

# Analysing panel flutter in supersonic flow by Hopf bifurcation

Z. Monfared\* and Z. Dadi

## Abstract

This paper is devoted to the study of a partial differential equation (PDE) governing panel motion in supersonic flow. This PDE can be transformed to an ODE by means of a Galerkin method. Here by using a criterion which is closely related to the Routh-Hurwitz criterion, we investigate the mentioned transformed ODE from Hopf bifurcation point of view. In fact we obtain a region for existence of simple Hopf bifurcation for it. With the aid of Matlab and Hopf bifurcation tool, flutter and limit cycle oscillations of panel are verified. Moreover, Hopf bifurcation theory is used to analyse the flutter speed of the system.

**Keywords:** Panel flutter; Limit cycle; Hopf bifurcation; Routh-Hurwitz criterion; Vibrations.

## 1 Introduction

Aerodynamics is a branch of dynamics concerned with studying the motion of air, particularly when it interacts with a solid object such as an Aircraft structure.

On the other hand, flow-induced structural vibration is one of the most technical problems affecting the reliability, cost and safety of aircraft structures. The vibration caused by a fluid flowing around a body is known as flow-induced vibration. Flow-induced vibrations best describe the interaction that occurs between the fluid's dynamic forces and a structure's inertial, damping, and elastic forces. The study of flow-induced vibrations has rapidly developed in aeronautical and nonaeronautical engineering. In aeronautics, flow-induced vibration is often referred to as flutter. Flutter is the instability of aeronautics structures under unsteady aerodynamic loadings. Panel flutter

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\*Corresponding author

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Z. Monfared

Department of Applied Mathematics, Faculty of Mathematical Sciences, Ferdowsi University of Mashhad, Mashhad, Iran. e-mail: monfared.zahra@gmail.com

Z. Dadi

Department of Mathematics, Faculty of Basic Sciences, University of Bojnord, Bojnord, Iran. e-mail: z.dadi@ub.ac.ir

is a phenomenon of self-exciting vibrations of skin panels of flight vehicle at high flight speeds resulting from the interaction between an elastic structure and the flow around the structure. Such vibrations typically have high amplitude and cause fatigue damage of skin panels. This flutter phenomenon was first observed during World War II; however, formal studies did not appear until the 1950s. Supersonic panel flutter is a key design consideration for some high-speed aerospace vehicles like spacecrafts and missiles, [14, 17, 20]. Moreover, for nonlinear systems flutter is usually interpreted as a limit cycle oscillation (LCO). If a limit cycle is created the system will oscillate forever.

Aeroelastic flutter is a catastrophic structural failure, which needs to be avoided within the flight envelope of an aircraft structure. Hence, engineering researchers have paid much attention to studying the flutter and limit cycle motions of thin panels in recent years (see [2, 12, 14, 15, 16, 17, 19, 20] ).

Furthermore, Hopf bifurcation theory can be utilized as an important tool for the determination of the flutter and limit cycle vibrations of panels. In addition, Hopf bifurcation tool can be used to analyze the flutter speed of the system. Hence, with the use of thin panels in shuttles and large space stations, nonlinear dynamics, bifurcations, and the chaos of thin panels have become more and more important. In the past decade, researchers have made a number of studies into nonlinear oscillations, bifurcations, and the chaos of thin panels. Holmes [5] studied flow-induced oscillations and bifurcations of thin panels and gave a finite-dimensional analysis. Then based on the analysis in [5], Holmes and Marsden [7] considered an infinite-dimensional analysis for flow-induced oscillations and pitchfork and fold bifurcations of thin panels. Holmes [6] then simplified this problem to a two-degrees-of-freedom nonlinear system and used center manifolds and the theory of normal forms to study the degenerate bifurcations. Yang and Sethna [18] used an averaging method to study the local and global bifurcations in parametrically excited, nearly square plates. From the von Karman equation, they simplified this system to a parametrically excited two-degrees-of-freedom nonlinear oscillators and analyzed the global behaviour of the averaged equations. Based on the studies in [18], Feng and Sethna [3] made use of the global perturbation method developed by Kovacic and Wiggins [8] to study further the global bifurcations and chaotic dynamics of a thin panel under parametric excitation, and obtained the conditions in which Silnikov-type homoclinic orbits and chaos can occur. Zhang et al. [19] investigated both the local and global bifurcations of a simply supported at the fore-edge, rectangular thin plate subjected to transversal and in-plane excitations simultaneously.

