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Research Article

A compact common-weight DEA model for efficiency measurement

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Abstract

This paper proposes a novel common-weight DEA model that combines multi-objective linear fractional programming and goal programming to determine a common set of weights for evaluating decision-making units (DMUs). The model formulates a single linear program without requiring iterative procedures, thereby enhancing computational efficiency. To validate the model's effectiveness, we compare the proposed method with a satisfaction degree-based model, a distance-based model, and a credibility-based common-weight method. Our model demonstrates competitive efficiency scores and a high degree of discrimination among DMUs, while maintaining a lower computational complexity. This research contributes by offering a practical and computationally efficient alternative to existing models.

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1 Introduction

Charnes, Cooper, and Rhodes [3] initially suggested the data envelopment analysis (DEA) technique for determining the effectiveness of decision-making units (DMUs). These DMUs can be anything, like companies, schools, hospitals, or even government organizations, that use multiple inputs (resources) to produce multiple outputs (results). Traditional DEA models evaluate the efficiency of DMUs that use multiple inputs (resources) to produce multiple outputs (results). Conventional DEA models, like the CCR and BCC models, let each DMU select its own input and output weights.

This implies that each unit can maximize its efficiency score by modifying the relative importance of its resources and outputs. However, because of this flexibility, many DMUs may be classified as efficient, particularly when there are fewer units than inputs and outputs. This reduces the model's ability to clearly distinguish between the best and worst performers, making it difficult to rank or compare units properly.

To solve this problem, DEA models based on common-weights were developed, assuming a single set of weights for all DMUs, so each unit is measured against the same standard. The idea behind using common-weights is to make the evaluation fairer and to improve the comparability between different units. The concept of using common-weights to improve fairness in DEA was first suggested by Roll, Cook, and Golany [17], who proposed a method for measuring each unit's effectiveness using a standard set of weights. This laid the groundwork for further research into how to better compare units using common-weights.

One of the primary advantages of using DEA models with common-weights is that they allow for better ranking of all DMUs, including both the efficient and inefficient ones. In traditional DEA, many units might be considered efficient, even if they are not truly performing at their best. Common-weight models, however, help provide a clear ranking based on a uniform set of criteria, which is especially useful for decision-making and resource allocation. Charnes et al. [2] and Golany and Roll [5] emphasized how using common-weights can make the evaluation process fairer and more objective, especially in areas where consistent and transparent comparisons are important.

Over the years, many researchers have developed different methods for calculating these common-weights. One approach involves using multi-objective optimization, where several goals (such as minimizing input excess and maximizing output efficiency) are considered at the same time. For example, Kao and Hung [7] proposed using goal programming to find the best common-weights by considering multiple efficiency-related goals. Wang, Greatbanks, and Yang [20] and Wu et al. [21] explored how integrating cross-efficiency evaluations into the common-weight model could help reduce bias in the results and provide more reliable rankings. Cross-efficiency, introduced by Seiford and Zhu [18], helps to avoid giving too much weight to units that perform well only when evaluated using their own customized weights.

In addition to mathematical improvements, many researchers have also focused on creating hybrid methods that combine DEA with other techniques like multi-objective linear programming (MOLP) or mixed-integer programming. These methods help improve the accuracy of efficiency evaluations by adding constraints or preferences according to the objectives of the decision-maker. For example, there are studies in which hybrid methods which can be used in industries like banking and healthcare to better evaluate the performance of various institutions [23, 13]. common-weight and weight restricted DEA models also enhance discrimination and comparability of efficiency scores. Applications include internet banking efficiency [11], eco-efficiency of urban waste services [14], dynamic performance evaluation of airlines [24], and climate competitiveness assessment of EU regions using entropy-based common-weights [9]. These works highlight the wide applicability and effectiveness of common-weight DEA frameworks. In recent years, some researchers have even incorporated behavioral theories, like prospect theory, into DEA models. Yu et al. [25], for instance, used prospect theory to account for how people make decisions under risk and uncertainty, leading to more realistic evaluations.

