Iranian Journal of Numerical Analysis and Optimization
Vol. 7, No. 1, (2017), pp 107-117
DOI: 10.22067/ijnao.v7i1.52283

Alternating direction method of multipliers for the extended trust region subproblem

M. Salahi* and A. Taati

Abstract

The extended trust region subproblem has been the focus of several research recently. Under various assumptions, strong duality and certain SOCP/SDP relaxations have been proposed for several classes of it. Due to its importance, in this paper, without any assumption on the problem, we apply the widely used alternating direction method of multipliers (ADMM) to solve it. The convergence of ADMM iterations to the first order stationary conditions is established. On several classes of test problems, the quality of the solution obtained by the ADMM for medium scale problems is compared with the SOCP/SDP relaxation. Moreover, the applicability of the method for solving large scale problems is shown by solving several large instances.

Keywords: Extended trust region subporblem; Alternating method; Nonconvex optimization; Semidefinite program; Second order cone program.

1 Introduction

Consider the following extended trust region subproblem with m linear inequalities (m-eTRS):

$$\begin{aligned} \min \quad & \frac{1}{2}x^TAx + a^Tx \\ & ||x||^2 \leq 1, \qquad\qquad\qquad\qquad (m\text{-eTRS}) \\ & b_i^Tx \leq \beta_i, \quad i = 1, \cdots, m, \end{aligned}$$

*Corresponding author
Received 19 December 2015; revised 31 July 2016; accepted 28 September 2016
M. Salahi
Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Guilan, Rasht, Iran. e-mail: salahim@guilan.ac.ir

A. Taati
Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Guilan, Rasht, Iran. e-mail: akram.taati@gmail.com

where $A \in \mathbb{R}^{n\times n}$ is a symmetric matrix but not necessarily positive definite, $a, b_i \in \mathbb{R}^n$ and $\beta_i \in \mathbb{R}$. Due to its importance and crucial role in solving general nonlinear optimization problems, several versions of it have been the focus of current research [6,7,11,13,17,18]. In [13], the authors have shown that the dimension condition, $\dim(\text{ Ker}(A-\lambda_1 I_n)) \geq s+1$, where λ_1 is the smallest eigenvalue of A and $s = \dim(\text{span}\{b_1,..,b_m\})$, together with the Slater condition ensure that a set of combined first and second-order Lagrange multiplier conditions are necessary and sufficient for the global optimality of (m-eTRS) and consequently for strong duality. In [11], the authors have proposed an induction technique that reduces (m-eTRS) to several small-sized trust region subproblems (TRS). When m is not too large, their method can be very efficient but requires finding local-nonglobal minimum (LNGM), which no efficient algorithm is known to find it. Moreover, no numerical evidences are reported by the authors to support their theoretical foundation. Also they have improved the dimension condition by Jeyakumar and Li under which (m-eTRS) admits an exact SDP relaxation. They proposed the following condition

$$\text{rank } ([A-\lambda_1 I_n \ \ b_1 \ b_2 \ \cdots \ b_m]) \leq n-1. \tag{1}$$

This rank condition implies that the global optimal solution of the (m-eTRS) does not happen at the (LNGM) of (TRS) [8,15]. In a most recent study [7], the authors have proposed the following SOCP/SDP relaxation for (m-eTRS) based on the following assumption:
Assumption 1: For all $i < j$, there exists no feasible point x for (m-eTRS) such that $b_i^T x = \beta_i$ and $b_j^T x = \beta_j$.

$$\begin{aligned} min_{x,X} \ \ & \frac{1}{2}trace(AX) + a^T x \\ \text{s.t. } & trace(X) \leq 1, \ X \succeq xx^T, \\ & ||\beta_i x - Xb_i|| \leq \beta_i - b_i^T x, \ \ 1 \leq i \leq m, \qquad\qquad (R_m) \\ & \beta_i\beta_j - \beta_j b_i^T x - \beta_i b_j^T x + b_i^T X b_j \geq 0, \ \ 1 \leq i < j \leq m. \end{aligned}$$

They also have proved that this relaxation is tight when we consider the following assumption instead of Assumption 1.
Assumption 2: For all $i < j$, there exists no x with $||x|| < 1$ such that $b_i^T x = \beta_i$ and $b_j^T x = \beta_j$.