In this paper, a problem of flow-induced oscillations, that of panel flutter is considered. In fact here we investigate a partial differential equation which describes panel motion and obtain a region for the existence of a special type of Hopf bifurcation for it. This type of Hopf bifurcation occurs where a pair of complex conjugate eigenvalues of the Jacobian matrix passes through the imaginary axis while all other eigenvalues have negative real parts. Furthermore, the existence of this type of Hopf bifurcation leads to flutter and limit

cycle motions of the panel which can cause failure of the structure. To the best of our knowledge, it is the first time that such a region for the existence of periodic solutions and Hopf bifurcation is being investigated. Moreover, by means of Matlab and the fourth and fifth-order Runge-Kutta (RK-45) method we do some numerical simulations. These simulations present our theoretical results and flutter and limit cycle oscillations of thin panel. Moreover, the flutter speed is obtained by using Hopf bifurcation tool.

## 2 Preliminaries

In this section, we state some mathematical concepts and basic results.

### 2.1 A criterion based on the Routh-Hurwitz criterion for the existence of simple Hopf bifurcation

Important criterion that gives necessary and sufficient conditions for all of the roots of the characteristic polynomial to lie in the left half of the complex plane is known as the Routh-Hurwitz Criterion. This criterion is stated in the next theorem, see [16].

**Theorem 2.1.** (*Routh-Hurwitz Criterion*). *Consider a polynomial of the form*

$$a_k z^k + a_{k-1} z^{k-1} + a_{k-2} z^{k-2} + \dots + a_0,$$

*the roots of this polynomial lie in the open left half-plane if and only if all the leading principal minors of the  $k \times k$  matrix*

$$Q = \begin{pmatrix} a_1 & a_0 & \dots & 0 \\ a_3 & a_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{2k-1} & a_{2k-2} & \dots & a_k \end{pmatrix},$$

*are positive and  $a_k > 0$ ; we assume that  $a_j = 0$  if  $j < 0$  or  $j > k$ .*

In dynamical systems, a bifurcation occurs when a small smooth change made to the parameter values (the bifurcation parameters) of a system causes a sudden qualitative or topological change in its behaviour. In general, at a bifurcation point, the local stability properties of equilibria, periodic orbits or other invariant sets change. Moreover, in many applications we are concerned with a special type of Hopf bifurcations, where a pair of complex conjugate eigenvalues of the Jacobian matrix passes through the imaginary axis while all other eigenvalues have negative real parts. These are called simple Hopf

bifurcations in [10], in order to distinguish them from the Hopf bifurcations with some other eigenvalues on the right half plane. Now consider the system

$$\dot{X} = f(X, \mu), \quad X \in \mathbb{R}^n, \quad \mu \in \mathbb{R}. \quad (1)$$

We use the notation in [4, 13] and mention the following theorem which states the sufficient conditions for existence of Hopf bifurcation.

**Theorem 2.2.** *Suppose that system (1), has an equilibrium  $(x_0, \mu_0)$  at which the following properties are satisfied:*

(SH1)  *$D_x f_{\mu_0}(x_0)$  has a simple pair of pure imaginary eigenvalues and other eigenvalues have negative real parts. Therefore, there exists a smooth curve of equilibria  $(x(\mu), \mu)$  with  $x(\mu_0) = x_0$ . The eigenvalues  $\lambda(\mu), \bar{\lambda}(\mu)$  of  $D_x f_{\mu_0}(x(\mu))$  which are imaginary at  $\mu = \mu_0$  vary smoothly with  $\mu$ . Furthermore, if,*

$$(SH2) \quad \left. \frac{d}{d\mu}(\operatorname{Re}\lambda(\mu)) \right|_{\mu=\mu_0} = d \neq 0,$$

*then Hopf bifurcation will occur.*

*Proof.* See [4]. □

Even though numerical computation of eigenvalues is feasible, it is ideal to have a criterion stated in terms of the coefficient of the characteristic polynomials rather than the traditional Hopf bifurcation criterion which is based on the property of eigenvalues. Specially for higher dimensional systems with many parameters, this criterion will be more convenient. See [10].

