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Research Article

ADI method of credit spread option pricing based on jump-diffusion model

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Abstract

As the main contribution of this article, we establish an option on a credit spread under a stochastic interest rate. The intense volatilities in financial markets cause interest rates to change greatly; thus, we consider a jump term in addition to a diffusion term in our interest rate model. However, this decision leads us to a partial integral differential equation. Since the integral part might bring some difficulties, we put forward a fairly new numerical scheme based on the alternating direction implicit method. In the remainder of the article, we discuss consistency, stability, and convergence of the proposed approach. As the final step, with the help of the MATLAB program, we provide numerical results of implementing our method on the governing equation.

AMS subject classifications (2020): 45D05, 42C10, 65G99.

Keywords: Interest rate; Option pricing; Jump-diffusion models; Alternating direction implicit, Convergence.

1 Introduction

Spread options have great importance in financial markets. Modern and specialized investment in contemporary markets is based on this reality that we can represent strategies and models to invest in particular markets such

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as the oil and exchange markets. These models are widely used in societies in which the inflation rate can change heavily during a short period, in short periods, and investors face less loss in these markets.

For example, we can consider an investment in the oil market. If the investor defines their call (buy) and put (sell) positions according to the oil assets and their derivatives, they do not face many volatility challenges. Because the asset price in the oil market changes over a period, the price of another asset will change equally in the same direction with a little increase or decrement. Another example of such markets, occurring in some countries, is intense volatilities of exchange. In this case, if investors investment to buy and sell currency (e.g., uses the difference between Dollar and Euro), they will not face intense price volatilities of this market from other markets.

The credit derivatives market has come under the spotlight recently. Credit derivatives pricing and valuation have received much attention over the last few years, and they are considered as one of the most significant new financial tools. The value of the portfolio with credit derivatives is extremely sensitive to changes in the spread between default-free bonds and defaultable bonds. Hence the investors can minimize and manage the exposure to credit risk. Credit default swaps and credit spread options are the most popular form of credit derivatives. Credit spread options payment depends on particular credit spread or credit-sensitive asset prices. The default or credit rating downgrade risks can be compensated by corporate bond credit spread; it is an additional remuneration paid to investors. Credit spread can protect the holders of corporate bonds from the loss of the rising yield spread. Deng, Johnson, and Sogomonian [1] studied the spread options in the energy market. In 2015, Zhou et al. [2] suggested a credit spread option pricing model that depends on the time, interest rate, and the logarithm of the credit spread, which was solved by GARCH and Longstaff–Schwartz methods. In 1995, Longstaff and Schwartz [3] studied that the credit spread logarithm follows the mean-reversion process and priced credit spread options on the assumption that the change in the distribution logarithm is normal. In 2012, Su and Wang [4] hypothesized that the interest rate would follow the Vasichek model and that the default intensity would follow the jump propagation trend. Tchuindjo [5], in 2011, proposed closed-form solutions for pricing credit risky discount bonds and their European call and put options in the intensity-based reduced-form framework, assuming the stochastic dynamics of both the risk-free interest rate and the credit spread are driven by two correlated Ho-Lee models.

These studies have not examined the abrupt changes in jumps and exchanges. In this study, a model investigates based on abrupt changes in the exchange. Also, the ADI numerical method with expected convergence is applied. The innovation of this article is in proving the sustainability of the method. Finally, implementing the model on the real data is one of the most significant tasks in this article. The ADI method is an appropriate method for solving large matrix equations and numerically solving parabolic and elliptic differential equations. This method is applied to prevent solving a large linear system. The advantages of this method are the optimal accuracy of stability, low computational volume, and low computer memory requirement.

This article is organized as follows: In section 2, the model, equation, and initial and boundary conditions will be introduced. In section 3, the ADI numerical method and its implementation on the model will be presented. Consistency, stability, and convergence of the method will be addressed in section 4. In Section 5, the proposed approach will be implemented on the real data. The last section provides some research titles for the followers.

2 Modeling of the credit spread option pricing

In this article, we consider the option pricing, according to the model presented by Longstaff and Schwartz [3]. That is if we take X_t as a credit spread option in a stock market, then its logarithm will satisfy the following stochastic differential equation [2]:

$$dX = (a - bX)dt + sdW_1,$$

also according to the Vasicek model, we have

$$dr = (\alpha - \beta r)dt + \sigma dW_2,$$

where the parameters a, b, s, α, β, and σ are constant numbers and W_1 and W_2 represent the Brownian motions and they are are independent variables with $corr(W_1, W_2) = \rho$. In mathematical modeling of financial topics, neglection of the jumps may lead to inaccurate results, by considering the economic and political problems, and also natural disasters such as earthquakes and storms as the main source of sudden movements, namely, jumps. So, in order to be able to present a pricing model for options, let us add the jump phrase to the Vasicek interest rate model. According to the information, we have

$$dX = (a - bX)dt + sdW_1,$$

$$dr = (\alpha - \beta r)dt + \sigma dW_2 + d(\sum_{i=1}^{M_t} Z_i),$$

where M_t is a Poisson process with the positive density rate λ that will be appeared soon, and the variables Z_i represent the jumps and are independent and identically distributed. The credit spread option price demonstrated by $P(X, r, t)$, equals to the solution of the following partial integral differential equation [6]:

