Iranian Journal of Numerical Analysis and Optimization
Vol. 16, No. 1, 2026, pp 220–242
https://doi.org/10.22067/ijnao.2025.94069.1670
https://ijnao.um.ac.ir/

Research Article

An improved search direction based on algebraic equivalent transformation technique for convex quadratic optimization

A. Kraria*, B. Merikhi and D. Benterki

Abstract

This work presents an improved interior point algorithm with full Newton step for convex quadratic optimization. Based on the technique of algebraic equivalent transformation, we first propose a new search direction for convex quadratic optimization with the aim of improving the algorithmic complexity of the proposed algorithm. We then perform a complete theoretical study of convergence and complexity, proving that our algorithm is well-defined, converge quadratically and achieves the best known polynomial complexity bounds established for primal-dual interior point methods. Following this, we conduct comparative numerical tests to evaluate the efficiency of the algorithm. The theoretical and numerical results are encouraging and clearly confirm our purpose.

*Corresponding author

Received 15 June 2025; revised 21 October 2025; accepted 22 November 2025

Aicha Kraria
Laboratory of Fundamental and Numerical Mathematics, Department of Mathematics, Faculty of Sciences, Setif 1 University-Ferhat Abbas, Setif, 19000, Algeria. e-mail: aicha.kraria@univ-setif.dz

Bachir Merikhi
Laboratory of Fundamental and Numerical Mathematics, Department of Mathematics, Faculty of Sciences, Setif 1 University-Ferhat Abbas, Setif, 19000, Algeria. e-mail: bmerikhi@univ-setif.dz

Djamel Benterki
Laboratory of Fundamental and Numerical Mathematics, Department of Mathematics, Faculty of Sciences, Setif 1 University-Ferhat Abbas, Setif, 19000, Algeria. e-mail: djbenterki@univ-setif.dz

How to cite this article
Kraria, A. Merikhi, B. and Benterki, D., An improved search direction based on algebraic equivalent transformation technique for convex quadratic optimization. *Iran. J. Numer. Anal. Optim.*, 2026; 16(1): 220-242. https://doi.org/10.22067/ijnao.2025.94069.1670

AMS subject classifications (2020): 90C51; 90C20.

Keywords: Interior point algorithm; Convex quadratic programming; Algebraic transformation; Search direction.

1 Introduction

Since the 1980s, interior point methods have seen great development and diffusion in the solution of optimization problems, and primarily for the solution of convex problems, such as linear, convex quadratic, and convex nonlinear programming, due to their theoretical and practical effectiveness. However, for nonconvex problems, several approaches have been developed, such as interior point algorithm for nonconvex optimization (see, e.g., [9, 22, 28]) and difference of convex functions algorithm (DCA) (see, e.g., [1, 20, 25]).

In this work, we are interested in solving a convex quadratic optimization (CQO) problem, using interior point methods.

One of the crucial steps in these methods is calculating the search direction. Several techniques have been developed for this purpose. One is based on the so-called self-regular and kernel barrier functions, which requires two types of iteration: inner iterations and outer iterations, and for each inner iteration an appropriate step size is calculated (see, e.g., [5, 6, 8, 21]). Another technique is based on the so-called algebraic equivalent transformation (AET). Recently, Darvay [10, 11] introduced a new interior point algorithm for linear optimization (LO) based on the AET technique, for the first time. The main idea of this technique is to transform the nonlinear equation in the system that defines the central path by $\psi\left(\frac{xs}{\mu}\right) = \psi(e)$, where ψ is a continuously differentiable and invertible function, then apply Newton's method to find new search directions. The advantage of this technique is that the algorithm uses a full Newton step size at each iteration. Later on, Achache [2] generalized this technique to CQO, while Zhang, Bai, and Wang [33] generalized it to linearly constrained convex optimization. Due to the effectiveness of this technique, Several other works have applied this idea to other optimization problems (e.g., [3, 4, 12, 13, 15, 17, 18, 23, 26, 29].

Another transformation was introduced by Darvay and Takács [14] based on applying the function $\psi(t) = t^2$ to both sides of the equation $\frac{xs}{\mu} = \sqrt{\frac{xs}{\mu}}$ instead of $\frac{xs}{\mu} = e$, which resulted in a new search direction for LO. Later, Zhang et al. [34] extended this work to $P_*(\kappa)$-linear complementarity problem. In [19], Kheirfam and Nasrollahi introduced a new interior point algorithm for LO based on the above AET technique using the parametric function $\psi(t) = t^q$, $q \geq 2$. Subsequently, Zaoui et al. [30, 32] proposed the new functions $\psi(t) = t^{\frac{3}{2}}$ and

$\psi(t) = t^{\frac{7}{4}}$, respectively, for LO, and it was proved that the algorithm using this first function is better than the other one. Recently, Zaoui, Benterki, and Khelladi [31] extended the algorithm given by Darvay and Takács [14] to CQO.

Inspired by the previous works, we first present an interior point algorithm for CQO based on a new function $\psi(t) = t^{\frac{5}{4}}$, to enhance the performance of primal-dual algorithms. Next, we prove that this algorithm is well-defined and has a polynomial complexity. Finally, we perform comparative numerical tests between our approach and that of Zaoui, Benterki, and Khelladi [31], to confirm our theoretical results and evaluate the effectiveness of our algorithm. Our aim is to propose, develop, and analyze an interior point algorithm based on a new search direction for CQO.

This paper is divided into six sections. In Section 2, we introduce some notions related to interior point methods for CQO problems. In Section 3, we focus on calculating the search direction using the AET technique, and present the corresponding algorithm. Section 4 is dedicated to analyzing the algorithm's convergence and complexity. In Section 5, we perform comparative numerical tests to show the efficiency of our algorithm. Finally, a general conclusion is given in Section 6.

2 Preliminaries

We consider the standard CQO problem (P) and its Lagrangian dual (D)

$$\min\Big\{\frac{1}{2}x^TQx+c^Tx : Ax = b, x \geq 0\Big\}, \tag{P}$$

$$\max\Big\{-\frac{1}{2}x^TQx + b^Ty : A^Ty + s = Qx + c, s \geq 0\Big\}, \tag{D}$$

where $Q \in \mathbb{R}^{n\times n}$ is a positive semidefinite symmetric matrix, $A \in \mathbb{R}^{m\times n}$ is a full rank matrix, $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$.