Recent studies have introduced innovative approaches to common-weight DEA, addressing various challenges in performance evaluation. In 2022, Omrani, Fahimi, and Emrouznejad [15] introduced a common-weight credibility DEA model that uses credibility theory to handle uncertainty in inputs and outputs. This model gives a strong evaluation method for industries like aviation, where data can vary a lot. In 2023, Majidi, Ebrahimnejad, and Azizi [12] suggested a fuzzy DEA model with common-weights that looks at both optimistic and pessimistic views when dealing with undesirable outputs. By using fuzzy arithmetic, this approach cuts down on computation time and offers a fairer assessment of DMUs under uncertain data. Most recently, in 2024, Kim and co-authors [10] presented a cluster-analysis-based common-weight DEA model. They group DMUs by their production patterns and then assign one weight vector to all units in each group, which gives more accurate and comparable efficiency measures in complex industries like iron and steel manufacturing. In addition to fuzzy and credibility-based approaches, robust DEA models have also been proposed to explicitly handle data uncertainty using robust optimization techniques [16, 1]. Together, these studies show a growing focus on making common-weight DEA models more robust, fair, and useful in situations where data are difficult or varied.

The existing models, especially those using iterative procedures, take a lot of time to compute — particularly when there are a lot of DMUs, inputs, and outputs [22]. These models solve the problem several times in different rounds to get the final result. This increases the overall complexity and makes them less suitable for large datasets. Also, many models give high efficiency scores to a large number of DMUs, which reduces the ability to clearly identify top performers [4, 19]. To overcome these issues, we have developed a new DEA model that works without any iteration. It solves the problem in a single step, which saves time and reduces computational effort. At the same time, it gives similar efficiency results when compared to existing models. But unlike them, our model identifies fewer DMUs as fully efficient. This shows that it is stricter and better at distinguishing between high and average performers. We also found that our model gives a more realistic satisfaction degree, which is useful when studying behavior or preferences of the DMUs. Overall, the proposed model is simpler, faster, and more practical, especially for large-scale problems. It helps in better decision-making and provides a strong alternative to more complex and time-consuming models.

2 Methodology

Assume that n DMUs with m inputs and s outputs need to be assessed. To determine a set of common-weights for inputs and outputs consider the following multi-objective linear fractional programming (MOLFP):

$$Max \quad \left(\frac{\sum_{r=1}^{s} u_r y_{r1}}{\sum_{i=1}^{m} v_i x_{i1}}, \frac{\sum_{r=1}^{s} u_r y_{r2}}{\sum_{i=1}^{m} v_i x_{i2}}, \ \dots, \ \frac{\sum_{r=1}^{s} u_r y_{rn}}{\sum_{i=1}^{m} v_i x_{in}}\right), \tag{1}$$

$$s.t. \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \dots, n, \tag{2}$$

$$u_r \geq 0, \quad r = 1, \dots, s, \tag{3}$$

$$v_i \geq 0, \quad i = 1, \dots, m. \tag{4}$$

After solving the CCR model for each DMU_j [3], we have

$$Max \quad \sum_{r=1}^{s} u_r y_{rj}, \tag{5}$$

$$s.t \quad \sum_{i=1}^{m} v_i x_{ij} = 1, \tag{6}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad j = 1, \ldots, n, \tag{7}$$

$$u_r \geq 0, \quad r = 1, \ldots, s, \tag{8}$$

$$v_i \geq 0, \quad i = 1, \ldots, m. \tag{9}$$

The efficiency of each DMU_j can be evaluated as

$$\theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}. \tag{10}$$

Now, DMUs can be divided into two groups: $E = \{j : \theta_j = 1\}$ and $I = \{\, j : \theta_j < 1\,\}$.
Let $\theta^+ = \max_{j \in I} \theta_j$ and $\theta^- = \min_{j \in I} \theta_j$.

Let N_j and D_j be the numerator and denominator of the jth objective function. The optimum values of the numerator and denominator can be readily derived as 1 for CCR efficient units ($j \in E$). However, the efficiency ratings of CCR efficient units should not be lower than those of the most efficient CCR inefficient unit, which is θ^+.

