given by

$$R_0 = \rho(-FV^{-1}).$$

After calculating the eigenvalues of the matrix $-FV^{-1}$, we find

$$\lambda_1 = \sqrt{\tfrac{\beta\alpha}{(\lambda+\tau+\mu_H)(k+\theta)}} \text{ and } \lambda_2 = -\sqrt{\tfrac{\beta\alpha}{(\lambda+\tau+\mu_H)(k+\theta)}}.$$

We then conclude

$$R_0 = \sqrt{\frac{\beta\alpha}{(\lambda+\tau+\mu_H)\,(k+\theta)}}. \tag{8}$$

The calculation of equilibrium points

Theorem 2. The system (1) admits two equilibrium points E_0 and E_1, for strictly positive parameters. They are given indeed as follows.

1. If $R_0 < 1$, then the point E_0 exists and it is given by

$$E_0 = \left(\frac{\lambda_H}{\mu_H}, 0, 0, N_M, 0\right).$$

2. If

$$R_0 > 1 \quad \text{and} \quad \alpha\beta + \alpha\mu_H > (k+\theta)\,(\tau+\lambda),$$

then the endemic point E_1 exists and it is given by

$$E_1 = (S_H^*, I_H^*, R_H^*, N_M - M_I^*, M_I^*),$$

with

$$\begin{cases} S_H^* = \dfrac{\lambda_H - (\tau+\mu_H+\lambda)\,I_H^*}{\mu_H}, \\ I_H^* = \dfrac{\beta\alpha\lambda_H - \lambda_H\,(k+\theta)\,(\lambda+\tau+\mu_H)\,N_M}{\alpha\beta\,(\tau+\mu_H+\lambda) + N_M\,(\lambda+\tau+\mu_H)\,(\alpha\mu_H - (k+\theta)\,(\tau+\lambda))}, \\ R_H^* = \dfrac{\lambda}{\mu_H} I_H^*, \\ M_U^* = N_M - M_I^*, \\ M_I^* = \dfrac{\alpha\mu_H N_M I_H^*}{((\alpha\mu_H - (k+\theta)\,(\tau+\lambda))\,I_H^* + \lambda_H\,(k+\theta))}. \end{cases}$$

Proof. The equilibriums of the system (1) are given by the solutions of the following system of algebraic equations:

$$\begin{cases} \lambda_H - \dfrac{\beta S_H^* M_I^*}{N_M} - \mu_H S_H^* = 0, \\ \dfrac{\beta S_H^* M_I^*}{N_M} - \lambda I_H^* - (\tau + \mu_H) I_H^* = 0, \\ \lambda I_H^* - \mu_H R_H^* = 0, \\ \lambda_M - \dfrac{\alpha M_U^* I_H^*}{S_H^* + I_H^*} + \theta M_I^* - k M_U^* = 0, \\ \dfrac{\alpha}{S_H + I_H} M_U^* I_H^* - (k + \theta) M_I^* = 0. \end{cases} \tag{9}$$

As the population of the material is constant $N_M = M_U + M_I$, then the system (9) is reduced to the following four equations:

$$\lambda_H - \frac{\beta S_H^* M_I^*}{N_M} - \mu_H S_H^* = 0, \tag{10}$$

$$\frac{\beta S_H^* M_I^*}{N_M} - \lambda I_H^* - (\tau + \mu_H) I_H^* = 0, \tag{11}$$

$$\lambda I_H^* - \mu_H R_H^* = 0, \tag{12}$$

and

$$\frac{\alpha (N_M - M_I^*) I_H^*}{S_H^* + I_H^*} - (k + \theta) M_I^* = 0. \tag{13}$$

The sum of (10) and (11) gives

$$S_H^* = \frac{\lambda_H - (\tau + \mu_H + \lambda) I_H^*}{\mu_H}. \tag{14}$$

From (12), it is clear that

$$R_H^* = \frac{\lambda}{\mu_H} I_H^*. \tag{15}$$

To determine M_I^*, we replace (14) in (13) and obtain

$$M_I^* = \frac{\alpha \mu_H N_M I_H^*}{\left(\left(\alpha \mu_H - (k + \theta)(\tau + \lambda)\right) I_H^* + \lambda_H (k + \theta)\right)}. \tag{16}$$