We call both problems (P) and (D) the primal-dual problem. Throughout this paper, we assume that the set of strictly feasible solutions $\mathcal{F}^0$ of the primal-dual problem is nonempty, where

$$\mathcal{F}^0 = \{(x, y, s) \in \mathbb{R}^{n+m+n} : Ax = b, A^Ty + s = Qx + c, x > 0, s > 0\}.$$

Solving the primal-dual problem is equivalent to solve the following nonlinear system given by Karush–Kuhn–Tucker conditions

$$\begin{aligned} Ax = b, &\quad x \geq 0, \\ -Qx + A^T y + s = c, &\quad s \geq 0, \\ xs = 0. & \end{aligned} \tag{1}$$

In primal-dual interior point method, the equation $xs = 0$ is replaced by the parametrized equation $xs = \mu e$, where $\mu > 0$ and $e = (1, 1, \ldots, 1)^T \in \mathbb{R}^n$. This gives the following system

$$\begin{aligned} Ax = b, &\quad x > 0, \\ -Qx + A^T y + s = c, &\quad s > 0, \\ xs = \mu e, & \end{aligned} \tag{2}$$

which is equivalent to the system given in (3);

$$\begin{aligned} Ax = b, &\quad x > 0, \\ -Qx + A^T y + s = c, &\quad s > 0, \\ \frac{xs}{\mu} = \sqrt{\frac{xs}{\mu}}, & \end{aligned} \tag{3}$$

with $\frac{xs}{\mu} = \left(\frac{x_1 s_1}{\mu}, \frac{x_2 s_2}{\mu}, \ldots, \frac{x_n s_n}{\mu}\right)^T$ and $\sqrt{\frac{xs}{\mu}} = \left(\sqrt{\frac{x_1 s_1}{\mu}}, \sqrt{\frac{x_2 s_2}{\mu}}, \ldots, \sqrt{\frac{x_n s_n}{\mu}}\right)^T$.

It is proved that system (2) has a unique solution $(x(\mu), y(\mu), s(\mu))$, for each $\mu > 0$. The set of all these solutions forms the so-called central path of (P) and (D). If $\mu \to 0$, then the central path converges to the optimal solutions of (P) and (D).

3 New search direction for CQO

Based on the work introduced by Darvay and Takács [14], we write system (3) as follows:

$$\begin{aligned} Ax = b, &\quad x > 0, \\ -Qx + A^T y + s = c, &\quad s > 0, \\ \psi\Big(\frac{xs}{\mu}\Big) = \psi\Big(\sqrt{\frac{xs}{\mu}}\Big), & \end{aligned} \tag{4}$$

where ψ is an invertible continuously differentiable function defined on $(L, +\infty)$, and L is determined by the condition $2t\psi'(t^2) - \psi'(t) > 0$.

Let $(x, y, s) \in \mathcal{F}^0$ be the current iterate. Applying Newton's method to system (4), we find the new iteration $x_+ = x + \Delta x$, $y_+ = y + \Delta y$, $s_+ = s + \Delta s$, where $(\Delta x, \Delta y, \Delta s)$ is the solution of the following system:

$$\begin{aligned} A\Delta x &= 0, \\ -Q\Delta x + A^T\Delta y + \Delta s &= 0, \\ \frac{1}{\mu}(s\Delta x + x\Delta s) &= \frac{\psi\left(\sqrt{\frac{xs}{\mu}}\right) - \psi\left(\frac{xs}{\mu}\right)}{\psi'\left(\frac{xs}{\mu}\right) - \frac{\sqrt{\mu}e}{2\sqrt{xs}}\psi'\left(\sqrt{\frac{xs}{\mu}}\right)}. \end{aligned} \tag{5}$$

We introduce the following notations:

$$v = \sqrt{\frac{xs}{\mu}}, \quad d_x = \frac{v\Delta x}{x}, \quad d_s = \frac{v\Delta s}{s}. \tag{6}$$

Hence, system (5) can be written as follows:

$$\begin{aligned} \bar{A}d_x &= 0, \\ -\bar{Q}d_x + \bar{A}^T\Delta_y + d_s &= 0, \\ d_x + d_s &= p_v, \end{aligned} \tag{7}$$

with

$$p_v = \frac{2\psi(v) - 2\psi(v^2)}{2v\psi'(v^2) - \psi'(v)}, \quad \bar{A} = \frac{1}{\mu}Adiag\left(\frac{x}{v}\right), \quad \bar{Q} = \frac{1}{\mu}diag\left(\frac{x}{v}\right)Qdiag\left(\frac{x}{v}\right),$$

and $diag\left(\frac{x}{v}\right)$ is the $n \times n$ diagonal matrix with diagonal entries $\frac{x_i}{v_i}$.

In this paper, we consider a new function $\psi : \left(\frac{1}{\sqrt[5]{16}}, +\infty\right) \to \mathbb{R}$, $\psi(t) = t^{\frac{5}{4}}$. Then we have

$$p_v = \frac{8v - 8v^{\frac{9}{4}}}{10v^{\frac{5}{4}} - 5e}. \tag{8}$$

At each iteration, we define a proximity measure

$$\delta(x, s, \mu) = \frac{\|p_v\|}{2} = \left\|\frac{4v - 4v^{\frac{9}{4}}}{10v^{\frac{5}{4}} - 5e}\right\|, \tag{9}$$

where $\|\cdot\|$ represents the Euclidean norm.

Remark 1. Let $(d_x, \Delta y, d_s)$ be a solution of system (7). Then

$$d_x^T d_s = d_x^T\bar{Q}d_x = \frac{1}{\mu}d_x^T diag\left(\frac{x}{v}\right)Qdiag\left(\frac{x}{v}\right)d_x \geq 0, \tag{10}$$

because Q is a positive semidefinite matrix.

We detail below the algorithm steps for this approach.

Algorithm 1 Primal-dual algorithm for CQO

Input
An accuracy parameter $\epsilon > 0$;
An update parameter $0 < \theta < 1$ (default $\theta = \frac{1}{7\sqrt{n}}$);
A barrier parameter $\mu^0 = \frac{(x^0)^T s^0}{n}$;
An initial point $(x^0, y^0, s^0) \in \mathcal{F}^0$, such that $\delta(x^0, s^0; \mu^0) < \beta = \frac{1}{4}$;
begin
$x := x^0$; $y := y^0$; $s := s^0$; $\mu := \mu^0$; $k := 0$;
While $x^T s > \epsilon$ **do**
$\mu := (1-\theta)\mu$;
Compute $(\Delta x, \Delta y, \Delta s)$ from system (5), then put
$x := x + \Delta x$; $y := y + \Delta y$; $s := s + \Delta s$; $k := k + 1$;
end While
end.

In order to facilitate the analysis of Algorithm 1, we introduce the following notation: $q_v = d_x - d_s$. From (7), we have

$$d_x = \frac{p_v + q_v}{2}, \qquad d_s = \frac{p_v - q_v}{2}, \tag{11}$$

and

$$\|p_v\|^2 = \|d_x\|^2 + \|d_s\|^2 + 2d_x^T d_s = \|q_v\|^2 + 4d_x^T d_s. \tag{12}$$

Using (10) and (12), we deduce that

$$\|p_v\|^2 \geq \|q_v\|^2. \tag{13}$$

Remark 2. In contrast to the case of CQO, in the LO case, the previous relation (13) becomes $\|p_v\|^2 = \|q_v\|^2$.