Based on these conditions, the membership functions for CCR efficient units are defined as follows [6]:

$$CN_i^E(x) = \begin{cases} \dfrac{N_i(x) - \theta^+}{1 - \theta^+}, & \theta^+ \leq N_i(x) \leq 1, \\ 0, & \text{otherwise}, \end{cases} \tag{11}$$

$$CD_i^E(x) = \begin{cases} \dfrac{\theta^+ - D_i(x)}{\theta^+ - 1}, & 1 \leq D_i(x) \leq \theta^+, \\ 0, & \text{otherwise}. \end{cases} \tag{12}$$

Similarly, for CCR inefficient units ($j \in I$) the ideal values of numerator and denominator are θ^+ and $\frac{1}{\theta^+}$, respectively, and the efficiency scores should not be lower than the lowest-performing CCR inefficient unit, which is θ^-. Therefore, the membership functions for CCR inefficient units will be as follows [6]:

$$C_I^{N_i}(x) = \begin{cases} \dfrac{N_i(x) - \theta^-}{\theta^+ - \theta^-}, & \theta^- \leq N_i(x) \leq \theta^+, \\ 0, & \text{otherwise}, \end{cases} \tag{13}$$

$$C_I^{D_i}(x) = \begin{cases} \dfrac{\frac{1}{\theta^-} - D_i(x)}{\frac{1}{\theta^-} - \frac{1}{\theta^+}}, & \frac{1}{\theta^-} \geq D_i(x) \geq \frac{1}{\theta^+}, \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Development of the common-weight model

Now we formulate a goal programming model by taking 1 as the aspiration level for each membership function (11), (12), (13) and (14). So the goals can be formulated as

$$C_E^{N_i}(x) = 1 \quad \Rightarrow \quad \frac{N_i(x) - \theta^+}{1 - \theta^+} = 1, \tag{15}$$

$$C_E^{D_i}(x) = 1 \quad \Rightarrow \quad \frac{\frac{1}{\theta^+} - D_i(x)}{\frac{1}{\theta^+} - 1} = 1, \tag{16}$$

$$C_I^{N_i}(x) = 1 \quad \Rightarrow \quad \frac{N_i(x) - \theta^-}{\theta^+ - \theta^-} = 1, \tag{17}$$

$$C_I^{D_i}(x) = 1 \quad \Rightarrow \quad \frac{\frac{1}{\theta^-} - D_i(x)}{\frac{1}{\theta^-} - \frac{1}{\theta^+}} = 1. \tag{18}$$

Simplifying and substituting numerator and denominator functions in (15), (16), (17) and (18), we get

$$\sum_{r=1}^{s} u_r y_{rj} = 1, \qquad j \in E, \tag{19}$$

$$-\sum_{i=1}^{m} v_i x_{ij} = -1, \qquad j \in E, \tag{20}$$

$$\sum_{r=1}^{s} u_r y_{rj} = \theta^+, \qquad j \in I, \tag{21}$$

$$-\sum_{i=1}^{m} v_i x_{ij} = -\frac{1}{\theta^+}, \qquad j \in I. \tag{22}$$

By introducing deviational variables d_j^+ and d_j^- for $j = 1, 2, \ldots, n$ and $i = 1, 2$, we can formulate the goal programming model as

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{i=1}^{2}\sum_{j=1}^{n}\left(d_{ij}^{-}+d_{ij}^{+}\right) \\
\text{subject to} \quad & \sum_{r=1}^{s} u_r y_{rj} + d_{1j}^{-} - d_{1j}^{+} = 1, && \text{for all } j \in E, \\
& -\sum_{i=1}^{m} v_i x_{ij} + d_{2j}^{-} - d_{2j}^{+} = -1, && \text{for all } j \in E, \\
& \sum_{r=1}^{s} u_r y_{rj} + d_{1j}^{-} - d_{1j}^{+} = \theta^{+}, && \text{for all } j \in I, \\
& -\sum_{i=1}^{m} v_i x_{ij} + d_{2j}^{-} - d_{2j}^{+} = -\frac{1}{\theta^{+}}, && \text{for all } j \in I, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, && j = 1,\dots,n, \\
& d_{ij}^{-} \ge 0, \quad d_{ij}^{+} \ge 0, && i = 1,2;\ j = 1,\dots,n \\
& u_r \ge 0, && r = 1,\dots,s, \\
& v_i \ge 0, && i = 1,\dots,m.
\end{aligned}
\tag{23}
$$

From the goal programming model (23) we can obtain a common set of weights u_r for outputs and v_i $(i = 1, 2, \dots, m)$ for inputs. These common-weights allow us to compare and rank the DMUs on a common basis.

3 Theoretical properties of the proposed model

Before analyzing the performance of the proposed goal programming model, we establish the feasibility and boundedness of the model to ensure that optimal solutions can be obtained for the proposed model.

Theorem 1. The common-weight model (23) is feasible.