Substituting (14) and (16) into (11), we find

$$\left(\frac{\beta \alpha \left(\lambda_H - (\tau + \mu_H + \lambda) I_H^*\right)}{\left(\left(\alpha \mu_H - (k + \theta)(\tau + \lambda)\right) I_H^* + \lambda_H (k + \theta)\right)} - (\lambda + \tau + \mu_H) N_M \right) I_H^* = 0. \tag{17}$$

According to (17), we distinguish two cases:

Case I $I_H^* = 0$, we then find

$$S_H = \frac{\lambda_H}{\mu_H}, R_H = M_I = 0 \text{ et } M_U = N_M.$$

Hence the existence of the first equilibrium point E_0 is as follows:

$$E_0 = \left(\frac{\lambda_H}{\mu_H}, 0, 0, N_M, 0\right).$$

Case II $I_H^* \neq 0$, we have

$$\frac{\beta\alpha\left(\lambda_H - (\tau + \mu_H + \lambda) I_H^*\right)}{\left((\alpha\mu_H - (k+\theta)(\tau+\lambda)) I_H^* + \lambda_H (k+\theta)\right)} - (\lambda + \tau + \mu_H) N_M = 0.$$

We can easily write the last equation in the form

$$BI_H^{*2} + AI_H^* = 0$$

with

$$\begin{cases} A = \beta\alpha(\tau + \mu_H + \lambda) + (\lambda + \tau + \mu_H) N_M (\alpha\mu_H - (k+\theta)(\tau+\lambda)), \\ B = -\lambda_H (k+\theta)(\lambda + \tau + \mu_H) N_M + \beta\alpha\lambda_H. \end{cases}$$

Hence,

$$I_H^* = \frac{\beta\alpha\lambda_H - \lambda_H (k+\theta)(\lambda + \tau + \mu_H) N_M}{\alpha\beta(\tau + \mu_H + \lambda) + (\lambda + \tau + \mu_H) N_M (\alpha\mu_H - (k+\theta)(\tau+\lambda))}. \tag{18}$$

Then, the existence of the endemic point E_1 is given by

$$E_1 = (S_H^*, I_H^*, R_H^*, N_M - M_I^*, M_I^*),$$

with

$$\begin{cases} S_H^* = \dfrac{\lambda_H - (\tau + \mu_H + \lambda) I_H^*}{\mu_H}, \\ I_H^* = \dfrac{\beta\alpha\lambda_H - \lambda_H (k+\theta)(\lambda + \tau + \mu_H) N_M}{\alpha\beta(\tau + \mu_H + \lambda) + (\lambda + \tau + \mu_H) N_M (\alpha\mu_H - (k+\theta)(\tau+\lambda))}, \\ R_H^* = \dfrac{\lambda}{\mu_H} I_H^*, \\ M_U^* = N_M - M_I^*, \\ M_I^* = \dfrac{\alpha\mu_H N_M I_H^*}{\left((\alpha\mu_H - (k+\theta)(\tau+\lambda)) I_H^* + \lambda_H (k+\theta)\right)}. \end{cases}$$

□

The positivity of the equilibrium points

It is clear that the point E_0 is positive (belong to the positive orthant) without condition. It then remains to show that the point E_1 is positive. This amounts to showing that $S_H^*, I_H^*, R_H^*, M_I^*$ are positive.