4 Analysis of convergence and complexity

In this section, we present a comprehensive theoretical study of the convergence and polynomial complexity of Algorithm 1 that we have conducted.

In the first lemma, we prove the strict feasibility of the iterates.

Lemma 1. Let $(x, y, s) \in \mathcal{F}^0$. The new iteration

$$(x_+, y_+, s_+) = (x, y, s) + (\Delta x, \Delta y, \Delta s)$$

is strictly feasible if $\delta(x, s, \mu) < 1$.

Proof. Let $0 \leq \alpha \leq 1$. We define $x_+(\alpha), s_+(\alpha)$ as follows:

$$x_+(\alpha) = x + \alpha\Delta x, \; s_+(\alpha) = s + \alpha\Delta s.$$

Hence

$$\frac{1}{\mu}x_+(\alpha)s_+(\alpha) = \frac{xs}{\mu} + \frac{\alpha}{\mu}(x\Delta s + s\Delta x) + \frac{\alpha^2}{\mu}\Delta x\Delta s.$$

Using (6), (7) and (11), we obtain

$$\begin{aligned}\frac{1}{\mu}x_+(\alpha)s_+(\alpha) &= v^2 + \alpha v(d_x + d_s) + \alpha^2 d_x d_s \\ &= v^2 + \alpha v p_v + \alpha^2\Big(\frac{p_v^2 - q_v^2}{4}\Big) \\ &= (1-\alpha)v^2 + \alpha(v^2 + vp_v) + \alpha^2\Big(\frac{p_v^2 - q_v^2}{4}\Big).\end{aligned} \tag{14}$$

We calculate the expression of $v^2 + vp_v$ by using (8) as follows:

$$v^2 + vp_v = \frac{2v^{\frac{13}{4}} + 3v^2}{10v^{\frac{5}{4}} - 5e} = e + \frac{2v^{\frac{13}{4}} + 3v^2 - 10v^{\frac{5}{4}} + 5e}{10v^{\frac{5}{4}} - 5e}. \tag{15}$$

After studying the function $\dfrac{2t^{\frac{13}{4}} + 3t^2}{10t^{\frac{5}{4}} - 5}$ for all $t > \frac{1}{\sqrt[5]{16}}$, we find that $\dfrac{2t^{\frac{13}{4}} + 3t^2}{10t^{\frac{5}{4}} - 5} \geq 1$. This is equivalent to

$$v^2 + vp_v \geq e. \tag{16}$$

From (14) and (16), we get

$$\begin{aligned}\frac{1}{\mu}x_+(\alpha)s_+(\alpha) &\geq (1-\alpha)v^2 + \alpha e + \alpha^2\Big(\frac{p_v^2 - q_v^2}{4}\Big) \\ &\geq (1-\alpha)v^2 + \alpha\bigg(e - \Big((1-\alpha)\frac{p_v^2}{4} + \alpha\frac{q_v^2}{4}\Big)\bigg).\end{aligned}$$

We have

$$\left\|(1-\alpha)\frac{p_v^2}{4} + \alpha\frac{q_v^2}{4}\right\|_\infty \leq (1-\alpha)\frac{\|p_v^2\|_\infty}{4} + \alpha\frac{\|q_v^2\|_\infty}{4}$$

$$\leq (1-\alpha)\frac{\|p_v\|^2}{4} + \alpha\frac{\|q_v\|^2}{4},$$

by using (13), we obtain

$$\left\|(1-\alpha)\frac{p_v^2}{4} + \alpha\frac{q_v^2}{4}\right\|_\infty \leq (1-\alpha)\frac{\|p_v\|^2}{4} + \alpha\frac{\|p_v\|^2}{4}$$
$$= \frac{\|p_v\|^2}{4} = \delta^2.$$

If $\delta < 1$, then $e - \left((1-\alpha)\frac{p_v^2}{4} + \alpha\frac{q_v^2}{4}\right) > 0$, which implies that

$$x_+(\alpha)s_+(\alpha) > 0.$$

As $x_+(\alpha), s_+(\alpha)$ are continuous functions, they do not change sign on the interval $[0,1]$, and since $x_+(0) = x > 0$, $s_+(0) = s > 0$, this implies that $x_+(1) = x + \Delta x = x_+ > 0$ and $s_+(1) = s + \Delta s = s_+ > 0$, which proves the lemma. □

Lemma 2. Let $v > \frac{1}{\sqrt[5]{16}}e$ and $v_+ = \sqrt{\frac{x_+s_+}{\mu}}$. If $\delta = \delta(x,s,\mu) < \frac{1}{4}$, then

$$v_+ > \frac{1}{\sqrt[5]{16}}e.$$

Proof. By replacing α with 1 in (14), we obtain

$$\frac{x_+s_+}{\mu} = v_+^2 = v^2 + vp_v + \frac{p_v^2}{4} - \frac{q_v^2}{4}, \tag{17}$$

from (16), we find

$$v_+^2 \geq e + \frac{p_v^2}{4} - \frac{q_v^2}{4} \geq e - \frac{q_v^2}{4},$$

which implies that

$$(v_+)_i^2 \geq 1 - \frac{\|q_v\|_\infty^2}{4} \geq 1 - \frac{\|q_v\|^2}{4}.$$

From (13), we deduce that

$$(v_+)_i^2 \geq 1 - \frac{\|p_v\|^2}{4} = 1 - \delta^2, \quad \text{for all } i = 1:n,$$

which implies that

$$\min(v_+) \geq \sqrt{1-\delta^2}. \tag{18}$$

Since $\delta = \delta(x, s, \mu) < \frac{1}{4}$, then

$$\min(v_+) > \frac{1}{\sqrt[5]{16}}. \tag{19}$$

$\square$

Lemma 3. [13]
Let us consider a decreasing function $f :]d, +\infty[\to \mathbb{R}_+^*$, with $d > 0$. If we take the vector $v \in \mathbb{R}_+^n$, where $\min(v) = \min\limits_i(v_i) \geq d$, then

$$\|f(v)(e - v^2)\| \leq f(\min(v))\|e - v^2\| \leq f(d)\|e - v^2\|.$$

In the following lemma, we prove the quadratic convergence of the iterate.

