

A parametric iteration method for solving Lane-Emden type equations

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Abstract

In this paper, an analytical method called the parametric iteration method (PIM) is presented for solving the second-order singular IVPs of Lane-Emden type, and its local convergence is discussed. Since it is often useful to have an approximate analytical solution to describe the Lane-Emden type equations, especially for ones where the closed-form solutions do not exist at all, therefore, an effective improvement of the PIM is further proposed that is capable of obtaining an approximate analytical solution. The improved PIM is finally treated as an algorithm in a sequence of intervals for finding accurate approximate solutions of the nonlinear Lane-Emden type equations. Also, we show how to identify an approximate optimal value of the convergence accelerating parameter within the frame of the method. Some examples are given to demonstrate the efficiency and accuracy of the proposed method.

Keywords: Piecewise-truncated parametric iteration method; Truncated parametric iteration method; Parametric iteration method; Nonlinear Lane-Emden type equations.

1 Introduction

Recently, a lot of attention has been focused on the study of singular initial value problems (IVPs) in the second-order ordinary differential equations (ODEs). Many problems in mathematical physics and astrophysics can be modelled by the so-called IVPs of the Lane-Emden type equation [2, 4, 14]:

$$\begin{cases} y'' + \frac{2}{x}y' + f(x, y) = g(x), & x > 0, \\ y(0) = a, \quad y'(0) = b, \end{cases} \quad (1)$$

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where a and b are constants, $f(x, y)$ is a continuous real valued function, and $g(x) \in C[0, \infty]$. When $f(x, y) = K(y)$, $g(x) = 0$, Equation(1) reduces to the classical Lane-Emden equation which, with specified $K(y)$, was used to model several phenomena in mathematical physics and astrophysics such as the theory of stellar structure, the thermal behavior of a spherical cloud of gas, isothermal gas sphere and theory of thermionic currents [2, 4, 14].

Since, the Lane-Emden type equations have significant applications in many fields of scientific and technical world, a variety of forms of $f(x, y)$ and $g(x)$ have been investigated by many researchers (e.g., [3, 16, 17]). A discussion of the formulation of these models and the physical structure of the solutions can be found in the literature. Though the numerical solution of the Lane-Emden Equation (1), as well as other various linear and nonlinear singular IVPs in quantum mechanics and astrophysics [9], is numerically challenging because of the singularity behavior at the origin $x = 0$, but analytical solutions are much needed for physical understanding. Recently, many analytical methods were used to solve the Lane-Emden equation [7, 8, 10, 18]. Those methods are based on either series solutions or perturbation techniques [1, 11–13]. However, the convergence region of the corresponding results is rather small.

The strategy that will be pursued in this work rests mainly on establishing useful algorithms based on the parametric iteration method (PIM) [5, 6, 15] for finding highly accurate solution of the Lane-Emden type equations that they

- Overcome the main difficulty arising in the singularity of the equation at $x = 0$.
- Provide us with a convenient way to modify the convergence region and rate of the solution.
- Are simple to implement, accurate when applied to the Lane-Emden type equations and avoid tedious computational works.

The examples analyzed in the present paper reveal that the newly developed algorithms are easy, effective and accurate to solve the singular IVPs of Lane-Emden type equation.

2 Analysis of methods

In this section, the PIM is described for solving Equation(1). This method provides the solution of Equation(1) as a sequence of iterations. The method gives rapidly convergent successive approximations of the exact solution if such a solution exists, otherwise approximations can be used for numerical purposes. The idea of the PIM is very simple and straightforward. To explain the basic idea of the PIM, we first consider Equation(1) as follows:

$$L[y(x)] + N[y(x)] = g(x), \quad (2)$$

with

$$L[y(x)] = y''(x) + \frac{2}{x}y'(x), \quad N[y(x)] = f(x, y), \quad (3)$$

where L denotes the so-called auxiliary linear operator with respect to y and N is a nonlinear operator with respect to y . The basic character of the PIM is to construct a family of iterative processes for Equation(1) as follows [15]:

$$y_{n+1}(x) = y_n(x) + h \int_0^x \left(t - \frac{t^2}{x} \right) \left\{ y_n''(t) + \frac{2}{t}y_n'(t) + f(t, y_n(t)) - g(t) \right\} dt, \quad (4)$$

where $y_0(x)$ is the initial guess and the subscript n denotes the n -th iteration, and $h \neq 0$ denotes the so-called auxiliary parameter which can be identified easily and efficiently by the technique proposed in this paper. Accordingly, the successive approximations $y_n(x)$, $n \geq 0$ of the PIM in the auxiliary parameter will be readily obtained by selecting the initial approximation. Consequently, the exact solution can be obtained by using

$$y(x) = \lim_{n \rightarrow \infty} y_n(x). \quad (5)$$

It is interesting to note that for the linear Lane-Emden type equations, its exact solution can be obtained easily by only one iteration step due to the fact that the multiplier can be suitably identified, as will be shown in this paper later.

Now we will have the following proposition for the iteration formula (4).

Proposition 1. *If $y(x) \in C^2[0, T]$, then, for $x \leq T$,*

$$\int_0^x \left(t - \frac{t^2}{x} \right) \left\{ y''(t) + \frac{2}{t}y'(t) \right\} dt = y(x) - y(0). \quad (6)$$

Proof. Simple integration by parts. □

In the light of (4) and (6), we will have the following simple iteration formula:

$$y_{n+1}(x) = (1 + h)y_n(x) - hy_0 + h \int_0^x \left(t - \frac{t^2}{x} \right) [f(t, y_n(t)) - g(t)] dt, \quad (7)$$

where $y_0 = y(0)$.

Note that the expressions (4) and (7) demonstrate that the variational iteration method (VIM) [7] is a special case of the PIM when $h = -1$. The fact that the PIM solves the Lane-Emden type equations without correction functional and restricted variation can be considered as an advantage of this method over the VIM.

The PIM (7) makes a recurrence sequence $\{y_n(x)\}$. Obviously, the limit of this sequence will be the solution of Equation (1) if this sequence is convergent.

In order to prove the sequence $\{y_n(x)\}$ is convergent, we construct a series

$$y_0(x) + [y_1(x) - y_0(x)] + \cdots + [y_n(x) - y_{n-1}(x)] + \cdots \quad (8)$$

Noticing that

$$S_{n+1} = y_0(x) + [y_1(x) - y_0(x)] + \cdots + [y_n(x) - y_{n-1}(x)] = y_n(x), \quad (9)$$

the sequence $\{y_n(x)\}$ will be convergent if the series is convergent.

Theorem 1. *If $N[y] = f(x, y)$ is Lipschitz-continuous in $[0, T]$ and $g(x) \in C[0, T]$ then the series of (8) is convergent, i.e., the sequence $\{y_n(x)\}$ is convergent for $x \in [0, T]$.*

Proof. According to (7), note that

$$|y_1(x) - y_0(x)| = \left| h \int_0^x \left(t - \frac{t^2}{x} \right) \{N[y_0(t)] - g(t)\} dt \right| \leq |h|MNx, \quad (10)$$

where

$$M = \max_{0 \leq t \leq x \leq T} \left| t - \frac{t^2}{x} \right| = \frac{T}{2}, \quad N = \max_{0 \leq t \leq x \leq T} |N[y_0(t)] - g(t)| \quad (11)$$