$$P_t + (a - bX - qs)P_X + (\alpha - \beta r - q\sigma)P_r + \rho\sigma s P_{Xr} + \frac{1}{2}s^2 P_{XX} + \frac{1}{2}\sigma^2 P_{rr} - (r+\lambda)P + \int_0^{+\infty} P(X, r+y, t) f(y) dy = 0,$$

where $f(y)$ is the Poisson distribution probability density function and y is the jump value.
By applying the change of variable $t = T - \tau$, for $t \in [0, T]$, the last equation changes into the following form:

$$P_\tau = (a - bX - qs)P_X + (\alpha - \beta r - q\sigma + \frac{1}{\lambda})P_r + \rho\sigma s P_{Xr} + \frac{1}{2}s^2 P_{XX} + (\frac{1}{2}\sigma^2 + \frac{1}{\lambda^2})P_{rr} - (r + \lambda - 1)P. \tag{1}$$

The initial and boundary conditions for $0 \leq X \leq K, 0 \leq r \leq H$, and $0 \leq \tau \leq T$ are as follows:

$$\frac{\partial^2 P}{\partial X^2}(0, r, \tau) = 0, \frac{\partial^2 P}{\partial X^2}(K, r, \tau) = 0,$$

$$\frac{\partial^2 P}{\partial r^2}(X, 0, \tau) = 0, \frac{\partial^2 P}{\partial r^2}(X, H, \tau) = 0,$$

$$\frac{\partial P}{\partial X}(K, r, \tau) = 0, \frac{\partial P}{\partial r}(X, H, \tau) = 0,$$

$$P(0, r, \tau) = P(X, 0, \tau) = 1, P(X, r, 0) = G(x),$$

where $G(x) = \max\{X - \bar{K}, r - \bar{K}, 0\}$ is the payoff function of an European put option (in Figure 1, we display payoff function based on data in Table 1) and $\bar{K}$ is the strike price.

3 Numerical Solution

In order to solve (1), we apply the ADI method. To do this, we define the operator LP as follows:

$$LP = L^X P + L^r P + L^{Xr} P + L^{XX} P + L^{rr} P + \Phi.$$

We define the last-mentioned components corresponding to the ones in (1) (see [7, 8]),

$$L^X P = (a - bX - qs)P_X, \tag{2}$$

$$L^r P = (\alpha - \beta r - q\sigma)P_r, \tag{3}$$

$$L^{Xr} P = (\rho\sigma s)P_{Xr}, \tag{4}$$

$$L^{XX} P = \frac{1}{2}(s^2)P_{XX}, \tag{5}$$

$$L^{rr} P = (\frac{1}{2}(\sigma)^2 + \frac{1}{\lambda^2})P_{rr}, \tag{6}$$

$$\Phi = (r + \lambda - 1)P, \tag{7}$$

while q is the market price of the risk. Approximation of these derivatives by finite differences results in [9, 16] as follows:

$$\begin{aligned}(2):&\frac{P_{ij}^{n+\frac{1}{6}} - P_{ij}^{n}}{\Delta\tau} = (a - bx_i - qs)\frac{P_{i+1j}^{n+\frac{1}{6}} - P_{ij}^{n+\frac{1}{6}}}{h}\\ &\Rightarrow P_{ij}^{n+\frac{1}{6}} - P_{ij}^{n} = A(P_{i+1j}^{n+\frac{1}{6}} - P_{ij}^{n+\frac{1}{6}}),\end{aligned}$$

where $A = \dfrac{(a - bX - qs)(\Delta\tau)}{h}$. By rearranging the last equation, one has $(1 + A)P_{ij}^{n+\frac{1}{6}} - AP_{i+1j}^{n+\frac{1}{6}} = P_{ij}^{n}$, $i = 0, 1, 2, \ldots, n$, and the following system is obtained:

$$\begin{cases} i = 0: & (1 + A)P_{0j}^{n+\frac{1}{6}} - AP_{1j}^{n+\frac{1}{6}} = P_{0j}^{n},\\ i = 1: & (1 + A)P_{1j}^{n+\frac{1}{6}} - AP_{2j}^{n+\frac{1}{6}} = P_{1j}^{n},\\ i = 2: & (1 + A)P_{2j}^{n+\frac{1}{6}} - AP_{3j}^{n+\frac{1}{6}} = P_{2j}^{n},\\ i = n-1: & (1 + A)P_{n-1j}^{n+\frac{1}{6}} - AP_{nj}^{n+\frac{1}{6}} = P_{n-1j}^{n},\\ i = n: & (1 + A)P_{nj}^{n+\frac{1}{6}} - AP_{n+1j}^{n+\frac{1}{6}} = P_{nj}^{n}. \end{cases}$$

The second term of the left in the last equation should be determined by known values as follows:

$$\frac{P_{n+1j}^{n+\frac{1}{6}} - 2P_{nj}^{n+\frac{1}{6}} + P_{n-1j}^{n+\frac{1}{6}}}{h^2} = 0 \Rightarrow P_{n+1j}^{n+\frac{1}{6}} = 2P_{nj}^{n+\frac{1}{6}} - P_{n-1j}^{n+\frac{1}{6}}.$$