Lemma 4. If $\delta = \delta(x, s, \mu) < \frac{1}{4}$, then

$$\delta(x_+, s_+, \mu) \leq \frac{160(2\sqrt{2} + \sqrt[8]{759375})(-15\sqrt[8]{15} + 4\sqrt{30})}{-200 + 375\sqrt[4]{15}}\delta^2.$$

Proof. Let $v > \frac{1}{\sqrt[5]{16}}e$. From (9), we have

$$\delta(x_+, s_+, \mu) = \frac{\|p_{v_+}\|}{2} = \left\| \frac{4v_+ - 4v_+^{\frac{9}{4}}}{\left(10v_+^{\frac{5}{4}} - 5e\right)(e - v_+^2)}(e - v_+^2) \right\|, v_+ \neq e.$$

We divide the expression of $\delta(x_+, s_+, \mu)$ into two terms:

1. $e - v_+^2$.

2. $f_1(v_+) = \dfrac{4v_+ - 4v_+^{\frac{9}{4}}}{\left(10v_+^{\frac{5}{4}} - 5e\right)(e - v_+^2)}$.

First, we compute the expression $e - v_+^2$ using (14) as follows:

$$\begin{aligned} e - v_+^2 &= e - \frac{x_+ s_+}{\mu} \\ &= e - \left((v^2 + vp_v) + \frac{p_v^2}{4} - \frac{q_v^2}{4}\right), \end{aligned}$$

from (15), we have

$$e - v_+^2 = \frac{q_v^2}{4} - \left(\frac{2v^{\frac{13}{4}} + 3v^2 - 10v^{\frac{5}{4}} + 5e}{10v^{\frac{5}{4}} - 5e} + \frac{p_v^2}{4}\right)$$

$$= \frac{q_v^2}{4} - \frac{p_v^2}{4}\left(\frac{36v^{\frac{9}{2}} - 12v^{\frac{13}{4}} + v^2 - (10v^{\frac{5}{4}} - 5e)^2}{16(v - v^{\frac{9}{4}})^2}\right). \tag{20}$$

It can be shown that $0 \leq \left(\frac{36v^{\frac{9}{2}} - 12v^{\frac{13}{4}} + v^2 - (10v^{\frac{5}{4}} - 5e)^2}{16(v - v^{\frac{9}{4}})^2}\right) \leq 3e$, for all $v > \frac{1}{\sqrt[5]{16}}e, v \neq e$. Therefore

$$\begin{aligned}\|e - v_+^2\| &\leq \frac{\|q_v^2\|}{4} + \left\|\frac{p_v^2}{4}\left(\frac{36v^{\frac{9}{2}} - 12v^{\frac{13}{4}} + v^2 - (10v^{\frac{5}{4}} - 5e)^2}{16(v - v^{\frac{9}{4}})^2}\right)\right\| \\ &\leq \frac{\|q_v\|^2}{4} + 3\frac{\|p_v\|^2}{4},\end{aligned}$$

by using (9) and (13), we get

$$\|e - v_+^2\| \leq \frac{\|p_v\|^2}{4} + 3\frac{\|p_v\|^2}{4} = 4\delta^2. \tag{21}$$

Second, we consider the function $f_1(t) = \dfrac{4t - 4t^{\frac{9}{4}}}{\left(10t^{\frac{5}{4}} - 5\right)(1 - t^2)}$. Since $f_1'(t) \leq 0$ for all $t > \frac{1}{\sqrt[5]{16}}$; $t \neq 1$, the function f_1 is decreasing on $\left]\frac{1}{\sqrt[5]{16}}, +\infty\right[\setminus \{1\}$. By applying Lemma 3, (18), and (21), we obtain

$$\delta(x_+, s_+, \mu) \leq f_1\left(\sqrt{1 - \delta^2}\right)\|e - v_+^2\| \leq 4f_1\left(\sqrt{1 - \delta^2}\right)\delta^2.$$

If $\delta < \frac{1}{4}$, then $f_1\left(\sqrt{1 - \delta^2}\right) \leq f_1\left(\frac{\sqrt{15}}{4}\right)$. Thus

$$\begin{aligned}\delta(x_+, s_+, \mu) &\leq 4f_1\left(\frac{\sqrt{15}}{4}\right)\delta^2 \\ &= \frac{160(2\sqrt{2} + \sqrt[8]{759375})(-15\sqrt[8]{15} + 4\sqrt{30})}{-200 + 375\sqrt[4]{15}}\delta^2 \\ &\approx 2.128\delta^2.\end{aligned}$$

□

In the next lemma, we give an upper bound of the duality gap.

Lemma 5. If $\delta = \delta(x, s, \mu) < \frac{1}{4}$, then the duality gap satisfies the following inequality:

$$x_+^T s_+ \leq \mu\left(n + \frac{3}{16}\right).$$

Proof. Let $v > \frac{1}{\sqrt[5]{16}}e$. We have

$$v_+^2 = \frac{x_+ s_+}{\mu}.$$

Then

$$x_+^T s_+ = \sum_{i=1}^n (x_+)_i (s_+)_i = \mu \sum_{i=1}^n (v_+^2)_i.$$

According to (17), we have

$$x_+^T s_+ = \mu \sum_{i=1}^n \left((v^2 + v p_v)_i + \frac{(p_v^2)_i}{4} - \frac{(q_v^2)_i}{4} \right). \tag{22}$$

Applying (15) yields

$$\begin{aligned} v^2 + v p_v &= e + \frac{2v^{\frac{13}{4}} + 3v^2 - 10v^{\frac{5}{4}} + 5e}{10v^{\frac{5}{4}} - 5e} \\ &= e + \left(\frac{20v^{\frac{9}{2}} + 20v^{\frac{13}{4}} - 15v^2 - (10v^{\frac{5}{4}} - 5e)^2}{16(v - v^{\frac{9}{4}})^2} \right) \frac{p_v^2}{4} \\ &= e + \left(\frac{36v^{\frac{9}{2}} - 12v^{\frac{13}{4}} + v^2 - (10v^{\frac{5}{4}} - 5e)^2}{16(v - v^{\frac{9}{4}})^2} - e \right) \frac{p_v^2}{4}. \end{aligned}$$

As shown in the proof of Lemma 4, $\dfrac{36v^{\frac{9}{2}} - 12v^{\frac{13}{4}} + v^2 - (10v^{\frac{5}{4}} - 5e)^2}{16(v - v^{\frac{9}{4}})^2} \leq 3e$, so

$$v^2 + v p_v \leq e + 2\frac{p_v^2}{4}. \tag{23}$$

By using (22) and (23), we deduce that

$$\begin{aligned} x_+^T s_+ &\leq \mu \sum_{i=1}^n \left(1 + 2\frac{(p_v^2)_i}{4} + \frac{(p_v^2)_i}{4} - \frac{(q_v^2)_i}{4} \right) \\ &= \mu \left(n + 3\frac{\|p_v\|^2}{4} - \frac{\|q_v\|^2}{4} \right) \\ &\leq \mu (n + 3\delta^2). \end{aligned}$$

If $\delta < \frac{1}{4}$, then

$$x_+^T s_+ \leq \mu \left(n + \frac{3}{16} \right).$$

□

The following lemma shows that the algorithm is well defined.