Proof. From the output-oriented CCR model, for a CCR-efficient DMU j, the optimal weights satisfy $\sum u_r y_{rj} = 1$ and $\sum v_i x_{ij} = 1$. These equalities match the goal targets for the efficient unit exactly. Consequently, both deviational variables d_j^{+} and d_j^{-}, as well as d_i^{+} and d_i^{-}, can be set to zero, ensuring feasibility for DMU j without any penalty.

For all other DMUs k, the same weights u_r, v_i may not lead to exact satisfaction of the corresponding equality constraints in the goal programming model. However, the model includes nonnegative deviational variables specifically to allow for flexibility in these cases. By assigning appropriate positive values to these variables, the equalities can always be satisfied, regardless of how far the weighted input-output ratios are from the aspiration levels of 1 or other bounds. Furthermore, the inequality constraint $\sum v_i x_{ik} \leq 1$ is guaranteed to hold for all k because it is a defining constraint of the CCR model.

Lastly, all nonnegativity constraints for the weights and deviational variables are respected by construction: the CCR model ensures nonnegative weights, and the deviational variables are explicitly constrained to be nonnegative in the goal programming formulation. □

Theorem 2. The optimal value of model (23) is bounded below, and the weights u_r and v_i are bounded above under the assumption that every input and output value is greater than zero.

Proof. Since the model minimizes the sum of nonnegative deviational variables and each variable in the model is restricted to be greater than or equal to zero, the objective function is clearly bounded below by zero. Now assume, for contradiction, that the weights u_r or v_i are unbounded above. Then, due to the positive input and output data, the expressions $\sum u_r y_{rj}$ or $\sum v_i x_{ij}$ could become arbitrarily large, violating the equality and inequality constraints of the model — particularly, those requiring sums to equal fixed finite values like 1. Thus, any increase in weights must be offset by changes in deviation variables, which increases the objective function value, preventing unbounded solutions from being optimal. Therefore, the optimal value of the model is bounded. □

4 Numerical example

To illustrate the applicability and effectiveness of the developed common-weight DEA model (23) based on MOLFP and goal programming, we consider a well-known dataset from Kao and Hung [7], originally from Kao and Yang [8], which assesses the performance of 17 forest districts across Taiwan. Each forest district is defined by four input variables, budget (x_1), initial stocking (x_2), labor (x_3), and land (x_4) and three output variables, main product (y_1), soil conservation (y_2), and recreation (y_3). The data for the 17 forest districts is given in Table 1 below:

Table 2 shows the common-weights given to each input and output used in our model. These weights represent how important each input (like budget, labor, land) and output (like main product, soil conservation, recreation) is when calculating the efficiency of the different forest districts. A special feature of our model is that it gives a nonzero weight to every input and output. This is different from many other models using common-weights, where sometimes some weights become zero. When a weight is zero, it means that particular input or output is ignored completely in the analysis, which can give an incomplete or unfair result.

Table 1: Input and Output Data for 17 Forest Districts

DMU	X_1 **(dollars)**	X_2 $(\mathbf{m}^3)$	X_3 **(persons)**	X_4 **(ha)**	Y_1 $(\mathbf{m}^3)$	Y_2 $(\mathbf{m}^3)$	Y_3 **(visits)**
1	51.62	11.23	49.22	33.52	40.49	14.89	166.71
2	85.78	123.98	55.13	108.46	43.51	173.93	6.45
3	66.65	104.18	257.09	13.65	139.74	115.96	0.00
4	27.87	107.60	14.00	146.43	25.47	131.79	0.00
5	51.28	117.51	32.07	84.50	46.20	144.99	0.00
6	36.05	193.32	59.52	8.23	46.88	190.77	822.92
7	25.83	105.80	9.51	227.20	19.40	120.09	0.00
8	123.02	82.44	87.35	98.80	43.33	125.84	404.69
9	61.95	99.77	33.00	86.37	45.43	79.60	52.62
10	80.33	104.65	53.30	79.06	27.28	132.49	42.67
11	205.92	183.49	144.16	59.66	14.09	196.29	16.15
12	82.09	104.94	46.51	127.28	44.87	108.53	0.00
13	202.21	187.74	149.39	93.65	44.97	184.77	0.00
14	67.55	82.83	44.37	60.85	26.04	85.00	23.95
15	72.60	132.73	44.67	173.48	5.55	135.65	24.13
16	84.83	104.28	159.12	171.11	11.53	110.22	49.09
17	71.77	88.16	69.19	123.14	44.83	74.54	6.14

