Iranian Journal of Numerical Analysis and Optimization
Vol 5, No. 1, (2015), pp 37-48

Numerical solution of multi-order fractional differential equations via the sinc collocation method

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Abstract

In this paper, the sinc collocation method is proposed for solving linear and nonlinear multi-order fractional differential equations based on the new definition of fractional derivative which is recently presented by Khalil, R., Al Horani, M., Yousef, A. and Sababeh, M. in A new definition of fractional derivative, *J. Comput. Appl. Math.* **264** (2014), 65–70. The properties of sinc functions are used to reduce the fractional differential equation to a system of algebraic equations. Several numerical examples are provided to illustrate the accuracy and effectiveness of the presented method.

Keywords: Sinc function; Fractional differential equations; Multi-order FDEs; Collocation method.

1 Introduction

One of the old fields of mathematics is fractional calculus which dates back to the time of Leibniz [1] and from then many studies were done in this field [14]–[12]. Fractional differential equations (FDEs) have attracted the interest of researchers in many areas such as Physics, Chemistry, Engineering and Social Sciences [22, 15]. The analytic results on the existence and uniqueness of solutions to the FDEs have been investigated by many authors [11, 22, 16]. Generally, most of the FDEs do not have analytic solutions, so one has to resort to approximation and numerical methods.

One class of FDEs is multi-order fractional differential equations. They have been used to model various types of visco-elastic damping [22] and are expressed as follows

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Received 12 August 2014; revised 14 January 2015; accepted 3 February 2015
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$$D^{(\alpha)}y(x) = F(x, y(x), D^{(\beta_1)}y(x), ..., D^{(\beta_k)}y(x)), \quad x \in I = [0, l], \tag{1}$$

with initial conditions

$$D^{(i)}(0) = d_i, \quad i = 0, 1, ..., m-1, \quad m \in N \tag{2}$$

where $m-1 < \alpha \leq m$, $0 < \beta_1 < \beta_2 < ... < \beta_k < \alpha$ and the values of d_i $(i = 0, 1, ..., m-1)$ describe the initial state of $y(x)$. $D^{(\alpha)}y$ indicates the fractional derivative of order α of y. Up to now, whenever this equation was under study, in most cases the fractional derivative was in the sense of Caputo definition. In this paper, we imply the new definition of conformable fractional derivative [18] which will be defined later. Depending on F, this equation classifies into linear and nonlinear.

In [14], it has been proved that equation (1) subject to the initial conditions (2) and under natural Lipschitz conditions imposed on F has a unique continuous solution.

Since the last decade, extensive research has been conducted on the development of numerical methods for equation (1). Doha et al.[25] proposed an efficient spectral tau and collocation method based on the Chebyshev polynomials for solving this equation. Extension of the tau method based on the shifted Legendre Gauss-Lobbato quadrature is used for solving equation (1) in [9]. In [12], this equation is converted into a system of FDEs and the shifted Chebyshev operational matrix method is used to solve the resultant system. Some other works on this problem are: piecewise polynomial collocation [17], Haar wavelet method [20], Lagrange wavelet method [23] and second kind Chebyshev wavelet method [30].

In this work, we apply the sinc collocation method for solving equation (1). The sinc method is an efficient method developed by Stenger [24]. It was widely used for the numerical solution of initial and boundary value problems [13, 19, 8], not only because of its exponential convergence rate but also due to its ability in handling problems with singularities. To the best of our knowledge, the sinc collocation method has not been used for solving FDEs directly. In this work, based on the new definition of fractional derivative [18], we compute the fractional derivative of the sinc function and apply it for solving equation (1).

The remainder of this paper is organized as follows: in Section 2, some definitions and theorems are presented that will be used in later sections. The proposed method is discussed in Section 3. Section 4 is devoted to numerical experiments. Finally some remarks are concluded.

2 Preliminaries

In this section, we recall some necessary definitions and mathematical preliminaries of the fractional theory and sinc method which will be used further in this paper.