Lemma 6. If $\delta < \frac{1}{4}$ and $\theta = \frac{1}{7\sqrt{n}}$, then

$$\delta(x_+, s_+, \mu_+) < \frac{1}{4}.$$

Proof. We introduce the notation $\bar{v} = \sqrt{\frac{x_+ s_+}{\mu_+}}$. Then, from (19) we have

$$\bar{v} = \frac{1}{\sqrt{1-\theta}} v_+ > \frac{1}{\sqrt[5]{16}} e, \ 0 < \theta < 1.$$

From the definition of δ, we have

$$\begin{aligned}\delta(x_+, s_+, \mu_+) &= \frac{\|p_{\bar{v}}\|}{2} \\ &= \left\| \frac{4\bar{v} - 4\bar{v}^{\frac{9}{4}}}{(10\bar{v}^{\frac{5}{4}} - 5e)(e - \bar{v}^2)} (e - \bar{v}^2) \right\| \\ &= \left\| f_1(\bar{v})(e - \bar{v}^2) \right\|.\end{aligned}$$

In addition, from (18) we have

$$\min(\bar{v}) = \frac{\min(v_+)}{\sqrt{1-\theta}} \geq \frac{\sqrt{1-\delta^2}}{\sqrt{1-\theta}}.$$

If $\delta < \frac{1}{4}$, then

$$\min(\bar{v}) > \frac{\sqrt{15}}{4}.$$

Since f_1 is decreasing (see Lemma 4), we get

$$\begin{aligned}\delta(x_+, s_+, \mu_+) &= \left\| f_1(\bar{v})(e - \bar{v}^2) \right\| \\ &\leq f_1\Big(\frac{\sqrt{15}}{4}\Big) \|e - \bar{v}^2\| \\ &= f_1\Big(\frac{\sqrt{15}}{4}\Big) \left\| e - \frac{v_+^2}{1-\theta} \right\| \\ &\leq \frac{1}{1-\theta} f_1\Big(\frac{\sqrt{15}}{4}\Big) \Big(\|\theta e\| + \|e - v_+^2\| \Big).\end{aligned}$$

By using (21), we obtain

$$\delta(x_+, s_+, \mu_+) \leq \frac{1}{1-\theta} f_1\Big(\frac{\sqrt{15}}{4}\Big) \Big(\theta\sqrt{n} + 4\delta^2\Big).$$

Since $\delta < \frac{1}{4}$, then

$$\delta(x_+, s_+, \mu_+) \leq \frac{1}{1-\theta} f_1\Big(\frac{\sqrt{15}}{4}\Big)\Big(\theta\sqrt{n} + \frac{1}{4}\Big). \tag{24}$$

Suppose that $\theta = \frac{1}{7\sqrt{n}}$. Then

$$1 - \theta \geq \frac{6}{7}, \tag{25}$$

for all $n \geq 1$.

Finally, by using (24) and (25), we deduce that

$$\begin{aligned} \delta(x_+, s_+, \mu_+) &\leq \frac{7}{6} f_1\Big(\frac{\sqrt{15}}{4}\Big)\Big(\frac{1}{7} + \frac{1}{4}\Big) \\ &< \frac{1}{4}. \end{aligned}$$

□

In the last lemma, we give a lower bound on the number of iterations required for optimality.

Lemma 7. Let $(x^0, y^0, s^0) \in \mathcal{F}^0$, let $\mu^{(0)} = \frac{(x^0)^T s^0}{n}$, let $\delta < \frac{1}{4}$, and let $x^{(k+1)}, s^{(k+1)}$ be the vectors obtained after $k+1$ iterations. The inequality $(x^{(k+1)})^T s^{(k+1)} \leq \epsilon$ is satisfied if

$$k \geq \frac{1}{\theta} \ln \frac{\mu^{(0)}(n + \frac{3}{16})}{\epsilon}.$$

Proof. After k iterations, we find $\mu^{(k)} = (1-\theta)^k \mu^{(0)}$. Using this and Lemma 5, we obtain

$$(x^{(k+1)})^T s^{(k+1)} \leq \mu^{(k)}\left(n + \frac{3}{16}\right) = \mu^{(0)}(1-\theta)^k\left(n + \frac{3}{16}\right).$$

Then, the inequality $(x^{(k+1)})^T s^{(k+1)} \leq \epsilon$ is satisfied if

$$\mu^{(0)}(1-\theta)^k\left(n + \frac{3}{16}\right) \leq \epsilon,$$

this is equivalent to

$$\ln(1-\theta)^k + \ln \mu^{(0)}\left(n + \frac{3}{16}\right) \leq \ln \epsilon,$$

that is,

$$-k\ln(1-\theta) \geq \ln \frac{\mu^{(0)}(n + \frac{3}{16})}{\epsilon}.$$

As $-\ln(1-\theta) \geq \theta$, for all $0 < \theta < 1$, so $-k\ln(1-\theta) \geq k\theta$. Thus, the inequality

$$-k\ln(1-\theta) \geq \ln\frac{\mu^{(0)}\left(n+\frac{3}{16}\right)}{\epsilon}$$

is satisfied if

$$k\theta \geq \ln\frac{\mu^{(0)}\left(n+\frac{3}{16}\right)}{\epsilon},$$

hence

$$k \geq \frac{1}{\theta}\ln\frac{\mu^{(0)}\left(n+\frac{3}{16}\right)}{\epsilon}.$$

□

Theorem 1. If $\theta = \frac{1}{7\sqrt{n}}$, then the algorithm gives an approximate solution of the primal-dual problem after at most

$$7\sqrt{n}\ln\frac{\mu^{(0)}\left(n+\frac{3}{16}\right)}{\epsilon}.$$

Proof. Replacing θ with $\frac{1}{7\sqrt{n}}$ in Lemma 7 gives the result. □

5 Comparative numerical experiments

To evaluate the effectiveness of Algorithm 1 using the new function $\psi(t) = t^{\frac{5}{4}}$, we conduct comparative numerical tests between our function and that of Zaoui, Benterki, and Khelladi [31]. We solve five fixed-size examples and two variable-size examples. Moreover, we test some real-examples from Maros-Mészáros test set [24]. The algorithm was implemented in MATLAB on a computer with an Intel i3 processor and 4 GB of RAM, with the following parameters: $\epsilon = 10^{-4}$ and $\theta = 0.5$. In the table of results, we denote by

- Nbr: the number of iterations required for optimality.
- Time: the computation time in seconds.
- $x^T s$: the duality gap at final iteration.