Substitution the recent formula in the last equation (for $i = n$) results

$$i = n: (1 - A)P_{nj}^{n+\frac{1}{6}} + P_{n-1j}^{n+\frac{1}{6}} = P_{nj}^{n+\frac{1}{6}}.$$

In the matrix representation, one gets

$$\begin{bmatrix} 1+A & -A & 0 & 0 & \cdots & 0 \\ 0 & 1+A & -A & 0 & \cdots & 0 \\ 0 & 0 & 1+A & -A & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1+A & -A \\ 0 & 0 & 0 & \cdots & A & 1-A \end{bmatrix} \begin{bmatrix} P_{0j}^{n+\frac{1}{6}} \\ P_{1j}^{n+\frac{1}{6}} \\ P_{2j}^{n+\frac{1}{6}} \\ \vdots \\ P_{n-1j}^{n+\frac{1}{6}} \\ P_{nj}^{n+\frac{1}{6}} \end{bmatrix} = \begin{bmatrix} P_{0j}^{n} \\ P_{1j}^{n} \\ P_{2j}^{n} \\ \vdots \\ P_{n-1j}^{n} \\ P_{nj}^{n} \end{bmatrix}$$

Now we apply the finite difference for (3) as follows:

$$\begin{aligned} \frac{P_{ij}^{n+\frac{1}{3}} - P_{ij}^{n+\frac{1}{6}}}{\Delta\tau} &= \frac{1}{2}(-^{r}P_{ij}^{n+\frac{1}{6}} + 3^{r}P_{ij}^{n+\frac{1}{3}}) \\ &= \frac{1}{2}(-B_j \frac{P_{ij+1}^{n+\frac{1}{6}} - P_{ij}^{n+\frac{1}{6}}}{k} + 3B_j \frac{P_{ij+1}^{n+\frac{1}{3}} - P_{ij}^{n+\frac{1}{3}}}{k}) \\ \Rightarrow (1+3B_j^{'})P_{ij}^{n+\frac{1}{3}} - 3B_j^{'}P_{ij+1}^{n+\frac{1}{3}} &= P_{ij}^{n+\frac{1}{6}} - B_j^{'}P_{ij+1}^{n+\frac{1}{6}} = P_{ij}^{n+\frac{1}{6}} - B_j^{'}P_{ij+1}^{n+\frac{1}{6}}, \end{aligned}$$

where $B_j^{'} = \frac{B_j\Delta\tau}{2k}$, $B_j = \alpha - \beta r_j - qs$, and $f_{ij} = P_{ij}^{n+\frac{1}{6}} - B_j^{'}P_{ij+1}^{n+\frac{1}{6}}$.

For $j = 0, 1, 2, \ldots, m$, we obtain the following system:

$$(1+3B_j^{'})P_{ij}^{n+\frac{1}{3}} - 3B_j^{'}P_{ij+1}^{n+\frac{1}{3}} = f_{ij}$$

$$\Rightarrow \begin{cases} j=0: & (1+3B_0^{'})P_{i0}^{n+\frac{1}{3}} - 3B_0^{'}P_{i1}^{n+\frac{1}{3}} = f_{i0}, \\ j=1: & (1+3B_1^{'})P_{i1}^{n+\frac{1}{3}} - 3B_1^{'}P_{i2}^{n+\frac{1}{3}} = f_{i1}, \\ j=2: & (1+3B_2^{'})P_{i2}^{n+\frac{1}{3}} - 3B_2^{'}P_{i3}^{n+\frac{1}{3}} = f_{i2}^{n}, \\ \vdots & \\ j=m-1: & (1+3B_{m-1}^{'})P_{im-1}^{n+\frac{1}{3}} - 3B_{m-1}^{'}P_{im}^{n+\frac{1}{3}} = f_{im-1}, \\ j=m: & (1+3B_m^{'})P_{im}^{n+\frac{1}{3}} - 3B_m^{'}P_{im+1}^{n+\frac{1}{3}} = f_{im}. \end{cases}$$

By applying the boundary conditions, the second term of the left in the last equation (for $j = m$) should be determined by known values:

$$P_{im+1}^{n+\frac{1}{3}} = 2P_{im}^{n+\frac{1}{3}} - P_{im-1}^{n+\frac{1}{3}}.$$

Substitution recent formula in the last equation (for $j = m$) results in:

$$j = m : (1+3B_m^{'})P_{im}^{n+\frac{1}{3}} - 3B_m^{'}(2P_{im}^{n+\frac{1}{3}} - P_{im-1}^{n+\frac{1}{3}}) = f_{im}$$

$$\Rightarrow (1-3B_m^{'})P_{im}^{n+\frac{1}{3}} + 3B_m^{'}P_{im-1}^{n+\frac{1}{3}} = f_{im}.$$

This system can be written as the following:

$$\begin{bmatrix} 1+3B_0^{'} & -3B_0^{'} & 0 & 0 & \cdots & 0 \\ 0 & 1+3B_1^{'} & -3B_1^{'} & 0 & \cdots & 0 \\ 0 & 0 & 1+3B_2^{'} & -3B_2^{'} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1+3B_{m-1}^{'} & -3B_{m-1}^{'} \\ 0 & 0 & 0 & \cdots & -3B_m^{'} & 1-3B_m^{'} \end{bmatrix} \begin{bmatrix} P_{i0}^{n+\frac{1}{3}} \\ P_{i1}^{n+\frac{1}{3}} \\ P_{i2}^{n+\frac{1}{3}} \\ \vdots \\ P_{n-1j}^{n+\frac{1}{3}} \\ P_{nj}^{n+\frac{1}{3}} \end{bmatrix} = \begin{bmatrix} f_{i0} \\ f_{i1} \\ f_{i2} \\ \vdots \\ f_{im-1} \\ f_{im} \end{bmatrix}$$