1. The positivity of S_H^* is according to (14).

   It is clear that the denominator of S_H^* is positive, so it suffices to study the positivity of the numerator

$$\lambda_H - (\tau + \mu_H + \lambda) I_H^* = \frac{\lambda_H \alpha \mu_H + \lambda_H (k+\theta) \mu_H}{\alpha\beta + \alpha\mu_H - (k+\theta)(\tau+\lambda)} = \frac{S_1}{S_2}.$$

   As the numerator S_1 is positive, then it remains to show that S_2 is positive; that is,

$$\alpha\beta + \alpha\mu_H > (k+\theta)(\tau+\lambda).$$

2. The positivity of I_H^* is according to (18). We let

$$I_H^* = \frac{A}{B}.$$

   We write A as a function of R_0 defined in (8),

$$A = \alpha\lambda_H\beta\frac{R_0^2 - 1}{R_0^2}.$$

   Therefore A is positive if $R_0 > 1$.

   We write B as a function of R_0,

$$B \geq \alpha\beta(\lambda+\tau)\frac{R_0^2 - 1}{R_0^2}.$$

   We note that B is positive if $R_0 > 1$.

3. The positivity of R_H^* is according to (15). We note that R_H^* is positive if I_H^* is positive.

4. The positivity of M_I^* is according to (16). We let

$$M_I^* = \frac{M_1}{M_2}.$$

It is clear that M_1 is positive if I_H^* is positive. Then, M_I^* is positive if M_2 is positive. Indeed

$$M_2 > 0 \Longrightarrow \frac{C_1}{C_2} > 0,$$

with

$$\begin{cases} C_1 = \alpha\mu_H\left(\lambda+\tau+\mu_H\right)\left(\alpha\beta+\alpha\mu_H-\left(k+\theta\right)\left(\tau+\lambda\right)\right) \\ \qquad +\lambda_H\left(k+\theta\right)\left(\alpha\beta\mu_H+\left(\tau+\mu_H+\lambda\right)\alpha\mu_H\right) > 0, \\ C_2 = \alpha\beta\left(\tau+\mu_H+\lambda\right)+\left(\lambda+\tau+\mu_H\right)\left(\alpha\mu_H-\left(k+\theta\right)\left(\tau+\lambda\right)\right) > 0. \end{cases}$$

We then deduce that M_2 is positive if

$$\alpha\beta+\alpha\mu_H > \left(k+\theta\right)\left(\tau+\lambda\right),$$

and therefore M_I^* is positive if I_H^* is positive and if

$$\alpha\beta+\alpha\mu_H > \left(k+\theta\right)\left(\tau+\lambda\right).$$

3.3 Stability

The stability of the equilibrium point [5] results from the stability of the Jacobian matrix of the system (10), (11), (12), (13) (i.e., its eigenvalues must be negative), which is given by

$$J(S_H, I_H, R_H, M_I) = \begin{pmatrix} -\frac{\beta M_I}{N_M}-\mu_H & 0 & 0 & -\frac{\beta S_H}{N_M} \\ \frac{\beta M_I}{N_M} & -\lambda-(\tau+\mu_H) & 0 & \frac{\beta S_H}{N_M} \\ 0 & \lambda & -\mu_H & 0 \\ \frac{-\alpha(N_M-M_I)I_H}{(S_H+I_H)^2} & \frac{\alpha(N_M-M_I)S_H}{(S_H+I_H)^2} & 0 & \frac{-\alpha I_H}{S_H+I_H}-(k+\theta) \end{pmatrix}$$

Theorem 3. It holds that E_0 is locally asymptotically stable (the solutions must approach an equilibrium point under initial conditions close to the equilibrium point) if and only if

$$R_0 < 1.$$

Proof. The Jacobian matrix at point E_0 is given by

$$J(E_0) = \begin{pmatrix} -\mu_H & 0 & 0 & -\dfrac{\beta\lambda_H}{N_M\mu_H} \\ 0 & -(\lambda+\tau+\mu_H) & 0 & \dfrac{\beta\lambda_H}{N_M\mu_H} \\ 0 & \lambda & -\mu_H & 0 \\ 0 & \dfrac{\alpha N_M\mu_H}{\lambda_H} & 0 & -(k+\theta) \end{pmatrix} = \begin{pmatrix} -A & 0 & 0 & -B \\ 0 & -C & 0 & B \\ 0 & D & -A & 0 \\ 0 & E & 0 & -F \end{pmatrix}.$$