2.1. The fractional derivative

The fractional calculus involves different definitions of fractional derivative operators such as Caputo and Riemmann-Lioville fractional derivative[22, 1]. One of the most recent works on the theory of derivatives of fractional order is done by Khalil et al. [18] which is the simplest definition. Up to now, some works were done based on this new definition [1, 2, 22]. In what follows, at first the conformable fractional derivative is defined and then some fantastic properties of this definition are presented.

Definition 1. [18] Let $\alpha \in (n, n+1]$, and f be an $n-$differentiable function at t, where $t > 0$. Then the conformable fractional derivative of f of order α is defined as

$$T_\alpha(f)(t) = \lim_{\varepsilon \to 0} \frac{f^{(\lceil \alpha \rceil - 1)}(t + \varepsilon t^{(\lceil \alpha \rceil - \alpha)}) - f^{(\lceil \alpha \rceil - 1)}(t)}{\varepsilon}, \tag{3}$$

where $\lceil \alpha \rceil$ is the smallest integer greater than or equal to α.

When the conformable fractional derivative of f of order α exists, we say f is $\alpha-$differentiable and we write $f^{(\alpha)}(t)$ for $T_\alpha(f)(t)$.

Remark 1. [18] As a consequence of Definition 1, one can easily show that

$$T_\alpha(f)(t) = t^{1+(\lceil \alpha \rceil - \alpha)} f^{(\lceil \alpha \rceil)}(t), \tag{4}$$

where $\alpha \in (n, n+1]$, and f is $(n+1)-$differentiable at $t > 0$.

Theorem 1. [18] Let $\alpha \in (0, 1]$, and f, g be $\alpha-$differentiable at a point $t > 0$. Then
1. $T_\alpha(af + bg) = aT_\alpha(f) + bT_\alpha(g), \quad for\ all\ a, b \in R,$
2. $T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f).$

In [1], Abdeljawad was defined the left and right conformable fractional derivative. Since the left fractional derivative on $[0, \infty)$ is the conformable fractional derivative, we can have the following theorems according to [1].

Theorem 2. (Chain Rule) Assume $f, g : (0, \infty) \longrightarrow R$ be $\alpha-$differentiable functions, where $0 < \alpha \leq 1$. Let $h(t) = f(g(t))$. Then $h(t)$ is $\alpha-$differentiable and for all t with $t \neq 0$ and $g(t) \neq 0$ we have

$$(T_\alpha h)(t) = (T_\alpha f)(g(t))(T_\alpha g)(t)g(t)^{\alpha-1}.$$

If $t = 0$, then

$$(T_\alpha h)(0) = \lim_{t\to 0^+}(T_\alpha f)(g(t))(T_\alpha g)(t)g(t)^{\alpha-1}.$$

Theorem 3. Let $f : (0,\infty) \longrightarrow R$ be twice differentiable on $(0,\infty)$ and $0 < \alpha, \beta \le 1$ such that $1 < \alpha + \beta \le 2$. Then

$$(T_\alpha T_\beta f)(t) = T_{\alpha+\beta}f(t) + (1-\beta)tT_\alpha f(t).$$

2.2. Sinc function

The sinc function is defined on the whole real line, $-\infty < x < \infty$, by

$$sinc(x) = \begin{cases} \frac{sin\pi x}{\pi x} & x \neq 0, \\ 1 & x = 0. \end{cases}$$

For each integer k and the mesh size h, the translated sinc basis function is defined as

$$s(k,h)(x) = sinc(\frac{x-kh}{h}).$$

If a function $f(x)$ is defined on the real axis, then for any $h > 0$, the Whittaker cardinal expansion of $f(x)$ is as follows

$$c(f,h)(x) = \sum_{k=-\infty}^{\infty} f(kh)sinc(\frac{x-kh}{h}),$$

whenever this series converges. The properties of Whittaker cardinal expansion are derived in the infinite strip D_s of the complex w-planes where for $d > 0$

$$D_s = \{w = t + is : |s| < d \le \frac{\pi}{2}\}.$$

These properties have been studied thoroughly in [24]. In order to approximate on the finite interval (a,b), which is used in this paper, we consider the one-to-one conformal map $w = \phi(z) = ln(\frac{z-a}{b-z})$, which maps the eye-shaped domain