Using the finite difference method and the boundary conditions, (4) will transform into the following form:

$$\begin{aligned} &\frac{P_{ij}^{n+\frac{1}{2}} - P_{ij}^{n+\frac{1}{3}}}{\Delta\tau} = \frac{1}{12}(5^{Xr}P_{ij}^{n+\frac{1}{6}} - 16^{Xr}P_{ij}^{n+\frac{1}{3}} + 23^{Xr}P_{ij}^{n+\frac{1}{2}}) \\ &\Rightarrow (1-46C)P_{ij}^{n+\frac{1}{2}} - 23CP_{i+1j+1}^{n+\frac{1}{2}} + 23CP_{i+1j}^{n+\frac{1}{2}} + 23CP_{ij+1}^{n+\frac{1}{2}} + 23CP_{i-1j}^{n+\frac{1}{2}} \\ &\qquad + 23CP_{ij-1}^{n+\frac{1}{2}} - 23CP_{i-1j-1}^{n+\frac{1}{2}} = f_{ij}, \end{aligned}$$

in which f_{ij} are the sentences in $n+\frac{1}{3}$ and $n+\frac{1}{6}$ steps and $C = \frac{\rho\sigma s\Delta\tau}{12hk}$. Note that the finite difference in (4) follows the following formula:

$$P_{Xr} = \frac{P_{i+1j+1} - P_{i+1j}^{n+\frac{1}{2}} - P_{ij+1}^{n+\frac{1}{2}} + 2P_{ij}^{n+\frac{1}{2}} - P_{i-1j}^{n+\frac{1}{2}} - P_{ij-1}^{n+\frac{1}{2}} + P_{i-1j-1}^{n+\frac{1}{2}}}{hk}.$$

For $i = 0, 1, 2, \ldots, n$, the following matrix equation is obtained:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots\cdots & \cdots & \cdots & 0 \\ 23C & 1-46C & 23C & 0 & \cdots\cdots & \cdots & \cdots & 0 \\ 0 & 23C & 1-46C & 23C & \cdots\cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots\ \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots\ 0 & 23C & 1-46C & 23C \\ 0 & 0 & 0 & 0 & \cdots\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_{0j}^{n+\frac{1}{2}} \\ P_{1j}^{n+\frac{1}{2}} \\ P_{2j}^{n+\frac{1}{2}} \\ \vdots \\ P_{n-2j}^{n+\frac{1}{2}} \\ P_{n-1j}^{n+\frac{1}{2}} \\ P_{nj}^{n+\frac{1}{2}} \end{bmatrix}$$

$$+ \begin{bmatrix} -23C & 23C & \cdots & \cdots & \cdots & 0 \\ -23C & 23C & \cdots & \cdots & \cdots & 0 \\ 0 & -23C & 23C & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & 23C & 0 \\ 0 & 0 & \cdots & \cdots & -23C & 23C \\ 0 & 0 & 0 & \cdots & -23C & 23C \end{bmatrix} \begin{bmatrix} P_{0j-1}^{n+\frac{1}{2}} \\ P_{1j-1}^{n+\frac{1}{2}} \\ \vdots \\ P_{n-2j-1}^{n+\frac{1}{2}} \\ P_{n-1j-1}^{n+\frac{1}{2}} \\ P_{nj-1}^{n+\frac{1}{2}} \end{bmatrix}$$

$$+\begin{bmatrix} 23C & -23C & 0 & 0 & \cdots & \cdots & 0 \\ 0 & 23C & -23C & 0 & \cdots & \cdots & 0 \\ 0 & 0 & 23C & -23C & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots & \\ 0 & 0 & 0 & \cdots & 23C & -23C & 0 \\ 0 & 0 & 0 & \cdots & 0 & 23C & -23C \\ 0 & 0 & 0 & \cdots & 0 & 23C & -23C \end{bmatrix} \begin{bmatrix} P_{0j+1}^{n+\frac{1}{2}} \\ P_{1j+1}^{n+\frac{1}{2}} \\ P_{2j+1}^{n+\frac{1}{2}} \\ \vdots \\ P_{n-2j+1}^{n+\frac{1}{2}} \\ P_{n-1j+1}^{n+\frac{1}{2}} \\ P_{nj+1}^{n+\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} f_{0j} \\ f_{1j} \\ f_{2j} \\ \vdots \\ f_{n-2j} \\ f_{n-1j} \\ f_{nj} \end{bmatrix}$$