By making sure that every factor has some weight, our model ensures that all aspects of performance are considered. This makes the evaluation more balanced and realistic. The weights assigned by the proposed model highlight the relative importance of each factor in shaping overall efficiency. For example, V_2 (Initial stocking) receives a higher weight of 0.005975, showing that this input strongly influences the efficiency of the 17 forest districts and plays a key role in distinguishing performance among them. In contrast, V_3 (Labor input) has a much smaller weight of 0.000832, suggesting that variations in labor have little impact on overall efficiency in this dataset. These differences illustrate the model's ability to identify which inputs and outputs most significantly affect efficiency, offering useful insight for decision-makers about where resources or focus can have the greatest effect. By looking at these weight patterns, practitioners can gain clearer insight into the practical implications of the evaluation and see which factors most strongly influence performance.

Table 2: Weights of inputs and outputs using proposed model

Weight	U_1	U_2	U_3	V_1	V_2	V_3	V_4
Value	0.002058	0.006144	0.000032	0.002091	0.005975	0.000832	0.001772

4.1 Comparison with satisfaction degree model

To evaluate the performance of the proposed common-weight model (23), in Table 3, we compare the efficiency scores and satisfaction degrees with those obtained using the iterative satisfaction degree-based approach [22].

Table 3: Efficiency scores comparison

DMU	Proposed Model	Rank	Satisfaction Degree Model	Rank
1	0.9793	5	1.0000	1
2	1.0000	1	0.9999	3
3	0.9997	3	0.6979	12
4	0.8867	8	0.9583	4
5	1.0000	1	1.0000	1
6	0.9898	4	0.9004	5
7	0.7093	16	0.8197	8
8	0.8743	9	0.8609	7
9	0.6711	17	0.7099	11
10	0.9479	7	0.8764	6
11	0.7543	15	0.6524	14
12	0.8614	10	0.7388	9
13	0.8368	13	0.6258	15
14	0.9547	6	0.7233	10
15	0.8598	11	0.6888	13
16	0.7640	14	0.5194	17
17	0.8405	12	0.5736	16

From the table, it is observed that the proposed model identifies two DMUs (DMU 2 and DMU 5) as fully efficient with scores slightly above or equal to 1.000, in close agreement with the satisfaction degree-based model. For the majority of the DMUs, our model delivers efficiency scores that are either comparable or significantly higher. For instance, DMU 3 receives a high score of 0.9997 from our model, while it was evaluated at only 0.6979 in the satisfaction model. Similarly, DMUs 13, 16, and 17 also see considerable improvement.

Examining Table 3 shows the effect of near-efficient DMUs under the proposed model. DMU 3 (0.9997), DMU 1 (0.9793), and DMU 6 (0.9898) are all close to the frontier, yet their ranks shift slightly compared to other methods. These small differences, typically within a few positions, reflect the model's enhanced discrimination among nearly identical units. Importantly, this effect influences ranking rather than the absolute efficiency classification, indicating that the magnitude is modest and does not compromise evaluation reliability.

To further evaluate the alignment between the models, satisfaction degrees derived from each are also compared in Table 4. The satisfaction degree reflects how closely a DMU's efficiency aligns with the ideal value (typically 1), interpreted through a satisfaction function [22].

Table 4: Satisfaction degree comparison

DMU	**Proposed Model**	**Satisfaction Model**
1	0.9793	1.0000
2	1.0000	0.9999
3	0.9997	0.6979
4	0.8867	0.9583
5	1.0000	1.0000
6	0.9898	0.8989
7	0.7093	0.8197
8	0.8743	0.8584
9	0.6711	0.6979
10	0.9479	0.9320
11	0.7543	0.6979
12	0.8614	0.8912
13	0.8368	0.7826
14	0.9547	0.9353
15	0.8598	0.9030
16	0.7640	0.6979
17	0.8405	0.8346

To statistically assess the difference in satisfaction degrees between our proposed model and the existing satisfaction degree-based model, the Wilcoxon signed-rank test was used. Pairs of ordinal or nonnormally distributed data can be compared using this nonparametric test.

The test yielded a test statistic $W = 89.000$ with a corresponding z-value of 0.592 and p-value $= 0.579$. Because the p-value is greater than the standard 0.05 threshold, we do not reject the null hypothesis, suggesting that there is insufficient statistical evidence to support a difference in satisfaction levels between the two models.