We denote the characteristic polynomial of the Jacobian matrix  $J(\mu)$  of (1) as:

$$P(\lambda; \mu) = \det(\lambda I_n - J(\mu)) = p_0(\mu) + p_1(\mu)\lambda + \dots + p_n(\mu)\lambda^n,$$

where every  $p_i(\mu)$  is a smooth function of  $\mu$ , and  $p_n(\mu) = 1$ . And we consider the case  $p_0(\mu) > 0$ , because there is not any nonnegative real root. Let

$$L_n(\mu) = \begin{pmatrix} p_1(\mu) & p_0(\mu) & \cdots & 0 \\ p_3(\mu) & p_2(\mu) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ p_{2n-1}(\mu) & p_{2n-2}(\mu) & \cdots & p_n(\mu) \end{pmatrix},$$

where  $p_i(\mu) = 0$  if  $i < 0$  or  $i > n$ . Moreover, when  $p_0(\mu) > 0$  by the R-H criterion the polynomial  $P(\lambda; \mu)$  of  $\lambda$  has all roots with negative real parts if and only if the following  $n$  principal sub determinants of  $L_n(\mu)$  are positive:

- $D_1(\mu) = \det(L_1(\mu)) = p_1(\mu) > 0$
- $D_2(\mu) = \det(L_2(\mu)) = \det \begin{pmatrix} p_1(\mu) & p_0(\mu) \\ p_3(\mu) & p_2(\mu) \end{pmatrix} > 0$
- $\vdots$

- $D_n(\mu) = \det(L_n(\mu)) > 0$ . Since  $D_n(\mu) = p_n(\mu)D_{n-1}(\mu)$  and in our case  $p_n(\mu) = 1$  the R-H criterion conditions can be stated as
- $p_0(\mu) > 0, \quad D_1 > 0, \quad D_2 > 0, \quad \dots, \quad D_{n-1} > 0$ .

**Theorem 2.3.** *Assume there is a smooth curve of equilibria  $(x(\mu), \mu)$  with  $x(\mu_0) = x_0$  for (1). Then conditions (SH1) and (SH2) for a simple Hopf bifurcation are equivalent to the following conditions on the coefficients of the characteristic polynomial  $P(\lambda; \mu)$ :*

- (1)  $p_0(\mu_0) > 0, \quad D_1(\mu_0) > 0, \quad \dots, \quad D_{n-2}(\mu_0) > 0, \quad D_{n-1}(\mu_0) = 0$
- (2)  $\frac{dD_{n-1}(\mu_0)}{d\mu} \neq 0$ .

*Proof.* see [10]. □

## 2.2 Galärkin method

Suppose we wish to solve the following boundary value problem of partial differential equations over the interval  $a \leq z \leq b$ ,

$$L[y(z, t)] + f(z, t) = 0,$$

$$y(a, t) = z_a, y(b, t) = z_b.$$

A Galärkin method is used to approximate the problem by a sequence of finite dimensional problems. In other words, we consider the problem as a flow defined on a space  $V$  and then choose the finite dimensional subspace  $V_N \subset V$  of dimensional  $N$  and project our problem onto  $V_N$ . Reducing the problem to a finite dimensional vector subspace allows us to numerically compute  $u_N$  (the solution of PDE) as a finite linear combination of the basis vectors in  $V_N$ .

Now, let  $\{\varphi_j\}_{j=1}^N$  be an orthonormal basis of the finite dimensional subspace  $V_N$  that satisfy the boundary conditions of the problem. Therefore, we can write

$$u_N(z, t) = \sum_{j=1}^N a_j(t)\varphi_j.$$

Through the use of orthogonal functions the error function  $E_N$ , representing the difference between the exact and approximate solution, is minimized such that

$$\int_a^b E_N(z, t) \cdot \varphi_j \, dz = 0, \quad \forall j = 1, 2, \dots, N,$$

where,

$$E_N(z, t) = L[u_N(z, t)] + f(z, t).$$

Each term in above equation gives an ODE in time for the  $N$  coefficients  $\{a_j(t)\}_{j=1}^N$  and these must be solved numerically. For more information see [5, 7].