Example 1. [16] Consider

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \end{pmatrix}, \quad Q = \begin{pmatrix} 4 & 2 & 2 & 0 \\ 2 & 4 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad c = \begin{pmatrix} -8 \\ -6 \\ -4 \\ 0 \end{pmatrix}.$$

The initial point is

$$x^0 = \begin{pmatrix} 0.5 & 0.5 & 0.5 & 1 \end{pmatrix}^T, \quad y^0 = \begin{pmatrix} -5 \end{pmatrix}, \quad s^0 = \begin{pmatrix} 1 & 2 & 8 & 5 \end{pmatrix}^T.$$

Example 2. [27] Consider

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \quad Q = \begin{pmatrix} 2 & -2 & 0 & 0 \\ -2 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad c = \begin{pmatrix} -2 \\ -6 \\ 0 \\ 0 \end{pmatrix}.$$

The initial point is

$$x^0 = \begin{pmatrix} 0.5145 & 0.8764 & 0.6090 & 0.7617 \end{pmatrix}^T, \quad y^0 = \begin{pmatrix} -5.4801 & -1.1789 \end{pmatrix}^T,$$
$$s^0 = \begin{pmatrix} 1.5776 & 4.3148 & 5.4801 & 1.1789 \end{pmatrix}^T.$$

Example 3. [8] Consider

$$A = \begin{pmatrix} 1 & 1.2 & 1 & 1.8 & 0 \\ 3 & -1 & 1.5 & -2 & 1 \\ -1 & 2 & -3 & 4 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 9.31 \\ 5.45 \\ 6.60 \end{pmatrix},$$

$$Q = \begin{pmatrix} 20 & 1.2 & 0.5 & 0.5 & -1 \\ 1.2 & 32 & 1 & 1 & 1 \\ 0.5 & 1 & 14 & 1 & 1 \\ 0.5 & 1 & 1 & 15 & 1 \\ -1 & 1 & 1 & 1 & 16 \end{pmatrix}, \quad c = \begin{pmatrix} 1 \\ -1.5 \\ 2 \\ 1.5 \\ 3 \end{pmatrix}.$$

The initial point is

$$x^0 = \begin{pmatrix} 2.4539 & 0.7875 & 1.5838 & 2.4038 & 1.3074 \end{pmatrix}^T,$$
$$y^0 = \begin{pmatrix} 20.5435 & 9.4781 & 4.3927 \end{pmatrix}^T,$$
$$s^0 = \begin{pmatrix} 7.1215 & 7.9763 & 8.3150 & 6.8686 & 7.9750 \end{pmatrix}^T.$$

Example 4. [16] Consider

$$A = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & -4 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 5 \\ 4 \\ -1.5 \end{pmatrix},$$

$$Q = \begin{pmatrix} 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad c = \begin{pmatrix} -1 \\ -3 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

The initial point is

$$\begin{aligned} x^0 &= \begin{pmatrix} 0.3599 & 0.5425 & 0.8142 & 2.0703 & 0.6706 & 2.8197 & 2.2993 \end{pmatrix}^T, \\ y^0 &= \begin{pmatrix} -2.2576 & -1.1007 & -1.5314 \end{pmatrix}^T, \\ s^0 &= \begin{pmatrix} 4.4655 & 1.6272 & 2.6719 & 3.0413 & 2.2577 & 1.1008 & 1.5315 \end{pmatrix}^T. \end{aligned}$$

Example 5. [8] Consider

$$A = \begin{pmatrix} 1 & -1 & 1.9 & 1.25 & 1.2 & 0.4 & -0.7 & 1.06 & 1.5 & 1.05 \\ 1.3 & 1.2 & 0.15 & 2.15 & 1.25 & 1.5 & 0.4 & 1.52 & 1.3 & 1 \\ 1.5 & -1.1 & 3.5 & 1.25 & 1.8 & 2 & 1.95 & 1.2 & 1 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 11.651 \\ 16.672 \\ 21.295 \end{pmatrix},$$

$$Q = \begin{pmatrix} 30 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 21 & 0 & 1 & -1 & 1 & 0 & 1 & 0.5 & 1 \\ 1 & 0 & 15 & -0.5 & -2 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & -0.5 & 30 & 3 & -1 & 1 & -1 & 0.5 & 1 \\ 1 & -1 & -2 & 3 & 27 & 1 & 0.5 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & 1 & 16 & -0.5 & 0.5 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0.5 & -0.5 & 8 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & 1 & 0.5 & 1 & 24 & 1 & 1 \\ 1 & 0.5 & 1 & 0.5 & 1 & 0 & 1 & 1 & 39 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 11 \end{pmatrix}, \quad c = \begin{pmatrix} -0.5 \\ -1 \\ 0 \\ 0 \\ -0.5 \\ 0 \\ 0 \\ -1 \\ -0.5 \\ -1 \end{pmatrix}.$$

The initial point is

$$x^0 = \left(0.9491\ 0.6121\ 1.8477\ 1.8115\ 1.2511\ 2.5211\ 1.5062\ 1.5658\ 0.8207\ 1.1289\right)^T,$$
$$y^0 = \left(4.3800\ 19.9367\ 4.5679\right)^T,$$
$$s^0 = \left(3.8902\ 4.4625\ 3.9788\ 3.6606\ 3.9018\ 3.5561\ 3.8760\ 3.7190\ 3.9138\ 4.3396\right)^T.$$

Table 1 summarizes the numerical results obtained for the problems described above.

Table 1: Numerical results obtained for fixed size examples.

	$\psi(t) = t^{\frac{5}{4}}$			$\psi(t) = t^2$		
Examples	Nbr	Time	$x^T s$	Nbr	Time	$x^T s$
Example 1	18	0.0080	5.28×10^{-5}	20	0.0289	7.12×10^{-5}
Example 2	17	0.0076	8.88×10^{-5}	20	0.0108	5.99×10^{-5}
Example 3	20	0.0082	8.03×10^{-5}	23	0.0164	5.62×10^{-5}
Example 4	18	0.0081	9.60×10^{-5}	21	0.0248	7.16×10^{-5}
Example 5	20	0.0082	6.79×10^{-5}	22	0.0105	9.15×10^{-5}

Example 6. [8] Consider

$$A(i,j) = \begin{cases} 1 & \text{if } (j = i \text{ or } j = i+m), \\ 0 & \text{otherwise.} \end{cases} \quad Q(i,j) = \begin{cases} 1 & \text{if } j = i, \\ 0 & \text{otherwise.} \end{cases}$$

$$c(i) = \begin{cases} -1 & \text{if } i = 1:m, \\ 0 & \text{if } i = m+1:n,\ n = 2m, \end{cases}$$
$$b(i) = 2 \quad \text{for } i = 1:m.$$

The initial point is

$$x^0(i) = 1 \qquad \text{for } i = 1:n,$$
$$y^0(i) = -1 \quad \text{for } i = 1:m,$$
$$s^0(i) = \begin{cases} 1 & \text{if } i = 1:m, \\ 2 & \text{if } i = m+1:n. \end{cases}$$

The results obtained for Example 6 are summarized in the following Table 2.