From (7) and (10), and the assumption that $|N[y_n] - N[y_{n-1}]| \leq L|y_n - y_{n-1}|$ where L denotes the Lipschitz constant of $N[y(x)]$, it follows that

$$\begin{aligned} |y_2(x) - y_1(x)| &= |1 + h||y_1(x) - y_0(x)| + |h|ML \int_0^x |y_1(t) - y_0(t)| dt \\ &\leq \frac{N}{L} \left[|1 + h| \frac{(|h|MLx)}{1!} + \frac{(|h|MLx)^2}{2!} \right] \\ &= \frac{N}{L} \sum_{k=0}^1 \binom{1}{k} |1 + h|^{1-k} \frac{(|h|MLx)^{k+1}}{(k+1)!}, \end{aligned} \quad (12)$$

$$|y_3(x) - y_2(x)| \leq \frac{N}{L} \sum_{k=0}^2 \binom{2}{k} |1+h|^{2-k} \frac{(|h|MLx)^{k+1}}{(k+1)!}, \quad (13)$$

$$|y_{n+1}(x) - y_n(x)| \leq \frac{N}{L} \sum_{k=0}^n \binom{n}{k} |1+h|^{n-k} \frac{[|h|MLx]^{k+1}}{(k+1)!}, \quad (14)$$

In view of (14), the convergence of the series (8) can be concluded for the solution domain $x < T$ and $|1+h| < 1$ with the help of the mathematical software such as Maple. Therefore the series of (8) is absolute convergence, i.e., the sequence $\{y_n(x)\}$ is convergent for $x \in [0, T]$. \square

3 A piecewise-truncated PIM

The successive iterations of the PIM may be very complex, so that the resulting integrals in the relation (4) may not be performed analytically. Also, the implementation of the PIM generally leads to calculation of unneeded terms, which more time is consumed in repeated calculations for series solutions. Here, an effective modification of the PIM is introduced to eliminate these repeated calculations. To completely stop these repeats in each step, provided that the integrand of (4) in each of iterations is expanded in multivariate Taylor series around $x = 0$. We propose the following improvement of the PIM (4), which is called the truncated PIM (TP):

$$y_{n+1}(x) = y_n(x) + h \int_0^x F_n(x, t) dt, \quad n \geq 0, \quad (15)$$

where

$$\left(t - \frac{t^2}{x}\right) \left\{ y_n''(t) + \frac{2}{t} y_n'(t) + f(t, y_n(t)) - g(t) \right\} = F_n(x, t) + O(x^{n+1}) + O(t^{n+1}). \quad (16)$$

It is noteworthy to point out that the TP formula (15) can cancel all the repeated calculations and terms that are not needed as will be shown below. Furthermore, it can reduce the size of calculations. Most importantly, however it is the fact that the TP algorithm (15) solves a Lane-Emden differential equation exactly if its solution is an algebraic polynomial up to some degree.

In general, by using the TP formula (15), we obtain a series solution, which in practice is a truncated series solution. This series solution gives a good approximation to the exact solution in a small region of x . An easy and reliable way of ensuring validity of the approximations (15) for large x , i.e. $I_i = [x_i, x_{i+1}]$ where $\Delta x = x_{i+1} - x_i$, $i = 0, 1, 2, \dots, N-1$, with $x_0 = 0$ and $x_N = T$. According to the relation (15), therefore, we can construct the following piecewise TP approximations (PTP) in the subintervals I_i . On

$[x_0, x_1]$, let

$$\left\{ \begin{array}{l} y_{1,m+1}(x) = y_{1,m}(x) + h \int_{x_0}^x F_{1,m}(x,t) dt, \quad m = 0, 1, \dots, n_1 - 1, \quad x \in [x_0, x_1], \\ y_{1,0}(x) = y(0) + y'(0)(x - x_0) = c_0 + c'_0(x - x_0), \\ \left(t - \frac{t^2}{x} \right) \left\{ y''_{1,m}(t) + \frac{2}{t} y'_{1,m}(t) + f(t, y_{1,m}(t)) - g(t) \right\} = \\ \qquad \qquad \qquad F_{1,m}(x,t) + O[(x - x_0)^{m+1}] + O[(t - x_0)^{m+1}]. \end{array} \right. \quad (17)$$

Then one can obtain the n_1 -order approximation $y_{1,n_1}(x)$ on $[x_0, x_1]$.