Applying the finite difference for (5), we get the following system of equations, $i = 0, 1, 2, \dots, n$:

$$\frac{P_{ij}^{n+\frac{2}{3}} - P_{ij}^{n+\frac{1}{2}}}{\Delta\tau} = \frac{1}{24}(-9L^{XX}P_{ij}^{n+\frac{1}{6}} + 37L^{XX}P_{ij}^{n+\frac{1}{3}} - 59L^{XX}P_{ij}^{n+\frac{1}{2}} + 559L^{XX}P_{ij}^{n+\frac{2}{3}})$$

$$\Rightarrow \begin{cases} i = 0: & P_{0j}^{n+\frac{2}{3}} = f_{0j}, \\ j = 1: & (1+110D)P_{1j}^{n+\frac{2}{3}} - 55P_{0j}^{n+\frac{2}{3}} - 55P_{2j}^{n+\frac{2}{3}} = f_{1j}, \\ j = 2: & (1+110D)P_{2j}^{n+\frac{2}{3}} - 55P_{1j}^{n+\frac{2}{3}} - 55P_{3j}^{n+\frac{2}{3}} = f_{2j}, \\ \vdots & \\ j = n-2: & (1+110D)P_{n-2j}^{n+\frac{2}{3}} - 55P_{n-3j}^{n+\frac{2}{3}} - 55P_{n-1j}^{n+\frac{2}{3}} = f_{n-2j}, \\ j = n-1: & (1+110D)P_{n-1j}^{n+\frac{2}{3}} - 55P_{n-2j}^{n+\frac{2}{3}} - 55P_{nj}^{n+\frac{2}{3}} = f_{n-1j}, \\ j = n: & P_{nj}^{n+\frac{2}{3}} = f_{nj}, \end{cases}$$

where f_{ij} are all the sentences in the steps $n+\frac{1}{2}$, $n+\frac{1}{3}$, $n+\frac{1}{6}$, and $D = \frac{s^2\Delta\tau}{48h^2}$.

The finite difference in (5) follows the following formula:

$$P_{XX} = \frac{P_{i+1j} - 2P_{ij} - P_{i-1j}}{h^2}.$$

Write the finite difference, regarding (6), and then, we obtain the following results for $j = 0, 1, 2, \dots, m$:

$$\frac{P_{ij}^{n+\frac{5}{6}} - P_{ij}^{n+\frac{2}{3}}}{\Delta\tau} = \frac{1}{720}(251L^{rr}P_{ij}^{n+\frac{1}{6}} - 1274L^{rr}P_{ij}^{n+\frac{1}{3}} + 2616L^{rr}P_{ij}^{n+\frac{1}{2}} - 2774L^{rr}P_{ij}^{n+\frac{2}{3}} + 1901L^{rr}P_{ij}^{n+\frac{5}{6}}).$$

The resulted matrix, by substitution $j = 0, 1, 2, \dots, m$ is as follows ($E = \frac{\Delta\tau}{720k^2}(\frac{1}{2}\sigma^2 + \frac{1}{\lambda^2})$):

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ -1901E & 1+3802E & -1901E & 0 & \cdots & \cdots & 0 \\ 0 & -1901E & 1+3802E & -1901E & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -1901E & 1+3802E & -1901E & 0 \\ 0 & 0 & \cdots & \cdots & -1901E & 1+3802E & -1901E \\ 0 & 0 & \cdots & \cdots & 0 & 0 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} P_{i0}^{n+\frac{5}{6}} \\ P_{i1}^{n+\frac{5}{6}} \\ P_{i2}^{n+\frac{5}{6}} \\ \vdots \\ P_{im-2}^{n+\frac{5}{6}} \\ P_{im-1}^{n+\frac{5}{6}} \\ P_{im}^{n+\frac{5}{6}} \end{bmatrix} = \begin{bmatrix} f_{i0} \\ f_{i1} \\ f_{i2} \\ \vdots \\ f_{im-2} \\ f_{im-1} \\ f_{im} \end{bmatrix}$$

The last step $(n+1)$ comes from (7) (see [6]) as follows:

$$\frac{P_{ij}^{n+1} - P_{ij}^{n+\frac{5}{6}}}{\Delta\tau} = \varphi = -(r+\lambda-1)P_{ij}^{n}$$
$$\Rightarrow P_{ij}^{n+1} = -(r+\lambda-1)\Delta\tau P_{ij}^{n} + P_{ij}^{n+\frac{5}{6}}.$$

4 Consistency, stability, and convergence

By proving the consistency and stability of the method, the convergence will be guaranteed via the Lax equivalence theorem [2].

4.1 Proof of consistency

Let Φ be the exact solution of (1), which depends on the independent variables r, X, and t (e.g., $L(\Phi) = 0$). Assume that the exact solution of the finite difference equation is ϕ (to write $F(\varphi) = 0$) and that v is an arbitrary continuous function of r, X, and t with a sufficient number of continuous derivatives such that $L(v)$ can be evaluated at $(i\eta, j\epsilon, kh)$. Then the error function at the point $v_{i,j,k} = v(i\eta, j\epsilon, hk)$ is defined as $\tau_{i,j,k}(v) = F(v_{i,j,k}) - L(v_{i,j,k})$. If $\tau_{i,j,k}(v) \to 0$ as η, ϵ , and h tend to zero, then the finite difference equation is consistent with the partial-differential equation as follows:

$$\begin{aligned}
&\frac{\varphi_{i,j,k+1}-\varphi_{i,j,k}}{h}\\
&=(a-bX_i-qs)\frac{\varphi_{i+1,j,k}-\varphi_{i,j,k}}{\eta}+(\alpha-\beta r_j-q\sigma+\frac{1}{\lambda})\frac{\varphi_{i,j+1,k}-\varphi_{i,j,k}}{\epsilon}\\
&\quad+\rho\sigma s\frac{\varphi_{i+1,j+1,k}-\varphi_{i+1,j,k}-\varphi_{i,j+1,k}+2\varphi_{i,j,k}-\varphi_{i-1,j,k}-\varphi_{i,j-1,k}+\varphi_{i-1,j-1,k}}{\eta\epsilon}\\
&\quad+\frac{1}{2}s^2\frac{\varphi_{i+1,j,k}-2\varphi_{i,j,k}+\varphi_{i-1,j,k}}{\eta^2}\\
&\quad+(\frac{1}{2}\sigma^2+\frac{1}{\lambda^2})\frac{\varphi_{i,j+1,k}-2\varphi_{i,j,k}+\varphi_{i,j-1,k}}{\epsilon^2}-(r_j+\lambda-1)\varphi_{i,j,k}. \qquad (8)
\end{aligned}$$

Let $X_i=X_0+i\eta$, let $r_j=r_0+j\epsilon$, let $A=a-bX_0-qs$, $B=\alpha+\frac{1}{\lambda}-\beta r_0-qs$, let $C=\rho\sigma s$, let $D=\frac{1}{2}s^2$, let $E=\frac{1}{2}\sigma^2+\frac{1}{\lambda^2}$, and let $F=r_0+\lambda-1$. By considering the Taylor expansion and previous assumptions, (8) changes into the following equation:

$$\begin{aligned}
\frac{h^2}{2}\frac{\partial^2\varphi}{\partial t^2}+\frac{h^2}{6}\frac{\partial^3\varphi}{\partial t^3}+\cdots=&(A-b\eta i)\frac{\eta}{2}\frac{\partial^2\varphi}{\partial X^2}+(B-\beta\epsilon j)\frac{\epsilon}{2}\frac{\partial^2\varphi}{\partial r^2}+\cdots\\
&+(A-b\eta i)\frac{\eta^2}{6}\frac{\partial^3\varphi}{\partial X^3}+(B-\beta\epsilon j)\frac{\epsilon^2}{3}\frac{\partial^3\varphi}{\partial r^3}+\cdots\ .
\end{aligned}$$

By considering the smallest power of the above equation and assuming $\eta=\epsilon=\kappa h$ (where κ is a constant number), as h approaches zero, τ will approach to zero too and the consistency of this method is proved.

4.2 Proof of stability

Proving stability directly from the definition is quite difficult, in general. Instead, it is easier to use tools from the Fourier analysis to evaluate the stability of finite difference schemes. In this section, we analyze the stability of the ADI method. To study the stability, we use the Von Neumann approach [13, 14, 17]. The Von Neumann analysis is based on calculating the amplification factor of schemes, G, and deriving conditions under which $|G|\le 1$. We have introduced the numerical amplification factor G as

$$G=\frac{E_{k,l}^{n+1}}{E_{k,l}^{n}}. \qquad (9)$$

From (9), we may relate the error $E_{k,l}^n$ at the nth time step with the initial error $E_{k,l}^0$ [8] by

$$E_{k,l}^n=G^nE_{k,l}^0, E_{k,l}^0=e^{ikx}e^{ily}.$$

For finding G, a simple procedure is to replace $\varphi_{k,l}^n$ in the scheme by $G^ne^{ikx}e^{ily}$, for each k,l, and n,

$$
\begin{aligned}
\varphi_{k,l}^{n+1} &= G^{n+1}e^{ikx}e^{ily} = G\varphi_{k,l}^{n},\\
\varphi_{k+1,l}^{n} &= G^{n}e^{ik(x+\Delta x)}e^{ily} = G^{n}e^{ik\Delta x}e^{ikx}e^{ily} = \varphi_{k,l}^{n}e^{ik\Delta x},\\
\varphi_{k,l+1}^{n} &= G^{n}e^{ikx}e^{il(y+\Delta y)} = G^{n}e^{ikx}e^{ily}e^{il\Delta y} = \varphi_{k,l}^{n}e^{il\Delta y},
\end{aligned}
\tag{10}
$$

where Δx and Δy are positive.
Without loss of generality of the problem, suppose that the coefficients of the relation (8) are assumed to be equivalent to constant A, where A is the largest value of the relation coefficients (8).
Substituting (10) in relation (8) leads to the following relation:

$$
\begin{aligned}
\frac{G-1}{\Delta t} =& A\Big(\frac{e^{ik\Delta x}-1}{\Delta x}+\frac{e^{il\Delta y}-1}{\Delta y}+\frac{e^{ik\Delta x}-2+e^{-ik\Delta x}}{(\Delta x)^2}+\frac{e^{il\Delta y}-2+e^{-il\Delta y}}{(\Delta y)^2}\\
&+\frac{e^{i(k\Delta x+l\Delta y)}-e^{ik\Delta x}-e^{il\Delta y}+2-e^{-ik\Delta x}-e^{-il\Delta y}+e^{-i(k\Delta x+l\Delta y)}}{\Delta x\Delta y}+()
\end{aligned}
\tag{11}
$$