Example 7. [7] Consider

Table 2: Numerical results obtained for Example 6.

	$\psi(t)=t^{\frac{5}{4}}$			$\psi(t)=t^2$		
Size	Nbr	Time	x^Ts	Nbr	Time	x^Ts
$n=100$	21	0.0454	9.43×10^{-5}	24	0.0673	7.13×10^{-5}
$n=200$	22	0.1284	9.43×10^{-5}	25	0.1304	7.36×10^{-5}
$n=600$	24	1.4369	7.07×10^{-5}	27	1.68704	5.86×10^{-5}
$n=1000$	25	5.9973	5.89×10^{-5}	27	6.6444	9.77×10^{-5}
$n=2000$	26	37.678	5.89×10^{-5}	29	48.034	5.17×10^{-5}
$n=4000$	27	172.71	5.89×10^{-5}	30	249.95	5.32×10^{-5}
$n=6000$	27	585.21	8.84×10^{-5}	30	670.68	7.98×10^{-5}
$n=7000$	28	959.09	5.15×10^{-5}	30	1059.28	9.31×10^{-5}

$$A(1,i)=1 \qquad \text{for all } i=1:n,$$

$$Q(i,j)=\begin{cases}2 & \text{if } \ j=i,\\ 0 & \text{otherwise,}\end{cases}$$

$$c(i)=-1 \qquad \text{for all } \ i=1:n,$$

$$b=1.$$

The results obtained for Example 7 are summarized in Table 3.

Table 3: Numerical results obtained for Example 7.

	$\psi(t)=t^{\frac{5}{4}}$			$\psi(t)=t^2$		
Size	Nbr	Time	x^Ts	Nbr	Time	x^Ts
$n=1000$	15	1.8535	7.00×10^{-5}	17	2.2843	7.62×10^{-5}
$n=2000$	15	13.850	7.01×10^{-5}	17	16.015	7.63×10^{-5}
$n=4000$	15	65.623	7.02×10^{-5}	17	71.749	7.64×10^{-5}
$n=6000$	15	185.75	7.05×10^{-5}	17	226.80	7.69×10^{-5}
$n=8000$	15	479.02	7.04×10^{-5}	17	562.85	7.65×10^{-5}
$n=10000$	15	1127.8	7.08×10^{-5}	17	1211.1	7.67×10^{-5}

We then tested some examples from Maros-Meszaros test set using different values of θ: $\{0.1, 0.3, 0.5\}$ and also its theoretical value $(\frac{1}{7\sqrt{n}})$, which is denoted by θ_{theo} in the table. The results for these examples are summarized in Table 4.

Table 4: Numerical results for some examples from Maros-Mészáros test set

		$\psi(t) = t^{\frac{5}{4}}$			$\psi(t) = t^2$		
Example	θ	Nbr	Time	$x^T s$	Nbr	Time	$x^T s$
Tame	$\theta = 0.1$	95	0.0100	9.04×10^{-5}	95	0.0100	9.11×10^{-5}
	$\theta = 0.3$	28	0.0088	9.79×10^{-5}	29	0.0096	7.89×10^{-5}
	$\theta = 0.5$	15	0.0079	8.04×10^{-5}	17	0.0080	8.72×10^{-5}
	θ_{theo}	94	0.0100	9.04×10^{-5}	94	0.0119	9.10×10^{-5}
Genhs28	$\theta = 0.1$	110	0.0128	9.30×10^{-5}	110	0.0129	9.37×10^{-5}
	$\theta = 0.3$	33	0.0100	8.23×10^{-5}	33	0.0103	9.47×10^{-5}
	$\theta = 0.5$	18	0.0093	5.03×10^{-5}	20	0.0099	6.40×10^{-5}
	θ_{theo}	250	0.0131	9.57×10^{-5}	250	0.0166	9.59×10^{-5}
Hs51	$\theta = 0.1$	103	0.0098	9.73×10^{-5}	103	0.0100	9.80×10^{-5}
	$\theta = 0.3$	31	0.0086	8.40×10^{-5}	29	0.0088	9.66×10^{-5}
	$\theta = 0.5$	17	0.0085	5.03×10^{-5}	19	0.0087	5.95×10^{-5}
	θ_{theo}	164	0.0100	9.94×10^{-5}	94	0.0114	9.97×10^{-5}
Zecevic2	$\theta = 0.1$	95	0.0098	9.04×10^{-5}	95	0.093	9.10×10^{-5}
	$\theta = 0.3$	28	0.0086	9.79×10^{-5}	29	0.0087	7.88×10^{-5}
	$\theta = 0.5$	15	0.0036	8.04×10^{-5}	17	0.0084	8.72×10^{-5}
	θ_{theo}	94	0.0094	9.03×10^{-5}	94	0.0098	9.10×10^{-5}
Lotschd	$\theta = 0.1$	112	0.0138	9.04×10^{-5}	112	0.0139	9.11×10^{-5}
	$\theta = 0.3$	33	0.0105	9.88×10^{-5}	34	0.0110	7.95×10^{-5}
	$\theta = 0.5$	18	0.0100	6.03×10^{-5}	20	0.0101	7.44×10^{-5}
	θ_{theo}	278	0.0202	9.88×10^{-5}	278	0.0193	9.89×10^{-5}

Notes

The numerical tests we have performed show that our new function $\psi = t^{\frac{5}{4}}$ is more efficient than the one proposed in [31]. In all examples, we record a reduction in the number of iterations and in the computation time for our function $\psi = t^{\frac{5}{4}}$ compared with $\psi = t^2$. This shows that the algorithm using our function is the fastest.

6 General conclusion

We are interested in solving a CQO problem using a primal-dual interior point method based on the AET technique. We proposed a new function ψ, and conducted a complete theoretical study on the convergence and complexity of the proposed algorithm. We then performed numerical experiments comparing our results with those of Zaoui, Benterki, and Khelladi [31]. The results obtained using our function are encouraging and reinforce the effectiveness of our algorithm.

Future research might aim to develop other functions with powers between $\frac{1}{2}$ and 1 in order to improve the algorithmic complexity and numerical behavior of primal-dual interior point methods for convex quadratic programming. In addition, the algorithm could be extended to other optimization problems, such as nonlinear programming problems, semidefinite problems, and complementarity problems.

Acknowledgements

The authors are very grateful and would like to thank the Editor-in-Chief and the anonymous referees for their comments and suggestions which helped to improve the presentation of this paper.

Funding

This research received no external funding.

Conflicts of Interest

All authors jointly worked on the results and they read and approved the final manuscript, and also declare that they have no competing interests.