We calculate the characteristic polynomial of $J(E_0)$,

$$\begin{aligned} \det(J(E_0) - XI_3) &= -(X+\mu_H)^2 (k\lambda + k\tau + k\mu_H + \theta\lambda \\ &\quad +\theta\tau - \alpha\beta + \theta\mu_H + (\theta+\lambda+\tau+\mu_H+k)X + X^2) \\ &= -(X+\mu_H)^2 P(X). \end{aligned}$$

We have the first eigenvalues

$$X_1 = X_2 = -\mu_H < 0,$$

and

$$P(X) = A + BX + CX^2,$$

with

$$\begin{aligned} A &= k\lambda + k\tau + k\mu_H + \theta\lambda + \theta\tau + \theta\mu_H - \alpha\beta, \\ B &= (\theta + \lambda + \tau + \mu_H + k), \\ C &= 1. \end{aligned}$$

Let us use Descartes' rule [16] to show that the coefficients of the polynomial P do not change signs.

It is clear that B and C are positive. It only remains to show that A is positive or equivalently

$$1 - R_0 > 0.$$

So, A is positive if $R_0 < 1$.

According to Descartes' rule, the polynomial does not admit any positive root. Hence, the stability of the point E_0. □

4 Method of resolution

4.1 Jacobi wavelets

The Jacobi polynomials $J_m^{(\alpha,\beta)}$ $(\alpha > -1, \beta > -1)$ are orthogonal polynomials on the interval $[-1,1]$ ([13, 29]) with the weight function

$$\omega(x) = (1-x)^{\alpha}(1+x)^{\beta}, \tag{19}$$

where m is a positive integer, which represents the degree of the polynomial. These polynomials belong to the weight space $L^2_{\omega}([-1,1])$. The Jacobi polynomials can be represented by the recursive formula given by

$$\begin{aligned} &J_m^{(\alpha,\beta)}(x) \\ &= \frac{(\alpha+\beta+2m-1)\left[\alpha^2-\beta^2+x(\alpha+\beta+2m)(\alpha+\beta+2m-2)\right]}{2m(\alpha+\beta+2m-2)(\alpha+\beta+m)} J_{m-1}^{(\alpha,\beta)}(x) \\ &\quad -\frac{(\alpha+m-1)(\beta+m-1)(\alpha+\beta+2m)}{m(\alpha+\beta+2m-2)(\alpha+\beta+m)} J_{m-2}^{(\alpha,\beta)}(x), \end{aligned} \tag{20}$$

where

$$J_0^{(\alpha,\beta)}(x) = 1,\ J_1^{(\alpha,\beta)}(x) = \frac{\alpha+\beta+2}{2}x + \frac{\alpha-\beta}{2}. \tag{21}$$

As the Jacobi polynomials are orthogonal with respect to the weight function ω, then

$$\left\langle J_n^{(\alpha,\beta)}, J_m^{(\alpha,\beta)} \right\rangle_{L^2_{\omega}} = h_m^{(\alpha,\beta)}\delta_{n,m}, \qquad \text{for all } n,m \in \mathbb{N}, \tag{22}$$

where

$$h_m^{(\alpha,\beta)} = \left\| J_m^{(\alpha,\beta)} \right\|^2 = \frac{2^{\alpha+\beta+1}\Gamma(\alpha+m+1)\Gamma(\beta+m+1)}{(2m+1+\alpha+\beta)m!\Gamma(\alpha+\beta+m+1)}, \tag{23}$$

$\delta_{n,m}$ represents the Kronecker symbol, Γ is the Euler gamma function, and $\langle\cdot,\cdot\rangle_{L^2_{\omega}}$ denotes the inner product of $L^2_{\omega}([-1,1])$.