Moreover, in several recent studies, when $m = 1$, efficient algorithms have been proposed to solve large instances of (m-eTRS) [17,18]. Therefore, finding an efficient algorithm to solve (m-eTRS) problems when $m > 1$ is still an active research area as the proposed methods either are using certain assumptions to simplify the analysis or using the SOCP/SDP relaxation, which both are not applicable practically, specially for large scale instances.

Recently, the *Alternating Direction Method of Multipliers* (ADMM) have been widely used to solve optimization problems arising in machine learning, signal processing, matrix factorization, financial optimization and etc. [2,3,5, 12,14,19,22]. Although the method exhibits faster convergence in practice, its global convergence is still the subject of research. Under various assumptions on the sequence generated by the method like "if the limit point exists" or "if the Lagrange multipliers are bounded", its global convergence to a stationary point is established in the above-mentioned papers. In this paper, we apply ADMM to (m-eTRS) without any assumption on the geometry of the feasible region. The convergence of the method to the first order necessary optimality conditions is proved. Finally, on several medium scale instances its performance is compared with the SOCP/SDP relaxation of [7] and then several large instances also are solved by ADMM. To the best of our knowledge, there is no algorithm in the literature for large scale (m-eTRS) which we could provide comparison.

2 ADMM for (m-eTRS)

One can write (m-eTRS) in the following equivalent form:

$$\begin{aligned} \min \quad & \frac{1}{2}x^T Ax + a^T x \\ & ||x||^2 \leq 1, \\ & b_i^T z \leq \beta_i, \quad i = 1, \cdots, m \\ & x = z. \end{aligned} \tag{2}$$

Now consider the following *augmented Lagrangian* for (2):

$$L(x, z, \lambda) = \frac{1}{2}x^T Ax + a^T x + \lambda^T (x - z) + \frac{\rho}{2}||x - z||^2,$$

where λ_i's are Lagrange multipliers and $\rho > 0$ is the penalty parameter. The ADMM iterations for the given x^k and λ^k are as follows [5]:

- **Step 1:** $z^{k+1} = \text{argmin}_{b_i^T z \leq \beta_i,\ i=1,\cdots,m} L(x^k, z, \lambda^k)$.
- **Step 2:** $x^{k+1} = \text{argmin}_{||x||^2 \leq 1} L(x, z^{k+1}, \lambda^k)$.
- **Step 3:** $\lambda^{k+1} = \lambda^k + \gamma\rho(x^{k+1} - z^{k+1})$, where $\gamma \in (0, 1)$ is a constant.

In what follows, we discuss the above steps. In Step 1, we need to solve the following convex quadratic optimization problem with m linear inequality constraints which can be efficiently solved using existing convex optimization software packages like CVX [9]:

$$\begin{aligned} \min \quad & \tfrac{\rho}{2} z^T z - (\lambda^k + \rho x^k)^T z \\ & b_i^T z \le \beta_i, \ i = 1, \cdots, m. \end{aligned} \tag{3}$$

In Step 2 we need to solve the following (TRS) problem:

$$\begin{aligned} \min \ & \tfrac{1}{2} x^T (A + \rho I) x + (a + \lambda^k - \rho z^{k+1})^T x \\ & ||x||^2 \le 1. \end{aligned} \tag{4}$$

The (TRS) problems are widely used and studied in the literature [8] and they can be solved efficiently using the exiting eigenvalue approaches even for large instances [1, 16]. Now, the ADMM algorithm for solving (m-eTRS) can be outlined as follows.