References

[1] Abbaszadehpeivasti, H., de Klerk, E. and Zamani, M. *On the rate of convergence of the difference-of-convex algorithm (DCA)*, J. Optim. Theory. Appl., 202 (2024), 475–496.

[2] Achache, M. *A new primal-dual path-following method for convex quadratic programming*, Comput. Appl. Math., 25(1) (2006), 97–110.

[3] Asadi, S. and Mansouri, H. *A new full-Newton step $O(n)$ infeasible interior-point algorithm for $\mathcal{P}_*(\kappa)$-horizontal linear complementarity problems*, Comput. Sci. J. Mold., 64 (2014), 37–61.

[4] Asadi, S., Mansouri, H. and Darvay, Z. *An infeasible full-NT step IPM for horizontal linear complementarity problem over Cartesian product of symmetric cones*, Optim., 66(2) (2016), 225–250.

[5] Bai, Y.Q., El Ghami, M. and Roos, C. *A new efficient large-update primal-dual interior-point method based on a finite barrier*, SIAM J. Optim., 13(3) (2002), 766–782.

[6] Bouafia, M., Benterki, D. and Yassine, A. *Complexity analysis of interior point methods for linear programming based on a parameterized kernel function*, RAIRO Oper. Res., 50(4-5) (2016), 935–949.

[7] Boudjellal, N. and Benterki, D. *A new full-Newton step feasible interior point method for convex quadratic programming*, Optimization, 73 (2024), 1571–1588.

[8] Boudjellal, N., Roumili, H. and Benterki, D. *Complexity analysis of interior point methods for convex quadratic programming based on a parameterized kernel function*, Bol. Soc. Parana. Mat., 40 (2022), 1–16.

[9] Chinchilla, R., Yang, G. and Hespanha, J.P. *Newton and interior-point methods for (constrained) nonconvex–nonconcave minmax optimization with stability and instability guarantees*, Math. Control Signals Syst., 36 (2024), 381–421.

[10] Darvay, Z. *A new algorithm for solving self-dual linear optimization problems*, Studia Univ. Babeș-Bolyai Informatica, 47 (2002), 15–26.

[11] Darvay, Z. *New interior point algorithms in linear programming*, Adv. Model. Optim., 5(1) (2003), 51–92.

[12] Darvay, Z., Papp, I.M. and Takács, P.R. *An infeasible full-Newton step algorithm for linear optimization with one centering step in major iteration*, Studia Univ. Babes-Bolyai Informatica, 59 (1) (2014), 28–45.

[13] Darvay, Z., Papp, I. M. and Takács P. R. *Complexity analysis of a full-Newton step interior-point method for linear optimization*, Period. Math. Hungar., 73 (2016), 27–42.

[14] Darvay, Z. and Takács, P. R. *New method for determining search directions for interior-point algorithms in linear optimization*, Optim. Lett., 12 (2018), 1099–1116.

[15] Grimes, W. and Achache, M. *A path-following interior-point algorithm for monotone LCP based on a modified Newton search direction*, RAIRO Oper. Res., 57(3) (2023), 1059–1073.

[16] Hock, W. *Test examples for nonlinear programming codes*. Springer Berlin, Heidelberg, 1981.

[17] Kheirfam, B. *A new infeasible interior-point method based on Darvay's technique for symmetric optimization*, Ann. Oper. Res., 211 (2013), 209–224.

[18] Kheirfam, B. *New complexity analysis of a full Nesterov–Todd step interior-point method for semidefinite optimization*, Asian-Eur. J. Math., 10(4) (2017), 1750070.

[19] Kheirfam, B. and Nasrollahi, A. *An extension for identifying search directions for interior-point methods in linear optimization*, Asian-Eur. J. Math., 13 (2020), 2050014.

[20] LeThi, H.A., Le, H.M., Phan, D.N. and Tran, B. *Novel DCA based algorithms for a special class of nonconvex problems with application in machine learning*, Appl. Math. Comput., 409 (2021), 125904.

[21] Li, X. and Zhang, M. *Interior-point algorithm for linear optimization based on a new trigonometric kernel function*, Oper. Res. Lett., 43(5) (2015), 471–475.

[22] Lukšan, L., Matonoha. C. and Vlček, J. *Interior-point method for non-linear non-convex optimization*, Numerical linear algebra with applications, 11(5-6) (2004), 431–453.

[23] Mansouri, H. and Pirhaji, M. *A polynomial interior-point algorithm for monotone linear complementarity problems*, J. Optim. Theory Appl., 157 (2013), 451–461.

[24] Maros, I. and Mészáros, C. *A repository of convex quadratic programming problems*, Optimization methods and software, 11(1-4) (1999), 671–681.

[25] Phan, D.N. and Thi, H.A. *Difference-of-convex algorithm with extrapolation for nonconvex, nonsmooth optimization problems*, Math. Oper. Res., 49(3) (2024), 1973–1985.

[26] Roumili, H. and Kebbiche, Z. *A weighted target-following algorithm for linearly constrained convex optimization*, Int. J. Open Problems Compt. Math., 5(4) (2012), 25–33.

[27] Tahmasebzadeh, S., Navidi, H. and Malek, A. *Novel interior point algorithms for solving nonlinear convex optimization problems*, Adv. Oper. Res., 2015 (2015).

[28] Vanderbei, R.J. and Shanno, D.F. *An interior-point algorithm for nonconvex nonlinear programming*, Comput. Optim. Appl., 13 (1999) , 231–252.

[29] Wang, G.Q. *A new polynomial interior-point algorithm for the monotone linear complementarity problem over symmetric cones with full NT-steps*, Asia-Pac. J. Oper. Res., 29(2) (2012), 1250015.

[30] Zaoui, B., Benterki, D. and Khelladi, S. *New efficient descent direction of a primal-dual path-following algorithm for linear programming*, Stat. Optim. Inf. Comput., 12 (2024), 1098–1112.

[31] Zaoui, B., Benterki, D. and Khelladi, S. *Efficient descent direction of a primal-dual interior point algorithm for convex quadratic optimization*, J. Inf. Optim. Sci., (2025). Available online.

[32] Zaoui, B., Benterki, D., Kraria, A. and Raouache, H. *Interior-point algorithm for linear programming based on a new descent direction*, RAIRO Oper. Res., 57 (2023), 2473–2491.

[33] Zhang, M., Bai Y.Q. and Wang G.Q. *A new primal-dual path-following interior-point algorithm for linearly constrained convex optimization*, Journal of Shanghai University (English Edition), 12 (2008), 475–480.

[34] Zhang, M., Huang K., Li M. and Lv Y. *A new full-Newton step interior-point method for $P_*(\kappa)$-LCP based on a positive-asymptotic kernel function*, J. Appl. Math. Comput., 64 (2020), 313–330.