On $[x_1, x_2]$, let

$$\left\{ \begin{array}{l} y_{2,m+1}(x) = y_{2,m}(x) + h \int_{x_1}^x F_{2,m}(x,t) dt, \quad m = 0, 1, \dots, n_2 - 1, \quad x \in [x_1, x_2], \\ y_{2,0}(x) = y_{1,n_1}(x_1) + y'_{1,n_1}(x_1)(x - x_1) = c_1 + c'_1(x - x_1), \\ \left(t - \frac{t^2}{x} \right) \left\{ y''_{2,m}(t) + \frac{2}{t} y'_{2,m}(t) + f(t, y_{2,m}(t)) - g(t) \right\} = F_{2,m}(x,t) \\ \qquad \qquad \qquad + O[(x - x_1)^{m+1}] + O[(t - x_1)^{m+1}]. \end{array} \right. \quad (18)$$

Also the n_2 -order approximation $y_{2,n_2}(x)$ on $[x_1, x_2]$ can be obtained

In a similar way, on $[x_i, x_{i+1}]$, $i = 2, \dots, N - 1$, let

$$\left\{ \begin{array}{l} y_{i+1,m+1}(x) = y_{i+1,m}(x) + h \int_{x_i}^x F_{i+1,m}(x,t) dt, \quad m = 0, 1, \dots, n_{i+1} - 1, \\ \qquad \qquad \qquad x \in [x_i, x_{i+1}], \\ y_{i+1,0}(x) = y_{i,n_i}(x_i) + y'_{i,n_i}(x_i)(x - x_i) = c_i + c'_i(x - x_i), \\ \left(t - \frac{t^2}{x} \right) \left\{ y''_{i+1,m}(t) + \frac{2}{t} y'_{i+1,m}(t) + f(t, y_{i+1,m}(t)) - g(t) \right\} = F_{i+1,m}(x,t) \\ \qquad \qquad \qquad + O[(x - x_i)^{m+1}] + O[(t - x_i)^{m+1}]. \end{array} \right. \quad (19)$$

Hence this implies the n_{i+1} -order approximation $y_{i+1,n_{i+1}}(x)$ on $[x_i, x_{i+1}]$.

Therefore, according to (17)-(19), the approximation of Equation (1) on the entire interval $[0, T]$ will be calculated. It should be emphasized that the PIM and TF algorithms provide analytical solutions on $[0, T]$, while the PTP technique provides analytical solutions in $[x_i, x_{i+1}]$, which are continuous at the end points of each interval, i.e., $y_{i,n_i}(x_i) = c_i = y_{i+1,n_{i+1}}(x_i)$ and $y'_{i,n_i}(x_i) = c'_i = y'_{i+1,n_{i+1}}(x_i)$, $i = 1, 2, \dots, N - 1$.

Remark 1. In the case of failure of convergence of the PIM, the presence of the parameter could play a very important role in the frame of the PIM. Although, we can find a valid region for every physical problem by plotting the solution or its derivatives versus the parameter in some points [6,15], but an approximate optimal value of the convergence accelerating parameter h can be determined at the order of approximation by residual error:

$$Re s(h) = \int_{x_0}^{x_f} \{L[y_n(t)] + N[y_n(t)] - g(t)\}^2 dt, \quad (20)$$

One can easily minimize (20) by imposing the requirement $\frac{dRes(h)}{dh} = 0$.

4 Implementations

To give a clear overview of the content of this study, the several Lane-Emden type equations will be studied. These equations will be tested by the above-mentioned algorithms, which will ultimately show the usefulness and accuracy of these methods. Moreover, the numerical results indicate that the approach is easy to implement. All the results here are calculated by using the symbolic calculus software Maple 17.