ADMM Algorithm for solving (m-eTRS)

Input parameters $tol > 0$, maxiter> 0. Choose appropriate penalty parameter $\rho > 0$ and $\gamma > 0$. Set $k = 0$ and choose appropriate x^k and λ^k
For $k = 1, \cdots$, maxiter **do**
Solve quadratic optimization problem (3) and let its solution be z^{k+1}.
Solve the (TRS) problem (4) and let its solution be x^{k+1}.
If $||x^{k+1} - z^{k+1}|| \le tol$, then exit with x^{k+1} as output.
end if
Set $\lambda^{k+1} = \lambda^k + \gamma\rho(x^{k+1} - z^{k+1})$ and $k = k + 1$.
end for.

As we see, the method is very easy to implement and subproblems of the Steps 1 and 2 are efficiently solvable even for large instances. In what follows, we discuss the convergence of the above algorithm to the stationary point of (m-eTRS). First we present the following lemma.

Lemma 1. *Suppose that* $\{\lambda^k\}$ *is bounded and* $\sum_{k=1}^{\infty} ||\lambda^{k+1} - \lambda^k||^2 < \infty$. *Then*

$$||x^{k+1} - x^k|| \to 0, \quad ||z^{k+1} - z^k|| \to 0 \quad \text{as} \quad k \to \infty.$$

Proof. Since x^{k+1} solves problem (4) at k-th iteration and $x^k - x^{k+1}$ is a feasible direction with respect to the feasible region of (4), then

$$\nabla_x L(x^{k+1}, z^{k+1}, \lambda^k)^T (x^k - x^{k+1}) \ge 0. \tag{5}$$

Now

$$L(x^k, z^{k+1}, \lambda^k) - L(x^{k+1}, z^{k+1}, \lambda^k) = \frac{1}{2}(x^k - x^{k+1})^T (A + \rho I)(x^k - x^{k+1})$$
$$+\nabla_x L(x^{k+1}, z^{k+1}, \lambda^k)^T (x^k - x^{k+1}) \geq \frac{\lambda_1 + \rho}{2} ||x^k - x^{k+1}||^2, \tag{6}$$

where the inequality follows from the definition of the smallest eigenvalue of A, λ_1, and (5). We also have

$$L(x^k, z^k, \lambda^k) - L(x^k, z^{k+1}, \lambda^k) \geq 0, \tag{7}$$

as z^{k+1} is the minimizer of $L(x^k, z, \lambda^k)$. On the other hand

$$\begin{aligned} L(x^{k+1}, z^{k+1}, \lambda^k) - L(x^{k+1}, z^{k+1}, \lambda^{k+1}) &= (\lambda^k - \lambda^{k+1})^T (x^{k+1} - z^{k+1}) \\ &= -\frac{1}{\gamma\rho} ||\lambda^k - \lambda^{k+1}||^2. \end{aligned} \tag{8}$$

Now using (5), (7), and (8) we have

$$\begin{aligned} L(x^k, z^k, \lambda^k) \ &- L(x^{k+1}, z^{k+1}, \lambda^{k+1}) = L(x^k, z^k, \lambda^k) - L(x^k, z^{k+1}, \lambda^k) + \\ L(x^k, z^{k+1}, \lambda^k) &- L(x^{k+1}, z^{k+1}, \lambda^k) + L(x^{k+1}, z^{k+1}, \lambda^k) - L(x^{k+1}, z^{k+1}, \lambda^{k+1}) \\ &\geq \frac{\lambda_1 + \rho}{2} ||x^{k+1} - x^k||^2 - \frac{1}{\gamma\rho} ||\lambda^{k+1} - \lambda^k||^2. \end{aligned} \tag{9}$$