### 2.3 Dynamic pressure

In fluid dynamics, dynamic pressure (indicated with  $q$ , or  $Q$ , and sometimes called velocity pressure) is the quantity of air measured by most airspeed instruments and defined by

$$q = \frac{1}{2}\rho v^2,$$

where  $\rho$  and  $v$  are density and velocity of the flow respectively. See [1], Section 3.5.

### 3 Formulation of the problem

Consider a supersonic stream of fluid passes above a thin plate with the length 1, fixed at the edges  $z = 0$  and  $z = 1$ . The panel is simultaneously subjected to an in-plane tensile load  $\Gamma$ . The fluid velocity is characterized in terms of the dynamic pressure  $q$ , see Figure 1. Using nondimensional quantities, and assuming that the panel bends in a cylindrical mode (so that  $w(z, y, t) = w(z, t)$  is independent of  $y$ ), the following nonlinear partial-differential equation, which is essentially a one dimensional version of the von Karman equations is considered for a thin plate:

$$w_{tt} + \alpha w_{tzzzz} + \sqrt{q}\delta w_t - \{\Gamma + k \int_0^1 w_z^2 dz + \sigma \int_0^1 w_z \cdot w_{tz} dz\} w_{zz} + w_{zzzz} + qw_z = 0, \quad (2)$$

see [4, 7]. Here  $w = w(z, t)$  is the transverse displacement of the panel,  $\alpha, \sigma \geq 0$  are (linear) viscoelastic damping parameters associated with the panel,  $\delta > 0$  represents fluid damping and  $k > 0$  is a measure of the nonlinear axial (membrane) restoring forces generated in the panel due to transverse displacement. Moreover, all these parameters are assumed to be fixed, except  $q$  which can vary.

Now, consider equation (2) with the following simply supported boundary conditions

$$w(0, t) = w_{zz}(0, t) = w(1, t) = w_{zz}(1, t) = 0. \quad (3)$$

A plate subjected to a compressive in-plane load with fluid flow over its surface may undergo complex motions resulting in dynamic instabilities (flutter) and associated limit cycle motions.

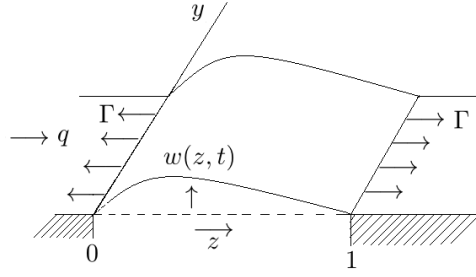


Figure 1: The panel flutter problem

#### 4 Bifurcation analysis

It does not seem possible to solve the equation (2) explicitly. As it is mentioned in [7] by using a Galérkin method the partial differential equation (2) together with the boundary conditions (3) can be transformed to a ordinary differential equation. In fact in [7] because of the simply supported boundary condition (3), the following family of orthogonal basis

$$\{\varphi_j(z)\}_{j=1}^N = \{\sin(j\pi z)\}_{j=1}^N$$

is chosen. Then by writing ,  $w(z, t) = \sum_{j=1}^N a_j(t)\varphi_j(z)$  and applying the Galérkin procedure for  $N = 2$  (two modes) to the governing Equation (2) and using the orthonormality of the bases and the relationships

$$\int_0^1 w_j'''' \cdot w_s dz = \int_0^1 w_j'' \cdot w_s'' dz,$$

$$\int_0^1 w_j'' \cdot w_s dz = - \int_0^1 w_j' \cdot w_s' dz,$$

the following ODE in the time dependent amplitude coefficients  $a_j(t)$  is obtained.