Example 1. As a first example, let consider the following linear, the non-homogeneous Lane-Emden equation, i.e., Equation (1) with $f(x, y) = y$ and $g(x) = x^2 e^{-x}$:

$$y'' + \frac{2}{x}y' + y = x^2 e^{-x}, \quad (21)$$

subject to conditions

$$y(0) = 1, \quad y'(0) = 0. \quad (22)$$

The PIM has a very simple approach. Its concepts begin with dividing the left hand (21) into two parts, i.e., the auxiliary linear operator L and the nonlinear operator N as:

$$L[y(x)] = y''(x) + \frac{2}{x}y'(x) + y(x) \quad \text{and} \quad N[y(x)] \equiv 0. \quad (23)$$

This will allow us to construct a family of iterative processes for Eq.(21) as follows [14]:

$$y_{n+1}(x) = y_n(x) + h \int_0^x \left[\frac{t}{x} \sin(x-t) \right] \left\{ y_n''(t) + \frac{2}{t}y_n'(t) - t^2e^{-t} \right\} dt. \quad (24)$$

Using simple integration by parts, similar to Proposition 1, implies that

$$\int_0^x \left[\frac{t}{x} \sin(t-x) \right] \left\{ y''(t) + \frac{2}{t}y'(t) + y(t) \right\} dt = y(x) - \frac{\sin(x)}{x}y(0), \quad (25)$$

In the light of (24) and (25), the following PIM is:

$$y_{n+1}(x) = (1+h)y_n(x) - hy_0 \frac{\sin(x)}{x} - h \int_0^x \left[\frac{t}{x} \sin(t-x) \right] (t^2e^{-t}) dt, \quad (26)$$

where $y_0 = y(0)$ and $y_0(x) = y(0) + y'(0)(x)$. According to (26), therefore the following approximations with starting the initial guess $y_0(x) = 1$ are:

$$y_1(x) = \frac{(2+2h)x + h \sin(x) - hx(3+3x+x^2)e^{-x}}{2x}, \quad (27)$$

$$y_2(x) = \frac{(2+4h+2h^2)x + (2h+h^2)\sin(x) - hx(6x+2x^2+6+3hx+hx^2+3h)e^{-x}}{2x}, \quad (28)$$

⋮

which the exact solution of Equation(21) yields for $h = -1$, i.e.,

$$y(x) = \frac{x(3+3x+x^2)e^{-x} - \sin(x)}{2x}. \quad (29)$$

Example 2. As other example, we consider the nonlinear and non-homogeneous Lane-Emden equation, i.e., Equation (1) with $f(x, y) = y^3$ and $g(x) = 6 + x^6$ [10]:

$$y'' + \frac{2}{x}y' + y^3 = 6 + x^6, \quad (30)$$

subject to conditions

$$y(0) = 0, \quad y'(0) = 0. \quad (31)$$

Now, our aim is to solve the equation (30) by means of the TP algorithm (15). According to (15), it is easily to obtain the following approximations of the TP with starting the initial approximation $y_0(x) = 0$:

$$y_1(x) = 0, \quad (32)$$

$$y_2(x) = -hx^2, \quad (33)$$

$$y_n(x) = [1 - (1 + h)^{n-1}]x^2, \quad n \geq 3. \quad (34)$$

If we choose $h = -1$, then the TP algorithm yields the exact solution

$$y(x) = x^2. \quad (35)$$

It is interesting to note that the TP algorithm (15) can solve a Lane-Emden type equation exactly if its solution is an algebraic polynomial up to some degree.