We compare the time complexity of our proposed model with that of the satisfaction degree-based model from previous studies. Both models aim to solve a linear programming (LP) problem, but they differ in their structure and the number of iterations required to achieve a satisfactory result.

Our model is formulated with a total of $4n + m + s$ variables and $7n + m + s$ constraints. It is solved as a single LP problem, meaning all the variables and constraints are processed simultaneously without the need for iterative procedures. This characteristic makes our model more computationally efficient, especially as the number of DMUs increases.

In contrast, the satisfaction degree-based model proposed in previous studies involves $3n + m + s + 1$ variables and $6n + m + s + 1$ constraints per iteration. A key difference is that this model requires multiple iterations to achieve a high level of precision. According to the authors of the satisfaction model, 14 iterations are required to reduce the approximation error to less than 0.0001. While this iterative process allows the model to fine-tune its results, it also increases computational time, making it less efficient for large-scale problems.

To compare the computational complexity of the two models, we analyze the overall time complexity involved in solving an LP problem. The computational effort required by algorithms such as the interior-point method or ellipsoid method is approximately

$$O\big((n+m)^3\big),$$

where n is the number of variables and m is the number of constraints. This cubic complexity arises due to the inherent structure of the LP problem and the algorithms used to solve it.

Assuming we denote by $v = m + s$ the total count of input and output variables, the computational load of each model can be expressed in closed form. Our proposed approach requires solving a single LP with $4n + v$ variables and $7n + v$ constraints, yielding a worst-case time complexity of

$$T_{\text{proposed}} = O\big((4n + v + 7n + v)^3\big) = O\big((11n + 2v)^3\big).$$

By contrast, the satisfaction degree model must solve $t = 14$ iterations of an LP that, in each pass, has approximately $3n + v$ variables and $6n + v$ constraints. Its per-iteration cost is

$$O\big((3n + v + 6n + v)^3\big) = O\big((9n + 2v)^3\big),$$

so that cumulatively:

$$T_{\text{iterative}} = 14 \times O\big((9n + 2v)^3\big).$$

Since $(11n + 2v)^3 < 14\,(9n + 2v)^3$ for all $n, v > 0$, our single-LP formulation always requires less total computation than 14 iterations of the satisfaction degree model. Indeed, one finds that if the iterative scheme needed as few as follows:

$$t > \left(\frac{11n + 2v}{9n + 2v}\right)^3 \approx 1.82$$

iterations (i.e., $t \geq 2$), the cumulative cost of the iterative model would exceed that of our method even after just two iterations.

This highlights the clear efficiency advantage of our proposed model, especially for medium- to large-scale DEA applications. In conclusion, our model provides a clear advantage by being faster and simpler to compute, while still delivering similar satisfaction levels as the satisfaction-based model. The satisfaction-based model requires multiple iterations to reach the desired accuracy, whereas our model achieves the same results in just one step. This makes our model more efficient and practical, especially for larger or time-sensitive problems. By reducing the complexity without sacrificing performance, our approach offers a more efficient solution for real-world applications.

4.2 Comparison with distance-based model

In this section, we further compare the proposed model with the results obtained from two alternative models found in the literature [19]. The models were developed to evaluate the efficiency of the same set of DMUs as in our study. Table 5 summarizes the efficiency scores obtained from both models and their respective rankings.

Table 5: Efficiency score comparison with distance-based model

DMU	Model 1	Model 2	Proposed Model
1	1.000	1.000	0.9793
2	1.000	1.000	1.0000
3	0.925	1.000	0.9997
4	1.000	1.000	0.8867
5	1.000	1.000	1.0000
6	1.000	0.966	0.9898
7	0.863	0.874	0.7093
8	0.834	0.847	0.8743
9	0.676	0.678	0.6711
10	0.883	0.878	0.9479
11	0.668	0.653	0.7543
12	0.702	0.717	0.8614
13	0.625	0.623	0.8368
14	0.710	0.712	0.9547
15	0.732	0.722	0.8598
16	0.666	0.670	0.7640
17	0.567	0.592	0.8405

In our model, only the best-performing DMUs are marked as efficient. Specifically, DMUs 2 and 5 are considered the most efficient. On the other hand, in the models used for comparison, a larger number of DMUs are identified as efficient, with five DMUs being marked as fully efficient.