$$\dot{x} = A_q x + f(x), \quad x \in \mathbb{R}^4, q \in \mathbb{R}, \quad (4)$$

where

$$x = \begin{pmatrix} a_1 \\ a_2 \\ \dot{a}_1 \\ \dot{a}_2 \end{pmatrix},$$

$$A_q = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \pi^2(\Gamma - \pi^2) & 8q/3 & -(\alpha\pi^4 + \sqrt{q}\delta) & 0 \\ -8q/3 & 4\pi^2(\Gamma - 4\pi^2) & 0 & -(16\alpha\pi^4 + \sqrt{q}\delta) \end{pmatrix},$$

and

$$f = \begin{pmatrix} 0 \\ 0 \\ -f_1 \\ -f_2 \end{pmatrix},$$

in which

$$f_1 = \frac{\pi^4}{2} \{k(a_1^2 + 4a_2^2) + \sigma(a_1\dot{a}_1 + 4a_2\dot{a}_2)\}a_1,$$

$$f_2 = 2\pi^4 \{k(a_1^2 + 4a_2^2) + \sigma(a_1\dot{a}_1 + 4a_2\dot{a}_2)\}a_2.$$

Moreover,  $a_i = x_i (i = 1, 2)$  are the amplitudes of normal two modes.

Now we investigate Hopf bifurcations of system (2) for the trivial equilibrium position  $x = 0$  or  $w(z, t) \equiv 0$  by using a criterion which is closely related to the Routh-Hurwitz (R-H) criterion to obtain a region of simple Hopf bifurcation parameters. For this purpose, we mention the sufficient conditions for existence of simple Hopf bifurcation of system (4) in the following theorem.

**Theorem 4.1.** *Suppose that for  $q = q_0$  and the trivial equilibrium position  $x = 0$  of (4), the following relations satisfy:*

1)  $p_0(q_0) > 0$

2)  $p_1(q_0) > 0$

3)  $\det \begin{pmatrix} p_1(q_0) & p_0(q_0) \\ p_3(q_0) & p_2(q_0) \end{pmatrix} > 0$

4)  $D_3(q_0) = \det \begin{pmatrix} p_1(q_0) & p_0(q_0) & 0 \\ p_3(q_0) & p_2(q_0) & p_1(q_0) \\ 0 & 1 & p_3(q_0) \end{pmatrix} = 0$

5)  $\frac{dD_3(q_0)}{dq} \neq 0,$

where



$$\begin{aligned}
p_0(q_0) &= 4\pi^2(\Gamma - \pi^2)(\Gamma - 4\pi^2) + \frac{64q_0^2}{9}, \\
p_1(q_0) &= -\pi^2\{(\Gamma - \pi^2)(16\alpha\pi^4 + \sqrt{q_0}\delta) + 4(\Gamma - 4\pi^2)(\alpha\pi^4 + \sqrt{q_0}\delta)\}, \\
p_2(q_0) &= (\alpha\pi^4 + \sqrt{q_0}\delta)(16\alpha\pi^4 + \sqrt{\pi}\delta) - \pi^2(5\Gamma - 17\pi^2), \\
p_3(q_0) &= 17\alpha\pi^4 + 2\sqrt{q_0}\delta.
\end{aligned}$$

Then  $q_0$  is a simple Hopf bifurcation value for system (4) at the trivial equilibrium position  $x = 0$ .

*Proof.* By computing the characteristic polynomial of (4) and using theorem 2.3., the above assertion can be proved.  $\square$

The panel transverse displacement and velocity for two modes ( $N = 2$ ) can be evaluated by the following equations

$$w(z, t) = \sum_{j=1}^2 a_j(t) \sin(j\pi z), \quad (5)$$

$$w_t(z, t) = \sum_{j=1}^2 \dot{a}_j(t) \sin(j\pi z). \quad (6)$$

Therefore, by the above theorem we can find a region for existence of simple Hopf bifurcation for equation (4) and therefore for equation (2).

