Iranian Journal of Numerical Analysis and Optimization
Vol. 16, No. 1, 2026, pp 19–37
https://doi.org/10.22067/ijnao.2025.94037.1667
https://ijnao.um.ac.ir/

Research Article

A fractional order model for malaria control using optimal strategies and numerical simulations

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Abstract

This paper develops and analyzes a fractional-order model for malaria transmission that captures memory effects in host-vector dynamics and enables a more flexible treatment of control strategies. The proposed model extends the classical SEIR framework by incorporating fractional-order derivatives, which offer a more accurate representation of the memory and hereditary effects in malaria transmission. The human population is divided into four compartments: people susceptible (S_P), people exposed (E_P), people infected (I_P), people recovered

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Received 11 June 2025; revised 1 September 2025; accepted 2 September 2025

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How to cite this article
Kouidere, A. El Bhih, A. Minifi, I. and Adnaoui, K., A fractional order model for malaria control using optimal strategies and numerical simulations. *Iran. J. Numer. Anal. Optim.*, 2026; 16(1): 19-37. https://doi.org/10.22067/ijnao.2025.94037.1667

(R_P), and the mosquitoes population is divided into two compartments: mosquitoes susceptible (S_M), mosquitoes infected (I_M), respectively. The model introduces two time-dependent control variables: public awareness campaigns, treatment. These controls aim to prevent new infections, reduce the number of individuals in infectious compartments, and mitigate long-term complications in recovered individuals. Pontryagin's maximum principle is employed to derive the necessary conditions for optimal control, and the resulting system is solved using an iterative numerical method. Simulation results, implemented in MATLAB, illustrate the influence of fractional-order derivatives on disease dynamics and demonstrate the comparative effectiveness of the control strategies. This work provides a novel fractional-order control framework for malaria and highlights the importance of integrated intervention strategies.

AMS subject classifications (2020): 49J15, 93C10, 92B05, 93A30

Keywords: Malaria, Optimal control, Malaria disease, Fractional-order model, Mathematical

1 Introduction

Malaria continues to exert a heavy burden on public health, especially in tropical and subtropical regions. It is triggered by *Plasmodium* parasites, transmitted through bites from infected *Anopheles* mosquitoes. Although treatments, vaccines, and mosquito control measures like insecticide-treated nets are available, the disease remains persistent in many endemic areas [25, 26, 27].

Classical compartmental modeling approaches, notably the Ross-Macdonald model and its derivatives, have been widely used to understand malaria dynamics. These models often assume Markovian processes, neglecting historical influence on current states. In contrast, fractional-order models accommodate memory effects, making them particularly well-suited for capturing the delayed responses and cumulative effects seen in real-world epidemiology.

This paper introduces a fractional-order SEIR framework for modeling malaria spread between human and mosquito populations. The model incorporates optimal control measures namely treatment and awareness campaigns to evaluate their impact on disease transmission. We employ fractional derivatives to account for hereditary characteristics of the system and apply Pontryagin's principle to formulate the optimal control problem.

Simulation results based on real-world parameter values demonstrate the effectiveness of fractional-order modeling in enhancing predictive accuracy and informing intervention strategies. This approach underscores the importance of integrating advanced mathematical tools in the design of public health policies against malaria.

Despite extensive global control efforts, malaria continues to be a significant public health burden. In 2022, the World Health Organization (WHO) reported approximately 249 million cases and 608,000 deaths worldwide, with the African region bearing the greatest share of the disease burden, followed by Southeast Asia and the Eastern Mediterranean [14, 15, 12, 9, 2, 28]. These figures underscore the urgent need for improved surveillance, equitable treatment access, and strengthened prevention strategies.

Various mathematical models have been employed to study malaria transmission, most commonly classical compartmental models based on integer-order differential equations. These models, including the Ross-Macdonald framework and extensions. Bounouiga, Basti, and Benhamidouche [10] studied a fractional-order model for malaria transmission under the effect of control strategies. Saha, Saha, and Podder [24] introduced a novel mathematical model to study the impact of drug-resistant strain, recovered-carrier, and relapse on malaria dynamics by implementing Caputo–Fabrizio fractional-order derivative. They begin by presenting theoretical results that are derived for our mode. Helikumi and Loliak [12] formulated and analyzed a fractional-order model for malaria disease transmission using Atangana–Beleanu–Caputo in sense to study the effects of heterogeneity vector biting exposure on the human population. To capture effects the heterogeneity vector biting exposure, they subdivided the human population into two subgroups, namely, the population in high and low risk areas.

Fractional-order calculus offers a more flexible mathematical framework by incorporating memory and hereditary effects inherent in biological systems. By employing fractional derivatives, it enables the modeling of time-dependent and nonlocal processes with greater accuracy, thereby improving the realism and predictive capacity of disease transmission dynamics [20, 19, 4, 5, 6]. To overcome the limitations of classical models, this study introduces a fractional-order model to better capture the complexities of malaria transmission dynamics of malaria transmission incorporating both human and mosquito populations. Human infection occurs via mosquito bites, while mosquitoes become infected by feeding on infectious humans. The model quantifies key transmission parameters and considers the impact of asymptomatic carriers, vector control, vaccination, and treatment access [3, 8, 7, 11, 12, 17].

Caputo-type fractional derivatives are used to extend the conventional SEIR (Susceptible exposed infected recovered) framework, integrating memory effects and allowing for more detailed representations of regional differences, behavioral patterns, and disease progression. The model also evaluates the effectiveness of various control strategies under varying levels of intervention coverage and compliance.