Example 3. As final example, we consider the nonlinear, homogeneous Lane-Emden-type equation, i.e., Equation(1) with $f(x, y) = 4(2e^y + e^{\frac{y}{2}})$ and $g(x) = 0$:

$$y'' + \frac{2}{x}y' + 4(2e^y + e^{\frac{y}{2}}) = 0, \quad (36)$$

subject to conditions

$$y(0) = 0, \quad y'(0) = 0. \quad (37)$$

Here, the aim to solve the equation (36) by means of the above-proposed methods. Since the integration of the nonlinear term $4(2e^y + e^{\frac{y}{2}})$ in Equation (36) is not easily evaluated, thus the PIM requires a large amount of computational work to obtain few iterations of the solution (we can replace the nonlinear term with a series of finite components). However, one can used the modified PIM method, i.e., the TP algorithm (15). According to (15), it is easily to obtain the following approximations of (36) with starting the initial approximation $y_0(x) = 0$:

$$y_1(x) = 0, \quad (38)$$

$$y_2(x) = 2hx^2, \quad (39)$$

$$y_3(x) = (4h + 2h^2)x^2, \quad (40)$$

$$y_4(x) = (6h + 6h^2 + 2h^3)x^2 + (2h^2 + h^3)x^4, \quad (41)$$

and so on. To investigate the influence of h on the convergence of the solution obtained via the truncated PIM, here we plot the curves of $y''_{20}(0)$ and $y_{20}^{(4)}$, as shown in Figure 1. According to these curves, it is easy to discover the

valid region h , which corresponds to the horizontal line segments. Now, in the light of (20), an approximate optimal value of h can be determined by the following residual error:

$$Res(h) = \int_0^1 \left\{ y_{20}''(t) + \frac{2}{t} y_{20}'(t) + 4(2e^{y_{20}(t)} + e^{\frac{y_{20}(t)}{2}}) \right\}^2 dt. \quad (42)$$

By imposing the requirement $dRes(h)/dh = 0$ and solving the resulting equation, we can obtain the approximate optimal value for $h = -1$. Fig. 2 shows a comparison of approximation obtained using the 20th-order TP algorithm for $h = -1$ with the exact solution of Eq.(36), i.e., $y(x) = -2 \ln(1 + x^2)$.

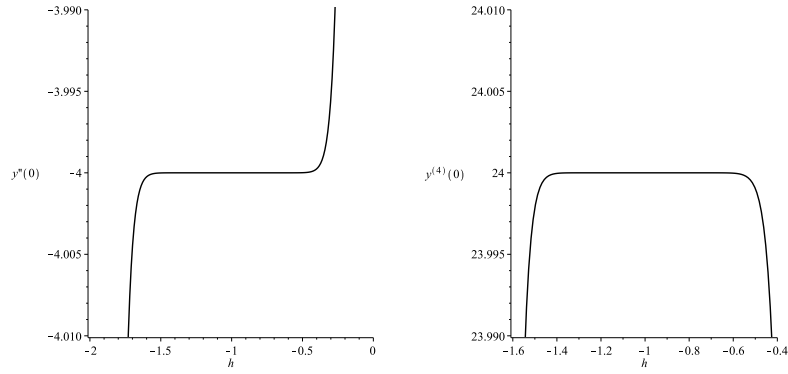


Figure 1: The valid region of h for Example 3 by using the 20th-order TP algorithm (15)

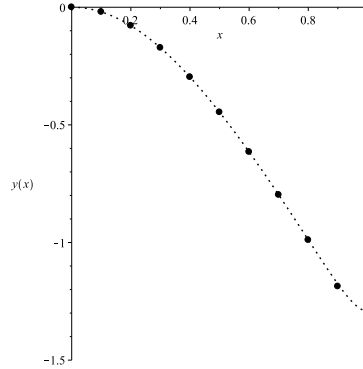


Figure 2: Approximate solution for Example 3 using the TP algorithm where the dotted-line: the 20th-order TP algorithm when $h = -1$ and symbol: the exact solution

As observed, the TP algorithm (15) in solving Equation (36) gives good approximations to the exact solution in a small region of x . In order to

enlarge the convergence region and rate of the series solution, here we implement the PTP (19) proposed in the section 3. According to (19), taking $N = 1000$ and $n_{i+1} = 3, n_{i+1} = 5, i = 0, 1, \dots, N - 1$, we can obtain the approximations of (36) on $[0, 100]$. Figure 3 shows the absolute errors (the differences between the approximate values and the exact values) of the PTP solution for $n_{i+1} = 3, n_{i+1} = 4, \Delta x = 0.1$ and the approximate optimal value $h = -1$. From Figure 3, it is easily to found that the present approximations are efficient for a larger interval.