Since $\{\lambda^k\}$ and $\{x^k\}$ are bounded, then from Step 3 of ADMM iterations, $\{z^k\}$ also is bounded. Therefore $\{L(x^k, z^k, \lambda^k)\}$ is bounded. Moreover, since by assumption $\sum_{k=1}^{\infty} ||\lambda^{k+1} - \lambda^k||^2 < \infty$, thus from (9), $\sum_{k=1}^{\infty} ||x^{k+1} - x^k||^2$ is a bounded series (in the sense that the sequence of partial sums is bounded)with nonnegative terms, thus it is convergent. Then $||x^k - x^{k+1}|| \to 0$, as $k \to \infty$. Moreover since by assumption $||\lambda^k - \lambda^{k+1}|| \to 0$, as $k \to \infty$, from the Step 3 we have $x^k - z^k \to 0$, as $k \to \infty$. Finally since

$$z^k - z^{k+1} = z^k - x^k + x^k - x^{k+1} + x^{k+1} - z^{k+1}$$

and we know $z^k - x^k \to 0$, $x^k - x^{k+1} \to 0$, $x^{k+1} - z^{k+1} \to 0$, as $k \to \infty$, then $z^k - z^{k+1} \to 0$. □

It should be noted the boundedness assumption of multipliers in the lemma is a standard assumption for convergence analysis of nonconvex optimization problems [14]. Similar assumptions are used to prove the convergence to the stationary point in [21, 22]. In what follows we prove the convergence to the first order stationary conditions

Theorem 1. *Let (x^*, z^*, λ^*) be any accumulation point of $\{(x^k, z^k, \lambda^k)\}$ generated by the ADMM Algorithm. Then by boundedness of $\{\lambda^k\}$ and $\sum_{k=1}^{\infty} ||\lambda^{k+1} - \lambda^k||^2 < \infty$, x^* satisfies the first order stationary conditions.*

Proof. Since (x^*, z^*, λ^*) is an accumulation point of $\{(x^k, z^k, \lambda^k)\}$, then there exists a subsequence $\{(x^k, z^k, \lambda^k)\}_{k \in I}$ that converges to (x^*, z^*, λ^*). Now consider subproblems that should be solved in Steps 1 and 2. As we mentioned, subproblem (3) in Step 1 is a convex quadratic optimization problem which its necessary and sufficient optimality conditions are as follows:

$$\begin{aligned}
&\rho z^{k+1} - (\lambda^k + \rho x^k) + \sum_{i=1}^{m} \nu_i^{k+1} b_i = 0,\\
&\nu_i^{k+1}(b_i^T z^{k+1} - \beta_i) = 0,\ b_i^T z^{k+1} \leq \beta_i, \quad i = 1, \cdots, m,
\end{aligned} \tag{10}$$

where ν_i^{k+1}'s are the Lagrange multipliers. Moreover, subproblem in Step 2 is a (TRS) as given in (4) that we have the following necessary and sufficient optimality conditions for it:

$$\begin{aligned}
&(A + \rho I_n + 2\mu^{k+1} I_n) x^{k+1} = -(a + \lambda^k - \rho z^{k+1}),\\
&\mu^{k+1}(||x^{k+1}||^2 - 1) = 0,\ ||x^{k+1}||^2 \leq 1,\\
&A + \rho I_n + 2\mu^{k+1} I_n \succeq 0_{n\times n}.
\end{aligned} \tag{11}$$

Now by taking the limit of both (10) and (11), we get

$$\begin{aligned}
&(A + \rho I_n + 2\mu^* I_n) x^* = -(a + \lambda^* - \rho z^*),\\
&\mu^*(||x^*||^2 - 1) = 0,\ ||x^*||^2 \leq 1,\\
&A + \rho I_n + 2\mu^* I_n \succeq 0_{n\times n},\\
&\rho z^* - (\lambda^* + \rho x^*) + \sum_{i=1}^{m} \nu_i^* b_i = 0,\\
&\nu_i^*(b_i^T z^* - \beta_i) = 0,\ b_i^T z^* \leq \beta_i, \quad i = 1, \cdots, m.
\end{aligned} \tag{12}$$