The Jacobi wavelets are defined by

$$\psi_{n,m}^{(\alpha,\beta)}(x) = \begin{cases} \frac{2^{\frac{k+1}{2}}}{\sqrt{h_m^{(\alpha,\beta)}}} J_m^{(\alpha,\beta)}\left(2^{k+1}x - 2n + 1\right), & \frac{n-1}{2^k} \le x < \frac{n}{2^k} \\ 0, & \text{otherwise,} \end{cases} \tag{24}$$

where $k \in \mathbb{N}$, $n = 1, \ldots, 2^k$ represents the number of decomposition levels, $m = 0, 1, \ldots, M$ is the degree of the Jacobi polynomials ($M \in \mathbb{N}^*$). The coefficient $\frac{2^{\frac{k+1}{2}}}{\sqrt{h_m^{(\alpha,\beta)}}}$ is for normality.

4.2 Decomposition in Jacobi wavelets basis

Since the Jacobi wavelets family $\left\{\psi_{n,m}^{(\alpha,\beta)}\right\}_{\substack{n=1,\ldots,2^k \\ m\geq 0}}$ forms an orthonormal basis in $L^2_\omega([0,1])$, we can express all functions f in $L^2_\omega([0,1])$ as a unique linear combination of elements of this basis:

$$f(x)=\sum_{n=1}^{2^k}\sum_{m=0}^{\infty}c_{n,m}\psi_{n,m}^{(\alpha,\beta)}(x), \tag{25}$$

where $c_{n,m}=\left\langle f,\psi_{n,m}^{(\alpha,\beta)}\right\rangle_{L^2_\omega([0,1])}$. From the point of view of the numerical analysis, we take the truncated sum (its projection on finite space)

$$f(x)=\sum_{n=1}^{2^k}\sum_{m=0}^{M}c_{n,m}\psi_{n,m}^{(\alpha,\beta)}(x). \tag{26}$$

Let

$$C=\left[c_{1,0},\ldots,c_{1,M},c_{2,0},\ldots,c_{2,M},\ldots,c_{2^k,0},\ldots,c_{2^k,M}\right]^T,$$

and let

$$\Psi^{(\alpha,\beta)}=\left[\psi_{1,0}^{(\alpha,\beta)},\ldots,\psi_{1,M}^{(\alpha,\beta)},\psi_{2,0}^{(\alpha,\beta)},\ldots,\psi_{2,M}^{(\alpha,\beta)},\ldots,\psi_{2^k,0}^{(\alpha,\beta)},\ldots,\psi_{2^k,M}^{(\alpha,\beta)}\right]^T. \tag{27}$$

We can find the following matrix notation:

$$f(x)=C^T\Psi^{(\alpha,\beta)}(x). \tag{28}$$

In this case, the $\Psi^{(\alpha,\beta)}$ are called the $2^k(M+1)$ Jacobi wavelets vector and C is a $2^k(M+1)$ vector.

The operational matrix of derivative

The derivative of the Jacobi wavelets vector $\Psi^{(\alpha,\beta)}$ from (27) can be expressed by [17]

$$\frac{d\Psi^{(\alpha,\beta)}(x)}{dx}=D^{(\alpha,\beta)}\Psi^{(\alpha,\beta)}(x),$$

where $D^{(\alpha,\beta)}$ denotes the $2^k(M+1)\times 2^k(M+1)$ operational matrix given by

$$D^{(\alpha,\beta)} = \begin{pmatrix} F^{(\alpha,\beta)} & 0 & \dots & 0 \\ 0 & F^{(\alpha,\beta)} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & F^{(\alpha,\beta)} \end{pmatrix},$$

$F^{(\alpha,\beta)}$ is $(M+1)\times(M+1)$ matrix, where its (i,j)th element is given by

$$F_{i,j}^{(\alpha,\beta)} = \begin{cases} 2^{k+1}\frac{\sqrt{h_{j-1}^{(\alpha,\beta)}}}{\sqrt{h_{i-1}^{(\alpha,\beta)}}}\gamma_{i-1,j-1}^{(\alpha,\beta)}, & \text{if } i > j \\ 0, & \text{otherwise,} \end{cases} \tag{29}$$