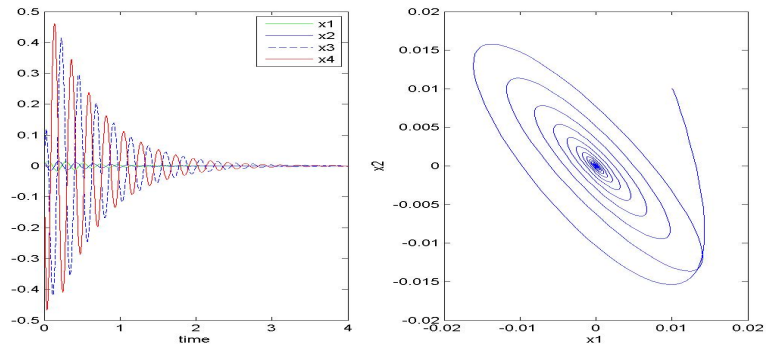
## 5 Numerical simulation

Here numerical simulations are carried out to support our theoretical results and show panel flutter.

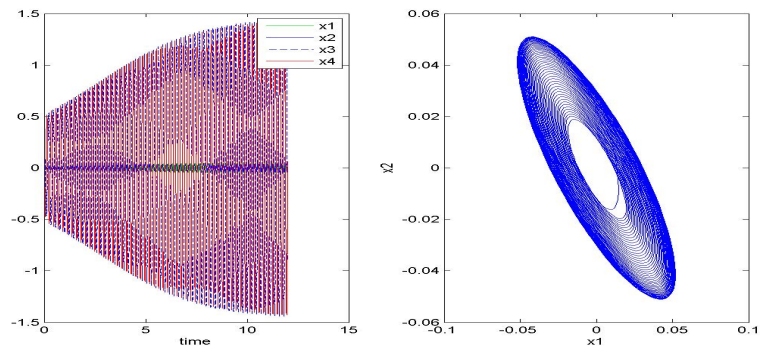
*Example.* Consider system (4), by the aid of software Auto for  $\delta = 1, \Gamma = 2, \alpha = 0$  and  $q_0 \simeq 323.24789021$  we can show that this system undergoes Hopf bifurcation at the equilibrium  $x = (x_1, x_2, x_3, x_4) = (0, 0, 0, 0)$ . Moreover, these values of parameters by [4, 5, 7] are physically meaningful. In addition, for the mentioned values of parameters the characteristic polynomial of the Jacobian matrix of (4) at  $x$  for  $q_0$  is :

$$P(\lambda; q_0) \simeq 857953.77463 + 27998.10001\lambda + 1880.50639\lambda^2 + 35.95819\lambda^3 + \lambda^4.$$

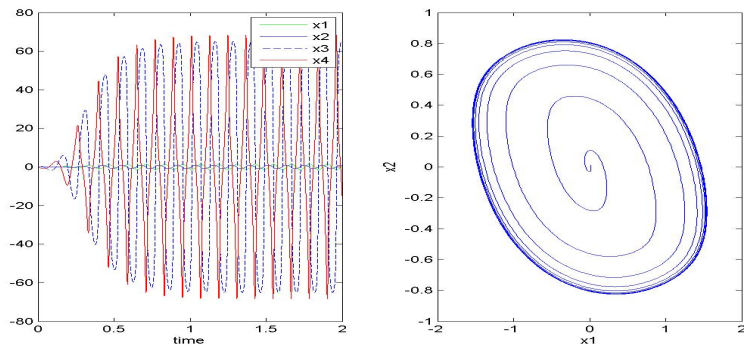
So



(a)

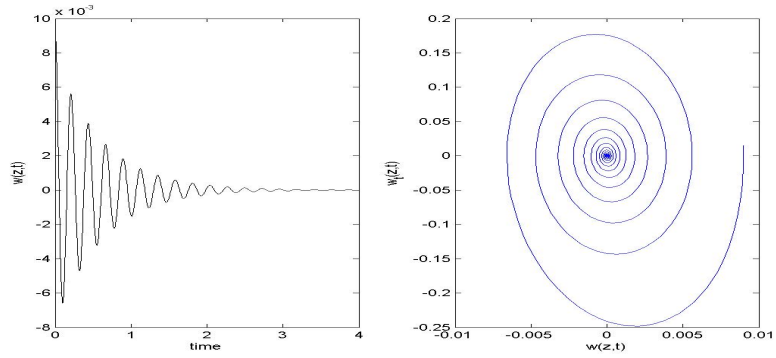


(b)