From the first and forth equations of (12), we get

$$(A + 2\mu^* I_n) x^* = -a - \sum_{i=1}^{m} \nu_i^* b_i,$$

which with the second and the last equations are the first order stationary conditions. □

3 Numerical experiments

In this section, we present several randomly generated test problems to assess the performance of ADMM for solving (m-eTRS). For small dimension problems, we compare ADMM with the SOCP/SDP relaxation (R_m). All computations are performed in MATLAB R2015a on a 2.50 GHz laptop with 4 GB of RAM. To solve the SOCP/SDP reformulation, we have used CVX 1.2.1. For all test problems, we set $tol = 10^{-6}$ and maxiter= 100. Our exten-

Table 1: Comparison between ADMM and SOCP/SDP relaxation for the first class of test problems with density=0.1.

Dimension	Algorithm	Time (s)
n=100	ADMM	1.4
	SOCP/SDP	1.9
n=300	ADMM	3.2
	SOCP/SDP	27.4
n=500	ADMM	4.8
	SOCP/SDP	163.5

sive testing showed that $\gamma = 0.9$ and $\rho = -2\lambda_1 + 1$ are appropriate choices. Moreover, we set $\lambda^0 = 2e$, where e denotes the all one vector and $x^0 = \frac{e}{\sqrt{n}}$. Finally, to solve the (TRS) problems within the algorithm we have used the algorithm in [1] and to solve the quadratic optimization problems we have used the 'quadprog' command of MATLAB.

- **First class of test problems:**
  For this class we consider $m = 2$ in (m-eTRS), $A = sprandsym(n, density)$, $a = randn(n, 1)$, $b_1 = e_1$, $b_2 = -e_1$, the unit vector in R^n and $\beta_1 = 0.1$ and $\beta_2 = 0.1$. As we see, the two linear inequality constraints are parallel, then the relaxation in [7] is exact. Results are summarized in Tables 1 to 3 for the average of 10 runs. In Tables 1 and 2 we compare ADMM with the SOCP/SDP relaxation for different densities. As we see, ADMM is much faster than SOCP/SDP relaxation. Moreover, for these two tables beside the time, we also have computed the relative objective function difference which is always below $O(10^{-8})$ for all test problems. This numerically verifies that ADMM converges to the same solution as SOCP/SDP relaxation does. In Table 3 we just report the results of ADMM as the relaxation is not applicable for these problems. In this table, KKT1 denotes the first order stationary condition, namely $||(A + 2\mu^* I_n)x^* + a + \sum_{i=1}^{m} \nu_i^* b_i||$.

- **Second class of test problems:**
  For this class we consider $m = 5$, the number of linear inequalities. As before, we consider $A = sprandsym(n, density)$ and $a = randn(n, 1)$ and consider $b = rand(n, m)$, $x = randn(n, 1)$ and $\beta = \frac{b^T x}{||x||}$. Obviously, the feasible region of (m-eTRS) is nonempty. Results of applying ADMM and SOCP/SDP relaxation to this class are summarized in Tables 4 to 6. Similar observations to the previous class hold here as well.

Table 2: Comparison between ADMM and SOCP/SDP relaxation for the first class of test problems with density=0.01.

Dimension	Algorithm	Time (s)	
n=100	ADMM	0.75	
	SOCP/SDP	1.1	
n=300	ADMM	2.6	
	SOCP/SDP	26.7	
n=500	ADMM	4.1	
	SOCP/SDP	141.5	

Table 3: Results of ADMM for the first class of test problems with density=0.001.

Dimension	KKT1	Time (s)	
n=3000	8.78×10^{-14}	21.5	
n=5000	5.51×10^{-14}	37.7	
n=8000	1.03×10^{-13}	70.8	

Table 4: Comparison between ADMM and SOCP/SDP relaxation for the second class of test problems with $m = 5$ and density=0.1.