$$D_E = \{z = x + iy : |arg\frac{z-a}{b-z}| < d \le \frac{\pi}{2}\},$$

onto the infinite strip D_s. The basis functions on (a,b) are taken to be the composite translated sinc functions

$$s_k(x) = s(k,h)o\phi(x) = sinc(\frac{\phi(x)-kh}{h}), \quad k \in Z \tag{5}$$

where $s(k,h)o\phi(x)$is defined by $s(k,h)(\phi(x))$.
Let $\psi = \phi^{-1}$. We define the range of ψ on the real line as

$$\Gamma = \{\psi(w) \in D_E : -\infty < w < \infty\}.$$

For the uniform grid $\{kh\}_{k=-\infty}^{\infty}$ on the real line, the image which corresponds to these nodes is denoted by

$$x_k = \psi(kh) = \frac{a + be^{kh}}{1 + e^{kh}}, k = 0, \pm 1, \pm 2, ... \tag{6}$$

For discretizing the problem we need the following definition and theorems.

Definition 2. [24] Let $L_\beta(D_E)$ be the set of all analytic functions, for which there exist a constant, C, such that

$$|y(z)| \leq C\frac{|\rho(z)|^\beta}{[1 + |\rho(z)|]^{2\beta}}, \quad z \in D_E, \quad 0 < \beta \leq 1,$$

where $\rho(z) = e^{\phi(z)}$.
Theorem 4. [21] Let $y \in L_\beta(D_E)$, N be a positive integer and h be selected by the formula

$$h = (\frac{\pi d}{\beta N})^{\frac{1}{2}}, \tag{7}$$

then there exists a positive constant c_1, independent of N, such that

$$sup_{z\in\Gamma}|y(z) - \sum_{j=-N}^{N} y(z_j)s(j,h)o\phi(z)| \leq c_1 e^{-(\pi d\beta N)^{\frac{1}{2}}}.$$

Theorem 5. [21] Let ϕ be a conformal one-to-one map of the simply connected domain D_E onto D_S.Then

$$\delta_{kj}^{(0)} = s_k(x)|_{x=x_j} = \begin{cases} 1 & k = j, \\ 0 & k \neq j. \end{cases}$$

$$\delta_{kj}^{(1)} = \frac{d}{d\phi}[s_k(x)]|_{x=x_j} = \frac{1}{h}\begin{cases} 0 & k = j, \\ \frac{(-1)^{(j-k)}}{j-k} & k \neq j. \end{cases}$$

$$\delta_{kj}^{(2)} = \frac{d^2}{d\phi^2}[s_k(x)]|_{x=x_j} = \frac{1}{h^2}\begin{cases} \frac{-\pi^2}{3} & k = j, \\ \frac{-2(-1)^{(j-k)}}{(j-k)^2} & k \neq j. \end{cases}$$

3 Method of Solution

Consider equation (1) in $I = [0,1]$ where $D^{\alpha}y$ denotes the fractional derivative which is defined in (3) i.e. $D^{(\alpha)}y = y^{(\alpha)}$.
The approximate solution of equation (1) based on the sinc basis functions (5), should satisfy the initial conditions (2). But this basis functions do not have a derivative when x tends to 0 or 1 so we modify them as

$$w(x)s_k(x), \tag{8}$$

where $w(x) = (x(1-x))^{(m-1)}$ [6].
In order to approximate the solution, we construct a polynomial $p(x)$ that satisfies initial conditions [6]. So the approximate solution is represented by

$$y_N(x) = u_N(x) + p(x), \tag{9}$$

where

$$u_N(x) = \sum_{k=-N}^{N} c_k w(x) s_k(x), \tag{10}$$

and

$$p(x) = a_0 + a_1 x + ... + a_m x^m, \quad m-1 < v \leq m. \tag{11}$$

The unknown coefficients $a_0, a_1, ..., a_m$ and $\{c_k\}_{k=-N}^{N}$ are determined by substituting $y_N(x)$ into equation (1) and evaluating the result at the sinc points