in which $h_{i-1}^{(\alpha,\beta)}$ and $h_{j-1}^{(\alpha,\beta)}$ are defined from (23), and $\gamma_{i-1,j-1}^{(\alpha,\beta)}$ are given by

$$\gamma_{i-1,j-1}^{(\alpha,\beta)} = \frac{\Gamma(i+\beta)}{2\Gamma(i+\alpha+\beta)}\frac{(2(j-1)+\alpha+\beta+1)\Gamma(\alpha+\beta+j)}{\Gamma(\alpha+j)} \times \left[\sum_{d=j-1}^{i-1}(-1)^{d-j-1}\frac{(2(d+1)+\alpha+\beta)\Gamma(\alpha+d+1)}{\Gamma(\beta+d+2)}\right]. \tag{30}$$

4.3 Description of the solution method

In this subsection, we describe how to apply the Jacobi wavelets to solve ODEs. Then, we use the DQLT with Jacobi wavelets method to solve a set of nonlinear differential equations. In the end, we present the formula of errors calculation.

Linear first order differential equation

Consider the linear first order differential equation with initial condition

$$\begin{cases} f'(x) + a(x)f(x) = g(x), & x \in]0,1], \\ f(0) = f_0, \end{cases} \tag{31}$$

where f_0 is arbitrary constant. To solve the problem (31), we decompose $f(x)$ in the Jacobi wavelets basis $\left\{\psi_{n,m}^{(\alpha,\beta)}\right\}_{\substack{n=1,\dots,2^k \\ m=0,\dots,M}}$ by estimating (28),

$$f(x) = C^T\Psi^{(\alpha,\beta)}(x), \tag{32}$$

where C denotes the solution vector of the problem. Then, we have

$$f'(x) = C^T D^{(\alpha,\beta)}\Psi^{(\alpha,\beta)}(x). \tag{33}$$

Now, by substituting (32)–(33) into problem (29), we get the following algebraic system:

$$C^T(D^{(\alpha,\beta)} + a(x_i)I)\Psi^{(\alpha,\beta)}(x_i) = g(x_i), \quad i = 1, \ldots, nc, \tag{34}$$

where I is the identity matrix and nc is the number of collocation points. We have to insert the initial condition

$$f_0 = C^T \Psi^{(\alpha,\beta)}(0). \tag{35}$$

Equations (34) and (35) generate $2^k(M+1)$ set of linear algebraic equations, which can easily be solved for the unknown C by using one of the method of resolution an algebraic system. Consequently, $f(x)$ given in (32) will be easily calculated.

Set of nonlinear differential equation

To solve a set of nonlinear differential equations, we will use the DQLT to transform this problem by iterative steps into a set of decoupled and linearized differential equations, where each equation can be written as the problem (31). Then we use the Jacobi wavelets method described in the previous subsection. Let us consider a set of p nonlinear differential equations. This iterative technique can be defined by

$$\begin{cases} \text{Given initial profile} f_1^{(0)}, f_2^{(0)}, \ldots, f_p^{(0)}, \\ (f_1'(x))^{(l+1)} + a_1(x) f_1^{(l+1)} = g_1\left(x, f_1^{(l)}, f_2^{(l)}, \ldots, f_p^{(l)}\right), \\ (f_2'(x))^{(l+1)} + a_2(x) f_2^{(l+1)} = g_2\left(x, f_1^{(l+1)}, f_2^{(l)}, \ldots, f_p^{(l)}\right), \\ \vdots \\ \left(f_p'(x)\right)^{(l+1)} + a_p(x) f_p^{(l+1)} = g_p\left(x, f_1^{(l+1)}, f_2^{(l+1)}, \ldots, f_p^{(l)}\right), \end{cases} \tag{36}$$

where $f_i^{(l+1)}$ and $f_i^{(l)}$ are the approximations of the solution f_i at the current and the precedent iteration, respectively. At each iteration, we apply the Jacobi wavelets method to solve p linear differential equation. Then, for $(l+1)$th iteration, we can calculate the decoupling error using the following formula:

$$E_{DQLT} = \max\left(\|f_1^l - f_1^{l+1}\|_2, \|f_2^l - f_2^{l+1}\|_2, \ldots, \|f_p^l - f_p^{l+1}\|_2\right). \tag{37}$$

The procedure is terminated when the error of decoupling is sufficiently small.