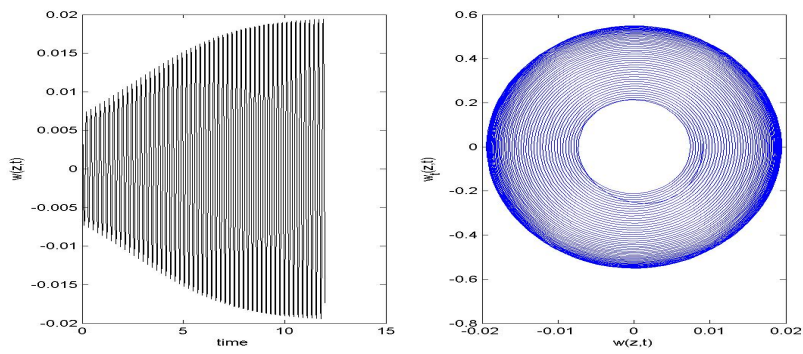


(c)

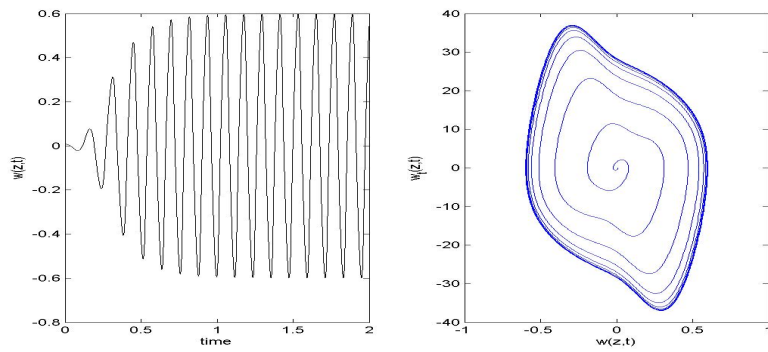
Figure 2: The related trajectories in the phase space  $x_1 - x_2$  and time history for different values of  $q$ . Initial condition is very close to the equilibrium  $x = 0$ : (a)  $q = 300$ ; (b)  $q = 327$ ; (c)  $q = 1000$



(a)



(b)



(c)

Figure 3: Phase Portrait in the space  $w - w_t$  and time History of the panel transverse displacement for different values of  $q$  : (a)  $q = 300$ ; (b)  $q = 327$ ; (c)  $q = 1000$

$$p_0(q_0) \simeq 857953.774634701 > 0,$$

$$D_1(q_0) = p_1(q_0) \simeq 27998.1000129 > 0,$$

$$D_2(q_0) = \det \begin{pmatrix} p_1(q_0) & p_0(q_0) \\ p_3(q_0) & p_2(q_0) \end{pmatrix} \simeq 21800139.664474831 > 0,$$

$$D_3(q_0) = \det \begin{pmatrix} p_1(q_0) & p_0(q_0) & 0 \\ p_3(q_0) & p_2(q_0) & p_1(q_0) \\ 0 & 1 & p_3(q_0) \end{pmatrix} = 0,$$

and

$$\frac{dD_3(q_0)}{d\omega} \simeq -4937513.3537057098 \neq 0.$$

Therefore, by Theorem 4.1.,  $q_0 \simeq 323.24789021876$  is a simple Hopf bifurcation value for (4) at  $x$ . Furthermore, for  $q_0$  the Jacobian matrix  $A_q$  at  $x$  has a pair of pure imaginary eigenvalues  $\lambda_1, \lambda_2 \simeq \pm 27.9039289667884189i$ , and a couple of complex conjugate eigenvalues with negative real part  $Re\lambda_3 = Re\lambda_4 \simeq -17.979095923287$ .

We solved the equation (4) by means of the fourth and fifth-order Runge-Kutta (RK-45) method. Then by numerical simulations we showed the occurrence of Hopf bifurcation. The related trajectories in the phase space  $x_1 - x_2$  (the space of the amplitudes of two modes) and the time history responses for different values of parameter  $q$  are presented in Figure 2.