$$x_j = \frac{e^{jh}}{1+e^{jh}}, j = -N-1, ..., N. \tag{12}$$

Notice that according to Theorem 1 and Remark 1, we have

$$(w(x)s_k(x))^{(\alpha)} = x^{1+[\alpha]-\alpha}(w(x)s_k(x))^{(1+[\alpha])}, \quad n < \alpha \leq n+1, \tag{13}$$

so

$$u_N^{(\alpha)}(x) = \Sigma_{k=-N}^{N} c_k (w(x)s_k(x))^{(\alpha)}. \tag{14}$$

Also it should be noted that when x tends to 1 or 0, we have

$$u_N(x) = u'_N(x) = ... = u_N^{(m-1)}(x) = 0.$$

Using equations (13) and (14), one can obtain

$$y_N^{(\alpha)}(x_j) = u_N^{(\alpha)}(x_j) + p^{(\alpha)}(x_j), \quad j = -N-1, ..., N. \tag{15}$$

Now by substituting this equation into equation (1), we obtain the following system of algebraic equations which can be solved for unknowns

$$y_N^{(v)}(x_j) = F(x_j, y_N(x_j), y_N^{(\beta_1)}(x_j), ..., y_N^{(\beta_k)}(x_j)), \quad -N-1 \leq j \leq N,$$
$$y_N^{(i)}(0) = d_i, \quad i = 0, 1, ..., m-1.$$

4 Applications and results

In this section, we solve some examples by the presented method and compare the numerical results with the exact solutions and some earlier works.

Example 1. As the first example, we consider the following nonlinear fractional initial value problem [5] on $[0, 1]$

$$y'''(x) + y^{(2.5)}(x) + y^2(x) = x^4, \quad y(0) = y'(0) = 0, \quad y''(0) = 2, \tag{16}$$

whose exact solution is $y(x) = x^2$. Following the procedure of the presented method, we consider the following approximate solution

$$y_N(x) = \sum_{k=-N}^{N} c_k w(x) s_k(x) + a_0 + a_1 x + a_2 x^2 + a_3 x^3,$$

where $w(x) = x^2(1-x)^2$. By substituting this approximate solution into equation (16) and evaluating at sinc points (12), we arrive at the following nonlinear system of algebraic equations which can be solved for unknown coefficients

$$6a_3 + \Sigma_{k=-N}^{N} c_k \{ w''_j \delta_{kj}^{(0)} + \delta_{kj}^{(1)} (3w''_j \phi'_j + 3w'_j \phi''_j + w_j \phi'''_j) + \delta_{kj}^{(2)} (3w'_j (\phi'_j)^2 +$$
$$3w_j \phi''_j \phi'_j) + \delta_{kj}^{(3)} w_j (\phi'_j)^3 \} + 6x_j^{0.5} a_3 + \Sigma_{k=-N}^{N} c_k \{ x_j^{0.5} w''' j \delta_{kj}^{(0)} + w_j x_j^{0.5} (\phi'''_j \delta_{kj}^{(1)} +$$
$$3\phi''_j \phi'_j \delta_{kj}^{(2)} + (\phi'_j)^3 \delta_{kj}^{(3)}) \} + (a_0 + a_1 x_j + a_2 x_j^2 + a_3 x_j^3 + \Sigma_{k=-N}^{N} c_k w_j \delta_{kj}^{(0)})^2 = x_j^4$$
$$j = -N-1, ..., N,$$
$$y(0) = 0 \Rightarrow a_0 = 0, \quad y'(0) = 0 \Rightarrow a_1 = 0, \quad y''(0) = 2 \Rightarrow a_2 = 1.$$

According to relation (7), by taking $d = \frac{\pi}{2}$ and $\beta = 2$, we have $h = \frac{\pi}{2\sqrt{N}}$. Then by applying the well known Newton method with starting points $c_k = 0, k = -N, ..., N,\ a_0 = a_1 = a_3 = 0, a_2 = 1$, we obtain $c_k = 0, k = -N, ..., N$ and $a_0 = a_1 = a_3 = 0, a_2 = 1$. So the approximate solution is $y_N(x) = x^2$, which is the exact solution.