Error estimation

Since the ODEs solutions are only known at collocation points, the most appropriate norm is the euclidean norm if the exact solution is given. The accuracy of the proposed method is estimated by

$$error = \|f(x) - f_{ex}(x)\|_2 = \sqrt{\sum_{i=1}^{nc} |f(x_i) - f_{ex}(x_i)|^2}, \tag{38}$$

where f_{ex} is the analytic solution, f is the approximate solution, and nc the number of collocation points.

5 Numerical simulations of model SIR-MI

In this section, we will study the stability of the point E_1 numerically. Then, we simulate our model to see the importance of studying the effect of the sterilization parameter infected material and management on the evolution of the human population. We apply the Jacobi wavelets with DQLT, which makes it possible to numerically evaluate the solutions of the ODEs and to build their graphs. We conclude our section with a discussion of the results obtained.

5.1 The study of the stability of the second equilibrium point E_1

The following table gives us the biological parameters that verify the conditions of existence and stability of the second point of equilibrium E_1:

Table 2: The parameters verifying the stability of E_1.

Equilibrium point	$E_1 = (1622, 9920, 1668, 18172, 11828)$								
Parameter	λ_H	β	μ_H	λ	λ_M	τ	α	k	θ
Value	230	0.23	0.05116	0.086	1500	0.011	0.6	0.05	0.3

1. For the conditions of existence, we have

$$(R_0 = 1.6313) > 1, \tag{39}$$

and

$$(\alpha\beta + \alpha\mu_H = 0.1687) > ((k + \theta)(\tau + \lambda) = 0.0339). \tag{40}$$

Hence, we have the existence of the point E_1.

2. For stability, the Jacobian matrix of system (34) at point E_1 after substitution of the parameters given in Table 2 is given by

$$J(E_1) = \begin{pmatrix} -0.1418 & 0 & 0 & -0.0124 \\ 0.0907 & -0.1482 & 0 & 0.0124 \\ 0 & 0.0860 & -0.0512 & 0 \\ -1.5836 & 2.5875 & 0 & -0.5778 \end{pmatrix}.$$

The eigenvalues of $J(E_1)$ are

$$\begin{cases} vp_1 = -0.0512, \\ vp_2 = -0.6842, \\ vp_3 = -0.0918 + 0.0515i, \\ vp_4 = -0.0918 - 0.0515i. \end{cases}$$

The eigenvalues of $J(E_1)$ have a negative real part, hence, the asymptotic stability of the second equilibrium point E_1.

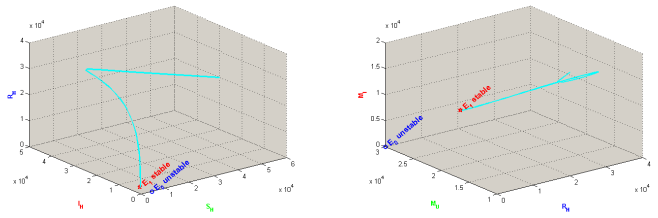


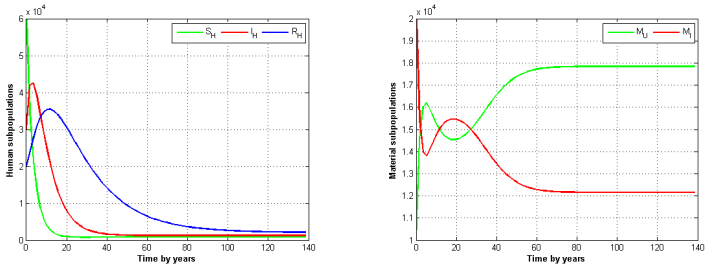
Figure 2: The convergence of the system toward E_1.