Suppose that $k\Delta x = l\Delta y = \theta$, that $\frac{\Delta t}{\Delta x} = \frac{\Delta t}{\Delta y} = h$, and that $\frac{\Delta t}{(\Delta x)^2} = \frac{\Delta t}{(\Delta y)^2} = \eta$. Then by simplifying relation (11), G is obtained as follows:

$$
G = 1 + A(-2h + 2h\cos\theta - 2\eta(1 - cos2\theta) + \Delta t) + 2Ah\sin\theta i.
$$

We calculate G^2 and establish $|G|^2 \le 1$ to obtain stability conditions from it (Because in this article, Δx and Δy are considered values less than 1, we consider $h < \eta$ in the $|G|^2$ formula) as follows:

$$
|G|^2 = (1 + A(-2\eta + 2\eta\cos\theta - 2\eta(1 - cos2\theta) + \Delta t))^2 + 4\eta^2A^2\sin^2\theta. \tag{12}
$$

If the following conditions are met, relation (12) will be stable [15, 5]:

$$
1 + A(-2\eta + 2\eta\cos\theta - 2\eta(1 - cos2\theta) + \Delta t) \le 2A\eta\cos\theta,\ 4\eta^2A^2 \le 1. \tag{13}
$$

The stability conditions of the problem, according to relations (13), are as follows:

$$
\frac{\Delta t}{\Delta x^2} \le \min\{\frac{1}{2|A|}, \frac{1}{A(2-\Delta x^2)}\},\ \frac{\Delta t}{\Delta y^2} \le \min\{\frac{1}{2|A|}, \frac{1}{A(2-\Delta y^2)}\}.
$$

Numerical results of this convergence and stability are given in the next section.

5 Numerical results

As an illustration, consider the credit spread and an interest rate to implement the ADI numerical method with $\frac{1}{6}$ step-size. Then, we obtain the changes in credit spread and an interest rate for these markets. Table 1 shows the parameters used in this study.

Table 1: Data

Parameters	definitions	values
K	$Max(X) : maximum\ value\ of$ X	30
H	$Max(r) : maximum\ value\ of$ r	30
q	$the\ market\ price\ of\ the\ risk$	1
σ	$volatility$	0.3
T	$Max(t) :\ maximum\ value\ of$ t	0.5
$a, b,$ and s	$constant$	$0.5, 0.5, and\ 0.25$
λ	$intensity\ rate\ of\ Poisson\ process$	0.1
ρ	$corr(W_1, W_2)$	0.5
r	$interest\ rate$	0.03

The payoff function can be illustrated in Figure 1

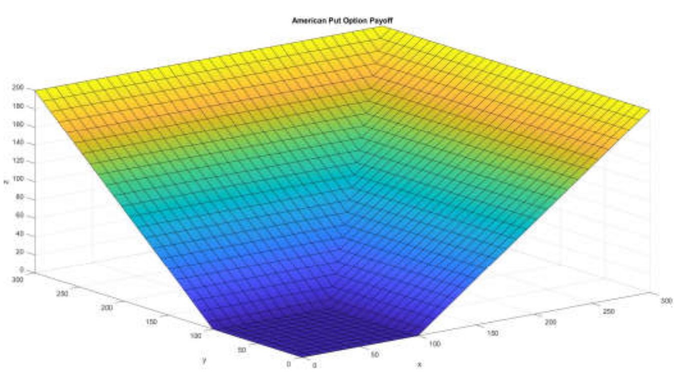

Figure 1: Payoff function.