Example 2. Consider the fractional Ricatti equation on $[0, 1]$

$$y^{(\alpha)}(x) = 2y(x) - y^2(x) + 1, \quad 0 < \alpha \leq 1, \quad y(0) = 0.$$

For $\alpha = 1$, the exact solution of this equation is $y(t) = 1 + \sqrt{2}tanh(\sqrt{2}t + \frac{1}{2}ln(\frac{\sqrt{2}-1}{\sqrt{2}+1}))$. Consider the following approximate solution based on the sinc collocation method

$$y_N(x) = \Sigma_{k=-N}^{N} c_k s_k(x) + a_0 + a_1 x.$$

Odibat and Momani [20], solve this equation by using the modified homotopy perturbation method. Also in [4], this equation is solved by the Chebyshev wavelet operational matrices of fractional integration. For comparison, the results of this method are presented in Tables 1 and 2 with 192–set of Block Pulse Functions (Chebyshev wavelets was expanded into an 192-term block pulse functions).

In Table 1, the results of the presented method with $N = 1$ for $\alpha = 0.5$ and

Table 1: Numerical results with comparison to [4, 20] for $\alpha = 0.5$ and $\alpha = 0.75$ in Example 2

	$\alpha = 0.5$			$\alpha = 0.75$		
x	Ours[N=1]	[4]	[20]	Ours[N=1]	[4]	[20]
0.1	0.3956920	0.592756	0.321730	0.2321153	0.310732	0.216866
0.2	0.9184524	0.9331796	0.629666	0.4961556	0.584307	0.428892
0.3	1.2973611	1.1739836	0.940941	0.7523005	0.822173	0.654614
0.4	1.5802323	1.3466546	1.250737	0.9998683	1.024974	0.891404
0.5	1.7987123	1.4738876	1.549439	1.2372036	1.198621	1.132764
0.6	1.9690794	1.5705716	1.825456	1.4604023	1.349150	1.370240
0.7	2.0982657	1.646199	2.066523	1.6619744	1.481449	1.594278
0.8	2.1867519	1.706880	2.260633	1.8278045	1.599235	1.794879
0.9	2.2352250	1.756644	2.396839	1.9347648	1.705303	1.962239
1.0	2.3926026	1.798220	2.466004	2.0825668	1.801763	2.087384

$\alpha = 0.75$ are compared with earlier works [4, 20]. We see that our results are in a good agreement with them. For $\alpha = 1$, the results are presented in Table 2. It is clear that by increasing N, the approximate solution becomes more and more accurate and for $N = 85$ the exact solution is obtained whereas Refs [4, 20] can not reach the exact solution. In Figure 1. the approximate solution for different values of α is shown. Numerical results show that as

Table 2: Numerical results with comparison to [4, 20] for $\alpha = 1$ in Example 2

x	Ours[N=10]	Ours[N=50]	Ours[N=80]	[4]	[20]	*Exact*
0.1	0.1134865	0.1103047	0.110295	0.1103111	0.110294	0.110295
0.2	0.2458331	0.2419881	0.241977	0.241995	0.241965	0.241977
0.3	0.3993884	0.3951178	0.395105	0.395123	0.395106	0.395105
0.4	0.5726929	0.5678265	0.567812	0.567829	0.568115	0.567812
0.5	0.7610790	0.7560297	0.756014	0.756029	0.757564	0.756014
0.6	0.9589295	0.9535820	0.953566	0.953576	0.958259	0.953566
0.7	1.1581332	1.1529646	1.152949	1.152955	1.163459	1.152949
0.8	1.3514117	1.3463785	1.366364	1.346365	1.365240	1.366364
0.9	1.5314497	1.5269249	1.526911	1.526909	1.554960	1.526911
1	1.6949935	1.6895135	1.689498	1.689494	1.723810	1.689498

α approaches to its integer value, the solution of fractional order differential equation approaches to the solution of integer order differential equation.