Figure 2: The convergence of the system toward E_1.

We note that the solutions obtained in Figure 2. All converge towards the equilibrium point E_1 when $t \to +\infty$.

Figure 3 shows that the five subpopulations converge after a fairly large time to the second equilibrium point E_1.

In what follows, we carried out simulation experiments with the parameters illustrated by Table 3.



(A) (B)

Figure 3: Evolution of human and material subpopulations: (A) represents the human subpopulations, (B) represents the material subpopulations.

Table 3: Variations in estimated values of biological data.

The time	40 years								
I.C	S_H	I_H		R_H		M_U		M_I	
Value	60000	30000		20000		10000		20000	
Parameter	λ_H	β	μ_H	λ	λ_M	τ	α	k	θ
Value	230	0.072-0.6	0.01-0.05116	0.006-0.235	1500	0.011	0.1-0.6	0.05	0.17-0.4

5.2 The impact of equipment sterilization on disease progression

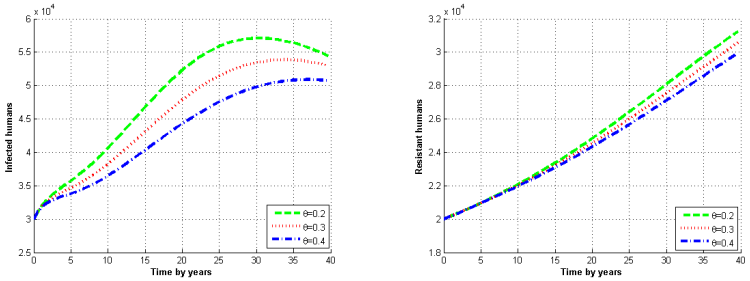


Figure 4: Evolution of human subpopulations for different values of θ.

For different values of $\theta = 0.2, 0.3, 0.4$, we see, in Figure 4, the positive effect played by the sterilization parameter to reduce the number of infections. This shows that better compliance with universal hygiene rules and recommendations for disinfection of nondisposable medical equipment and the development of equipment for use single should allow in the long term a quasi-disappearance of infections.

5.3 The impact of the transition rate from I_H to R_H

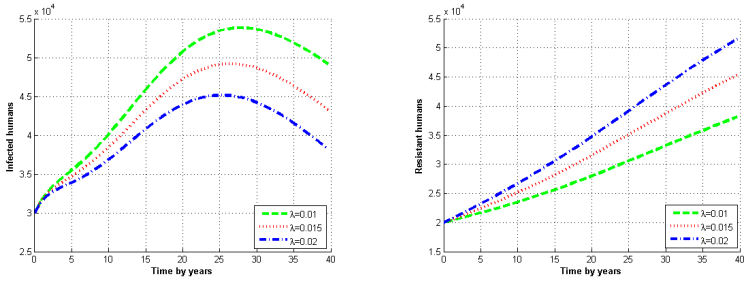


Figure 5: Evolution of human subpopulations for different values of λ.

For different values of $\lambda = 0.01, 0.015, 0.02$, the curves obtained in Figure 5 have made it possible to understand the important role of good care for infected people. Being infected with HCV does not protect against the risk of a new infection, which could worsen the medical situation. The development of a better therapeutic strategy can significantly improve the quality of life of people infected with hepatitis C.

6 Discussions and Conclusion

World Health Organization recommends that countries develop national strategies to reduce the burden of disease associated with hepatitis C hampered by weak or lacking national surveillance systems and unreliable estimates of the burden of hepatitis C morbidity.

In this work, we described and mathematically analyzed the dynamics of hepatitis C. The different numerical simulations were presented to see the behavior of the model at infinity, and the results obtained showed that the trends related to the prevention and management of infection considerably influence the subpopulations. We also applied the Jacobi wavelets method associated with the DQLT to obtain a numerical solution, which gave a very satisfactory results.

Due to the lack of data, our model has not been validated for the case of Algeria. Nevertheless the results of this modest work constitute the bases of work to be continued and improved for a much more in-depth study.

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