Dimension	Algorithm	Time (s)	
n=100	ADMM	2.5	
	SOCP/SDP	12.4	
n=300	ADMM	3.9	
	SOCP/SDP	345.6	
n=500	ADMM	5.2	
	SOCP/SDP	1235.8	

Table 5: Comparison between ADMM and SOCP/SDP relaxation for the second class of test problems with $m = 5$ and density=0.01.

Dimension	Algorithm	Time (s)	
n=100	ADMM	2.1	
	SOCP/SDP	7.4	
n=300	ADMM	3.3	
	SOCP/SDP	287.3	
n=500	ADMM	4.5	
	SOCP/SDP	933.4	

Table 6: Results of ADMM for the second class of test problems with $m = 5$ and density=0.001.

Dimension	KKT1	Time (s)	
n=3000	9.83×10^{-14}	37.6	
n=5000	7.90×10^{-14}	56.9	
n=8000	3.56×10^{-14}	110.7	

4 Conclusions

In this paper, we have applied ADMM for solving the extended trust region subproblem. The convergence of the method to the first order stationary conditions is established and the quality of the solutions for small dimensions are compared with the known SOCP/SDP relaxation showing that ADMM converges to the global solution in significantly shorter time. Moreover, several large instances also are solved by ADMM to show its capability. The second order convergence analysis of the method is an interesting future research direction which one may follow.

5 Acknowledgments

The authors would like to thank both reviewers for their useful comments and questions which improved the paper and University of Guilan for supporting this research.

References

1. Adachi, S., Iwata, S., Nakatsukasa, Y. and Takeda, A. *Solving the trust region subproblem by a generalized eigenvalue problem,* Mathematical Engineering Technical Report(METR 2015-14), Department of Mathematical Informatics, Graduate School of Information Science and Technology, The University of Tokyo.

2. Bai, X. and Scheinberg, K. *Alternating direction methods for non convex optimization with applications to second-order least-squares and risk parity portfolio selection,* Optimization-Online, 2015.

3. Bai, X., Sun, J., Sun, S. and Zheng, X. *An alternating direction method for chance-constrained optimization problems with discrete distributions,* Optimization-Online, 2012.

4. Beck, A. and Eldar, Y.C. *Strong duality in nonconvex quadratic optimization with two quadratic constraints,* SIAM Journal on Optimization, 17(3), 844-860, 2006.

5. Boyd, S., Parikh, N., Chu, E., Peleato, B. and Eckstein, J. *Distributed optimization and statistical learning via the alternating direction method of multipliers,* Foundations and Trends in Machine Learning, 3(1), 1-122, 2011.

6. Burer,S. and Anstreicher, K.M. *Second-order-cone constraints for extended trust-region subproblems,* SIAM Journal on Optimization, 23(1), 432-451, 2013.

7. Burer,S. and Yang, B. *The trust region subproblem with non-intersecting linear constraints,* Mathematical Programming, 149, 253-264, 2015.

8. Conn, A.R., Gould, N.I. and Toint, P.L. *Trust Region Methods,* SIAM, Philadelphia, PA, 2000.

9. Grant, M. and Boyd, S. *CVX: Matlab software for disciplined convex programming,* version 2.0 beta. http://cvxr.com/cvx, September 2013.

10. Fortin, C. and Wolkowicz, H. *The trust region subproblem and semidefinite programming,* Optimization Methods and Software, 19(1), 41-67, 2004.

11. Hsia, Y. and Sheu, R.L. *Trust region subproblem with a fixed number of additional linear inequality constraints has polynomial complexity,* arXiv preprint arXiv, 1312.1398, 2013.

12. Hong, M. , Luo, Z.Q. and Razaviyan, M. *Convergence analysis of alternating direction method of multipliers for a family of nonconvex problems,* SIAM Journal on Optimization, 26(1), 337-364, 2016.