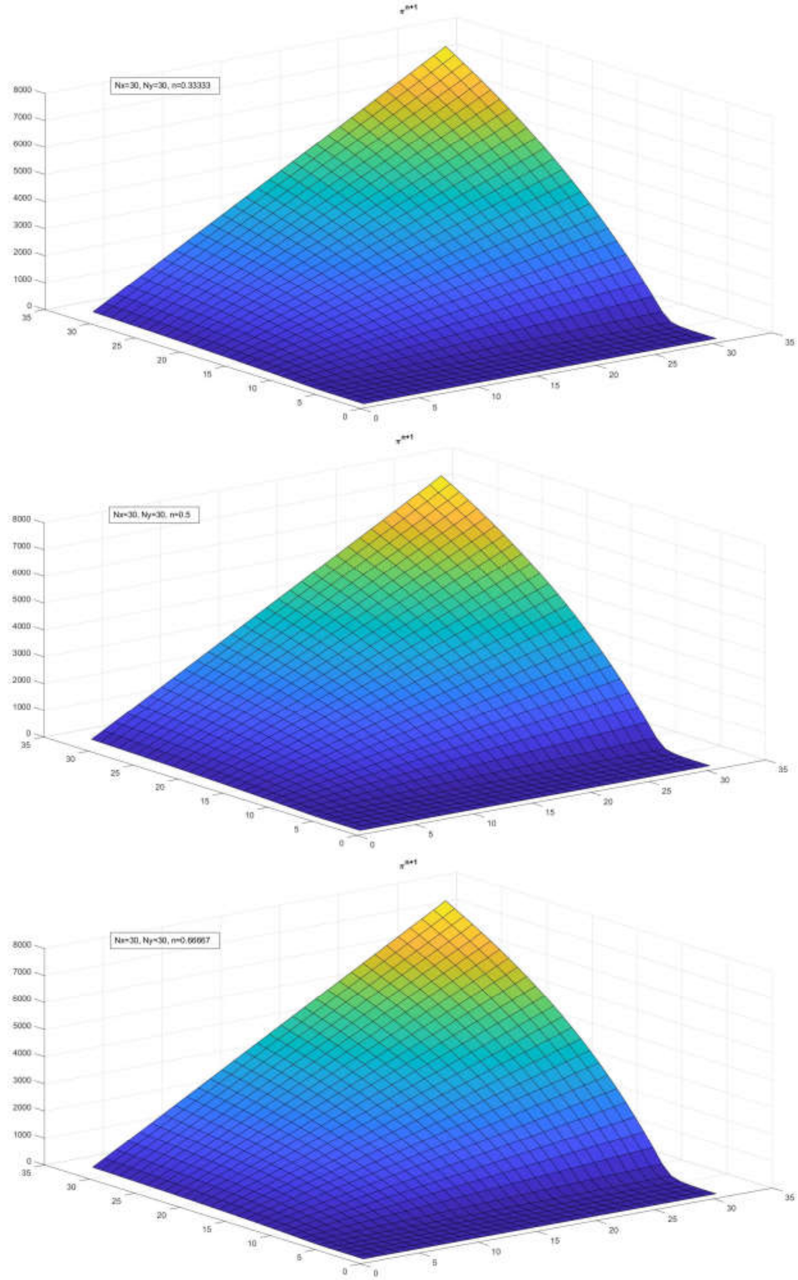

Figure 2: Numerical results using the ADI method with a step-size of $\frac{1}{6}$ for $n = 0.3333, 0.5$, and 0.66667

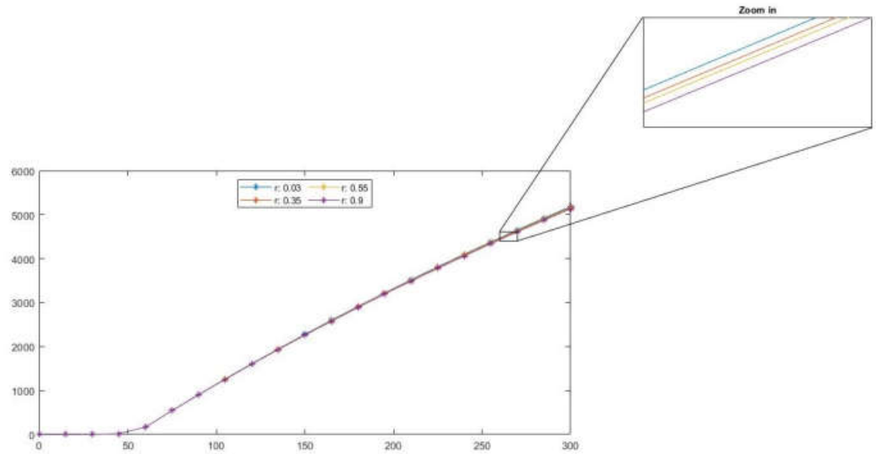

Figure 3: Comparison of option prices for different interest rates such as $r = 0.03, 0.35, 0.55$, and 0.9

Eventually, regarding our model and the suggested numerical method, we can obtain the credit spread option price. As can be seen from the following figures, the price of the credit spread option varies for different amounts of the variable n (Figure 2 shows the credit spread option price functions at $t = 0.33333, 0.5, 0.66667$). Some option values for a different base on asset prices, for example, $70, 110, 170, 220$, and 270, are listed in Table 2 for different interest rates, for instance, $r = 0.03, 0.35, 0.55$, and 0.9. In addition, Figure 3 shows a comparison of option prices for the mentioned interest rates, which shows that interest rates and option prices are inversely related.

By selecting $\Delta x = \Delta y = 1$ and $\Delta t = \frac{1}{4}$, the error results of time steps were obtained $0.6515, 0.4798, 0.3173, 0.0213$, and 0, respectively. As the time steps got smaller, these errors became smaller (by selecting $\Delta x = \Delta y = \frac{1}{2}$ and $\Delta t = \frac{1}{16}$, the error results of time steps were obtained $0.3173, 0.0030, 0.0009, 0.0006$, and 0, respectively), but in both cases, because our selected condition has been extracted from the stability range, the result of the error confirms the correctness of this range. As you can see, the resulting errors decrease and tend to zero. So, this method is convergent and stable [9].

Table 2: Option values for the different underlying asset prices

$P(x,r,\tau)$	x				
	70	110	170	220	270
r=0.03	925	1386.9	2079.8	2657.2	3234.7
r=0.35	922.2	1382.8	2073.7	2649.4	3225.3
r=0.55	920.6	1380.3	2069.9	2644.6	3219.3
r=0.9	917.6	1375.9	2063.2	2636.1	3209

6 Conclusions

The main goal of this article was to propose a pricing model, which is based on the credit spread and the interest rate, such that some jumps on the second are possible. Though the bringing jump in the modeling seems an interesting idea, it can bring some difficulties when it comes to implementing a numerical method. Therefore, to deal with this problem, as the second step, we postulate a fairly new numerical method call, ADI with step-size $\frac{1}{6}$ combined with Adams–Bashforth formula [9]. To continue this work, one good idea is to use other methods such as the machine learning, Chebyshev cardinal wavelets, and wavelets Galerkin method to obtain the results [4, 3]. Definitively, comparing the results of different methods could be highly instructive.

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