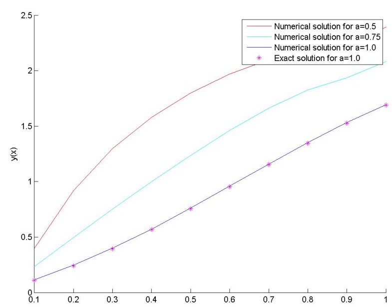

Figure 1: Approximate solution of Example 2 for different values of α

Example 3. [3] As the last example, consider the following inhomogeneous Bagley-Torvik equation

$$y''(x) + y^{(1.5)}(x) + y(x) = 1 + x,$$

subject to initial conditions

$$y(0) = y'(0) = 1.$$

The exact solution of this equation is $y(x) = 1 + x$.
In a same manner of last examples, by considering the approximate solution as

$$y_N(x) = \Sigma_{k=-N}^{N} c_k w(x) s_k(x) + a_0 + a_1 x + a_2 x^2,$$

where $w(x) = x(1-x)$, one can obtain $y_N(x) = 1 + x$ which is the exact solution.

5 Conclusion

In this work the sinc-collocation method is used to approximate the solution of multi-order fractional differential equations with initial conditions. This method converts the FDEs into a system of algebraic equations which can be solved more easier. In this work, the fractional derivatives are described in the sense of new definition which makes us able to solve fractional differential equation directly by the sinc method for the first time. Also this method can be applied to other types of FDEs easily. Several examples are included to demonstrate the reliability and efficiency of our method.

6 Acknowledgments

The authors are indebted to the unknown referees for their constructive suggestions toward improving this paper.

References

1. Abdeljawad, T. *On conformable fractional calculus*, J. Comput. Appl. Math. (2014), http://dx.doi.org/10.1016/j.cam.2014.10.016.

2. Abu Hammad, M. and Khalil, R. *Conformable fractional Heat differential equation*, ijpam. 94 (2014) 215–221.

3. Amairi, M., Aoun, M., Najar, S. and Abdelkrim, M.N. *A constant enclosure method for validating existence and uniqueness of the solution of an initial value problem for a fractional differential equation*, Appl. Math. Comput. 217 (2010) 2162–2168.

4. Atabakzadeh, M.H., Akrami, M.H. and Erjaee, G.H. *Chebyshev operational matrix method for solving multi-order fractional ordinary differential equations*, Appl. Math. Model. 37 (2013) 8903–8911.

5. Bhrawy, A.H., Alofi, A.S. and Ezz–Eldien, S. S. *A quadrature tau method for fractional differentials equations with variable coefficients*, Appl. Math. Lett. 24 (2011) 2146–2152.

6. Deng, J. and Ma, L. *Existence and uniqueness of solutions of initial value problems for nonlinear fractional differential equations*, Appl. Math. Lett. 23 (2010) 676–680.

7. Diethelm, K. and Ford, N.J. *Multi-order fractional differential equations and their numerical solutions*, Appl. Math. Comput. 154 (2004) 621–640.

8. Diethelm, K. and Ford, N.J. *Numerical solution of the Bagley-Torvik equation*, BIT. 42 (2002) 490–507.

9. Doha, E.H., Bhrawy, A.H. and Ezz-Eldien, S.S. *A Chebyshev spectral method based on operational matrix for initial and boundary value problems of fractional order*, Comput. Math. Appl. 62 (2011) 2364–2373.

10. Doha, E.H., Bhrawy, A.H. and Ezz-Eldien, S.S. *Efficient Chebyshev spectral methods for solving multi-term fractional orders differential equations*, Appl.Math. Modell. 35 (2011) 5662–5672.

11. El–Gamel, M., Cannon, J.R. and Zayed, A.I. *Sinc–Galerkin method for solving linear Sixth order boundary-value problems*, Math. Comp. 73 (2004) 1325–1343.

12. Garrappa, R. and Popolizio, M. *On the use of matrix functions for fractional partial differential equations*, Math. Comput. Simulation. 81 (2011) 1045–1056.

13. Jafari, H., Das, S. and Tajadodi, H. *Solving a multi-order fractional differential equation using homotopy analysis method*, J. King Saud Univ. Sci. 23 (2011) 151–155.