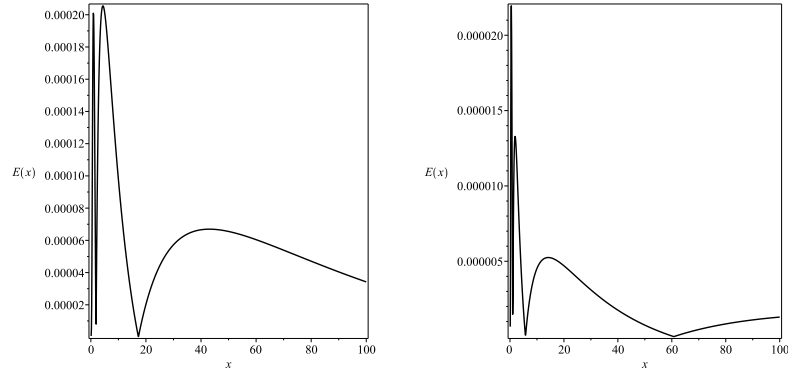


Figure 3: Shows the absolute errors ($E_k(x) = |\mu_{Exact}(x) - \mu_k(x)|$, $k = 4, 5$) of the PTP solution with $\Delta x = 0$ for Example 3 where left: the Abs. Err. of the 4th-order PTP solution for the approximate optimal value $h = -1$ and right: the Abs. Err. of the 5th-order PTP solution for the approximate optimal value $h = -1$

In closing our analysis, we point out that three concreated modeling equations of second-order singular IVPs of the Lane-Emden type equation were investigated by using the algorithms proposed. The obtained results have shown noteworthy performance.

5 Conclusions

Application of the methods based on the PIM presented in this paper to the three Lane-Emden type equations indicates that for the linear Lane-Emden type equations, its exact solution, if such a solution exists, can be obtained easily by only one iteration step due to the fact that the multiplier can be suitably identified, that the TP algorithm can solve a nonlinear Lane-Emden differential equation exactly if its solution is an algebraic polynomial up to some degree, and that for nonlinear Lane-Emden type equations can be useful in general. The numerical results demonstrate that the PIM is a useful analytic tool for solving the Lane-Emden type equations.

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روش تکرار پارامتریک بهینه برای حل معادلات نوع Lane-Emden

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چکیده : در این مقاله روش تحلیلی ای به نام روش تکرار پارامتریک (PIM) به منظور حل IVP های منفرد درجه دوم نوع Lane-Emden ارائه شده است و همگرایی داخلی اش مورد بحث قرار گرفته است. از آنجایی که دارا بودن راه حل تحلیلی تقریبی برای توصیف معادلات نوع Lane-Emden بویژه برای مواردی که راه حل های دقیقی برای شان موجود نیست، مفید است بنابراین، یک اصلاح (پیشرفت) موثر PIM نیز پیشنهاد شده است که قادر است راه حل تحلیلی تقریبی ای را بدست آورد. در نهایت، PIM اصلاح شده به عنوان الگوریتمی در دنباله ی فواصل برای یافتن راه حل تقریبی دقیق معادلات غیرخطی نوع Lane-Emden مورد توجه قرار می گیرد. همچنین نشان می دهیم که چگونه یک میزان بهینه ی تقریبی پارامتر افزایش همگرایی را در چارچوب این روش شناسایی می کنیم. مثال هایی ارائه شده است تا کارایی و درستی روش پیشنهادی را نشان دهد.

کلمات کلیدی : روش تکرار پارامتری piecewise-truncated ؛ روش تکرار پارامتریک truncated (برشی)؛ روش تکرار پارامتریک؛ معادلات غیرخطی نوع Lane-Emden .