While having more efficient DMUs might seem beneficial, it can create confusion and make it harder to clearly distinguish between the best-performing units and the others. By limiting the efficient DMUs to just those that truly perform well, our model focuses on the top performers, which helps to identify where excellence lies.

This method has the advantage of being clearer, as it avoids labeling too many units as efficient, which could reduce the importance of those that genuinely stand out. When fewer DMUs are marked as efficient, it makes it easier to focus on areas that need improvement and allows others to learn from the best. In simple terms, our model is more selective, making the results more meaningful and useful for decision-making. It highlights the best performers, making the analysis clearer and more actionable for anyone looking to improve efficiency.

4.3 Comparison with credibility-based common-weight model

In recent literature, many common-weight DEA models are developed for decision-making problems with uncertain data. Therefore, along with the comparison with satisfaction degree-based and distance-based models, we also compare the proposed method with a recent credibility-based common-weight DEA model, namely, Omrani, Fahimi, and Emrouznejad [15].

Although the proposed model is formulated for crisp data, this comparison is included to examine how the proposed compact common-weight structure performs when incorporated into a recent uncertainty-based framework. For this purpose, all credibility calculations and uncertainty modeling steps of [15] are retained, and only the final common-weight model is replaced by the proposed common-weight formulation.

For comparison, we use the numerical example of Omrani, Fahimi, and Emrouznejad [15], which evaluates the efficiency of 14 Iranian airlines under fuzzy conditions. The results of the comparison between Omrani's common-weight model (for $\lambda = 0.8$) and the proposed compact CSW model are given in Table 6.

In the results of Omrani et al., the efficiency score equal to 1 is assigned to more than one DMU, leading to repetition of fully efficient units. This limits the ability of the model to clearly distinguish the best-performing DMUs. In contrast, the proposed model avoids repetition of efficiency value 1 and produces a more spread-out set of efficiency scores. Omrani et al.'s model also shows repetition in the rankings. In particular, DMUs 2, 9, and 14 share the same rank (Rank 4), indicating insufficient discrimination. The proposed model clearly differentiates these DMUs by assigning distinct efficiency scores and ranks.

Table 6: Efficiency scores and ranks comparison with Omrani et al. model

DMU	**Omrani et al. Efficiency**	**Omrani et al. Rank**	**Proposed Model Efficiency**	**Proposed Model Rank**
1	0.866	11	0.8689	10
2	0.946	4	0.9285	8
3	0.938	7	0.9127	9
4	0.849	12	0.9448	6
5	0.967	3	1.0000	1
6	1.000	1	0.9522	4
7	0.769	14	0.9678	2
8	1.000	1	0.9461	5
9	0.946	4	0.7439	13
10	0.874	10	0.8689	11
11	0.906	9	0.9427	7
12	0.838	13	0.6817	14
13	0.946	4	0.8501	12
14	0.941	8	0.9583	3

Overall, the results in Table 6 indicate that replacing the final common-weight stage of Omrani et al.'s method with the proposed compact common-weight model leads to better discrimination without altering the underlying credibility-based efficiency evaluation framework.

5 Conclusion

This study introduced a noniterative common-weight DEA model capable of generating efficiency scores in a single computational step. By avoiding iterative procedures, the model enhances computational efficiency, especially when working with large number of inputs, outputs, and DMUs. Additionally, by detecting fewer perfectly efficient units, it increases the analysis's discriminatory power, thus enabling a more precise ranking of performance.

The results produced by the proposed model are comparable to those of existing methods but require less computational time and effort, making it a practical choice for real-world applications where resources are limited.

Although some DMUs achieve scores very close to one, the uniform evaluation framework of the proposed model highlights small differences among these near-efficient units, resulting in small shifts in their ranking. This behavior reflects the model's ability to provide finer discrimination. At the same time, the study has been conducted on selected datasets, and future research could examine the model's performance across other sectors and contexts. While the present study shows that the proposed model can be used within uncertainty-based DEA frameworks at the common-weight stage, a full extension to directly handle fuzzy data, as well as the inclusion of weight restrictions and decision-maker preferences, can be taken up in future research.

Overall, the proposed model offers a fast, straightforward, and effective alternative for common-weight DEA, contributing to improved performance evaluation and supporting future methodological developments in the field.

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Conflicts of Interest

The authors declare no conflicts of interest.

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