14. Khalil, R., Al Horani, M., Yousef, A. and Sababeh, M. *A new definition of fractional derivative*, J. Comput. Appl. Math. 264 (2014) 65–70.

15. Kilbas, A.A., Srivastava, H.M. and Trujillo, J.J. *Theory and Applications of Fractional Differential Equations*, Elsevier, San Diego, 2006.

16. LI, Y. *Solving a nonlinear fractional differential equation using Chebyshev wavelets*, Commun. Nonlinear Sci. Numer. Simul. 15 (9) (2010), 2284–2292.

17. Lund, J. and Bowers, K.L. *Sinc methods for quadrature and differential equations*, PA, Philadelphia, SIAM, 1992.

18. Narasimhan, S., Majdalani, J. and Stenger, F. *A first step in applying the Sinc collocation method to the nonlinear Navier Stokes Equations*, Numer. Heat Transfer Part B. 41 (2002) 447–462.

19. Nurmuhammad, A., Muhammad, M., Mori, M. and Sugihara, M. *Double exponential transformation in the Sinc-collocation method for a boundary value problem with fourth-order ordinary differential equation*, J. Comput. Appl. Math. 182 (2005) 32–50.

20. Odibat, Z. and Momani, S. *Modified homotopy perturbation method: application to quadratic Riccati differential equation of fractional order*, Chaos Solitons Fract. 36 (2008) 167–74.

21. Oldham, K.B. and Spanier, J. *The Fractional Calculus*, Academic Press, New York, 1974.

22. Ortigueiraa, M.D. and Machado, J.A.T. *What is a fractional derivative?*, J. Comput. Phys. (2014), http://dx.doi.org/10.1016/j.jcp.2014.07.019.

23. Pedas, A. and Tamme, E. *Piecewise polynomial collocation for linear boundary value problems of fractional differential equations*, J. Comput. Appl. Math. 236 (2012) 3349–3359.

24. Podlubny, I. *Fractional Differential Equations*, in: Mathematics in Science and Engineering, vol. 198, Academic Press Inc., San Diego, CA, 1999.

25. Rehman, M.U. and Khan, R.A. *A numerical method for solving boundary value problems for fractional differential equations*, Appl. Math. Model. 36 (2012) 894–907.

26. Rehman, M.U. and Khan, R.A. *The Legendre wavelet method for solving fractional differential equations*, Commun. Nonlinear Sci. Numer. Simulat. 16 (11) (2011) 4163–4173.

27. Saadatmandi, A. and Dehghan, M. *The use of Sinc-Collocation Method for Solving multi-point boundary value problems*, Commun. Nonlinear Sci. Numer. Simul. 17 (2) (2012) 593–601.

28. Stenger, F. *Numerical Methods Based on Sinc and Analytic Functions*, New York: Springer, 1993.

29. Sugihara, M. *Double exponential transformation in the Sinc-collocation method for two-point boundary value problems*, J. Comput. Appl. Math. 149 (2002) 239–250.

30. Wang, Y. and Fan, Q. *The second kind Chebyshev wavelet method for solving fractional differential equations*, Appl. Math. Comput. 218 (2012) 8592–8601.

31. Yuanlu, L. *Solving a nonlinear fractional differential equation using Chebyshev wavelets*, Commun. Nonlinear Sci. Numer. Simul. 15 (9) (2010) 2284–2292.

روش هم محلی سینکبرای حل عددی معادلات دیفرانسیل کسری با مرتبه چندگانه

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چکیده : در این مقاله روش هم محلی سینک را برای حل عددی معادلات دیفرانسیل کسری با مرتبه چندگانه بر پایه ی تعریف جدید مشتق کسری که اخیراً توسط خلیل و همکارانش ارائه شده است، به کار می بریم. از خواص تابع سینک استفاده کرده و معادله دیفرانسیل کسری را به یک سیستم از معادلات جبری تبدیل می کنیم. چند مثال عددی که مؤید دقت و کارایی این روش می باشد نیز ارائه گردیده است.

کلمات کلیدی : تابع سینک؛ معادلات دیفرانسیل کسری؛ معادلات دیفرانسیل کسری با مرتبه چندگانه؛ روش هم محلی.