Moreover, by equations (5) and (6), the panel transverse displacement ( $w(z, t)$ ) and velocity ( $w_t(z, t)$ ) for two modes can be determined. So, the related trajectories in the phase space  $w - w_t$  and the time history responses of the panel transverse displacement for different values of parameter  $q$  are illustrated in Figure 3.

As it is illustrated in Figures 2 and 3, the equilibrium point  $x = 0$  or  $w(z, t) \equiv 0$  is a stable focus when the dynamic pressure of flow is less than  $q_0$ . In this case by passing the time the amplitude of panel vibrations will vanish. While for  $q > q_0$  the equilibrium point  $x = 0$  or  $w(z, t) \equiv 0$  turns out to be an unstable focus surrounded by a stable limit cycle. That means that by passing the time, the amplitude of panel oscillations will increase and finally panel will vibrate with a fixed period for ever. Furthermore, in this case for large values of the dynamic pressure  $q$ , panel vibrations can have high amplitude and cause catastrophic failure of the structure. This increase of amplitude will be faster for larger values of  $q$ .

In addition, since the Hopf bifurcation occurs for the dynamic pressure  $q_0 \simeq 323.24789021876 Pa$ , due to Section 2.3., by knowing the density of flow (air) the flutter speed can be obtained. For example at sea level and at  $15^\circ C$ , air has a density of approximately  $1.225 \frac{kg}{m^3}$ . Hence, for this density of air the flutter speed is approximately 22.97284609 in the absence of linear

viscoelastic damping parameter  $\alpha$ . In Figure 3 our simulations show flutter and limit cycle oscillations (LCO) of the panel without considering the viscoelastic damping parameter  $\alpha$ .

## 6 Conclusion

In this paper, we extended the previous results in [5, 6, 7, 8, 17, 18, 19] to study the vibrations of a thin panel fixed at two edges. Indeed we found a region for existence of simple Hopf bifurcation for a partial differential equation governing panel motion. Because, the existence of simple Hopf bifurcation can lead to flutter and limit cycle oscillations of the panel. Numerical simulations were carried out by using the fourth and fifth-order Runge-Kutta method, to support our analytical results. In fact by simulations and Hopf bifurcation theory, we showed the occurrence of flutter and limit cycle motions of thin panel. Then Hopf bifurcation tool was used to calculate the flutter speed of the system. Moreover, numerical simulations presented vibrations of thin panel can have high amplitude which cause damage of panel.

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## تحلیل فلاتر پانل در جریان فرا صوت بوسیله انشعاب هاف

زهرا منفرد<sup>۱</sup> و زهره دادی<sup>۲</sup>

<sup>۱</sup> دانشگاه فردوسی مشهد، دانشکده علوم ریاضی، گروه ریاضی کاربردی

<sup>۲</sup> دانشگاه بجنورد، دانشکده علوم پایه، گروه ریاضی

**چکیده:** این مقاله به مطالعه یک معادله دیفرانسیل مشتقات جزئی حاکم بر حرکت پانل در جریان فراصوت اختصاص داده شده است. این معادله دیفرانسیل مشتقات جزئی می تواند بوسیله یک روش گالرکین به یک معادله دیفرانسیل معمولی تبدیل شود. در اینجا با استفاده از معیاری که وابسته به معیار روت-هورویتز می باشد، معادله دیفرانسیل معمولی ذکر شده را از نقطه نظر انشعاب هاف مورد بررسی قرار می دهیم. در حقیقت ما ناحیه ای برای وجود انشعاب هاف ساده برای آن را می یابیم. به کمک نرم افزار مطلب و ابزار انشعاب هاف، فلاتر ونوسانات چرخه حدی پانل بررسی شده اند. همچنین نظریه انشعاب هاف برای تحلیل سرعت فلاتر سیستم مورد استفاده قرار گرفته است.

**کلمات کلیدی:** فلاتر پانل؛ چرخه حدی؛ انشعاب هاف؛ معیار روت-هورویتز؛ نوسانات.