13. Jeyakumar, V. and Li, G.Y. *Trust-region problems with linear inequality constraints: exact SDP relaxation, global optimality and robust optimization,* Mathematical Programming, 147, 171-206, 2014.

14. Luo, H., Sun, X. and Wu, H. *Convergence properties of augmented Lagrangian methods for constrained global optimization,* Optimization Methods and Software, 23(5), 763-778, 2008.

15. Martinez, J.M. *Local minimizers of quadratic functions on Euclidean balls and spheres,* SIAM Journal on Optimization, 4(1), 159-176, 1994.

16. Rendl, F. and Wolkowicz, H. *A semidefinite framework for trust region subproblems with applications to large scale minimization,* Mathematical Programming, 77(1), 273-299, 1997.

17. Salahi, M. and Fallahi, S. *Trust region subproblem with an additional linear inequality constraint,* Optimization Letters, 10(4), 821-832, 2016.

18. Salahi, M. and Taati, A. *A fast eigenvalue approach for solving the trust region subproblem with an additional linear inequality,* Compuational and Applied Mathematics, DOI: 10.1007/s40314-016-0347-3, 2016.

19. Shen, Y., Wen, Z. and Zhang, Y. *Augmented Lagrangian alternating direction method for matrix separation based on low-rank factorization,* Optimization Methods and Software, 29(2), 239-263, 2014.

20. Sturm, J.F. and Zhang, S. *On cones of nonnegative quadratic functions,* Mathematics of Operations Research, 28(2), 246-267, 2003.

21. Xu, Y., Yin, W., Wen, Z. and Zhang, Y. *An alternating direction algorithm for matrix completion with nonnegative factors,* Journal of Frontiers of Mathematics in China, Special Issues on Computational Mathematics, 365-384, 2011.

22. Xu, L., Yu, B. and Zhang, Y. *An alternating direction and projection algorithm for structure-enforced matrix factorization,* Technical Reports, Department of Computational and Applied Mathematics, Rice University, 2013.

روش جهت متناوب ضرایب برای زیر مساله ناحیه اطمینان توسیع یافته

مازیار صلاحی و اکرم طاعتی

دانشگاه گیلان، دانشکده علوم ریاضی، گروه ریاضی کاربردی

دریافت مقاله ۱۸ آذر ۱۳۹۴، دریافت مقاله اصلاح شده ۱۰ مرداد ۱۳۹۵، پذیرش مقاله ۷ مهر ۱۳۹۵

چکیده : اخیراً زیرمساله ناحیه اطمینان توسیع یافته در تحقیقات متعددی مورد توجه قرار گرفته است. تحت مفروضات گوناگون، دوگانی قوی و آزاد سازی مخروطی درجه دو–نیمه معین برای کلاس های مختلف آن ارایه شده است. با توجه به اهمیت زیر مساله ناحیه اطمینان توسیع یافته، در این مقاله بدون در نظر گرفتن هیچ فرضی روی مساله، روش جهت متناوب ضرایب را که بسیار مورد استفاده قرار گرفته است، برای حل آن بکار می‌بریم. همگرایی روش به شرایط ایستایی مرتبه اول ثابت می‌شود. همچنین روی کلاس های مختلفی از مسایل آزمون، کیفیت جواب محاسبه شده توسط این روش در ابعاد متوسط با آزاد سازی مخروطی درجه دو–نیمه معین مقایسه می شود. علاوه بر این، کاربرد روش در حل مسایل در ابعاد بزرگ با حل چندین مثال مقیاس بزرگ نشان داده می شود.

کلمات کلیدی : زیر مساله ناحیه اطمینان توسیع یافته؛ روش متناوب؛ بهینه سازی نامحدب؛ برنامه ریزی نیمه معین؛ برنامه ریزی مخروطی درجه دو.