The paper is structured as follows: Section 2 introduces foundational concepts of fractional calculus, emphasizing the Caputo derivative's relevance to epidemiology. Section 3 presents a mathematical model of malaria disease. Section 4 formulates an optimal control problem using

Pontryagin's maximum principle. Section 5 presents simulation results under multiple scenarios. Section 6 concludes with key insights, recommendations, and policy implications.

By incorporating fractional calculus into malaria modeling, this study offers a more accurate and adaptable framework for understanding disease dynamics and optimizing public health interventions in endemic regions.

2 Definitions and preliminaries

We will follow the same methodology adopted in the papers [1, 22, 21].

Fractional calculus is a generalization of classical calculus where differentiation and integration are extended to noninteger (fractional) orders. This framework has been increasingly applied in physics, engineering, biology, and epidemiology due to its ability to capture memory and hereditary effects.

Among the different definitions of fractional derivatives, the Caputo derivative is particularly suitable for modeling real-world problems since it allows for the use of standard initial conditions expressed in terms of integer-order derivatives. The Caputo fractional derivative of order α $(0 < \alpha \leq 1)$ for a sufficiently smooth function $f(t)$ is defined as

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f'(s)}{(t-s)^{\alpha}}\, ds,$$

where $\Gamma(\cdot)$ is the Gamma function.

When $\alpha = 1$, the Caputo derivative reduces to the classical first-order derivative, while values of α between 0 and 1 interpolate between no memory and full memory effects. This property makes fractional calculus a powerful tool for studying systems where the past influences the present, such as disease progression and transmission.

In the following section, we employ the Caputo fractional derivative to formulate a malaria transmission model that incorporates these memory effects, providing a more accurate framework for analyzing disease dynamics.

3 Model presentation

We categorize the total population N is divided into six compartments $S_P, E_P, I_P, R_P, S_M, I_M$: susceptible, exposed, infected, and recovered people, along with susceptible and infected mosquitoes. The interaction between these compartments is described by a system of fractional differential

equations. These equations incorporate Caputo derivatives to account for the influence of historical infection trends.

Figure 1 illustrates the flow between compartments. The human population is replenished by recruitment, exposed through contact with infectious agents, and progresses through disease stages with recovery or mortality. Mosquitoes become carriers upon biting infected individuals.

The complete system is given below:

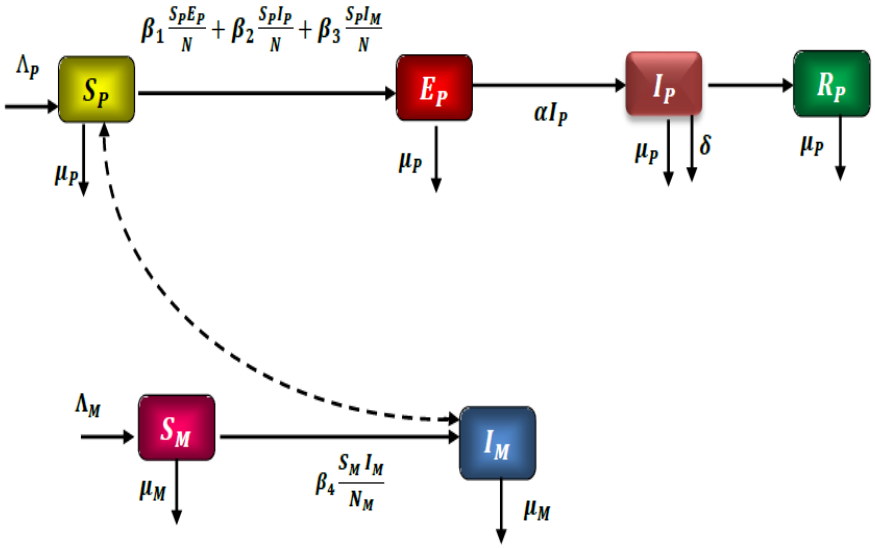

Figure 1: Figure compartments model

Accordingly, we formulate a fractional optimal control model for malaria transmission, which is described by the following system of fractional differential equations:

$$\begin{cases} D^\alpha S_P(t) = \Lambda_P - \mu_P S_P(t) - \beta_1 \frac{S_P(t)E_P(t)}{N} - \beta_2 \frac{S_P(t)I_P(t)}{N} - \beta_3 \frac{S_P(t)I_M(t)}{N}, \\ D^\alpha E_P(t) = \beta_1 \frac{S_P(t)E_P(t)}{N} + \beta_2 \frac{S_P(t)I_P(t)}{N} + \beta_3 \frac{S_P(t)I_M(t)}{N} - (\mu_P + \alpha)E_P(t), \\ D^\alpha I_P(t) = \alpha E_P(t) - (\mu_P + \sigma + \delta)I_P(t), \\ D^\alpha R_P(t) = \sigma I_P(t) - \mu_P R_P(t), \\ D^\alpha S_M(t) = \Lambda_M - \mu_M S_M(t) - \beta_4 \frac{S_M(t)I_M(t)}{N}, \\ D^\alpha I_M(t) = \beta_4 \frac{S_M(t)I_M(t)}{N} - \mu_M I_M(t), \end{cases} \tag{1}$$

where the initial state are $S_P(0) \geq 0$, $E_P(0) \geq 0$, $I_P(0) \geq 0$, $R_P(0) \geq 0$, $S_M(0) \geq 0$ and $I_M(0) \geq 0$, where

Λ_P: Recruitment rate of new susceptible people entering the population.

Λ_M: Recruitment rate of new susceptible malaria vectors (mosquitoes).
μ_P: Per capita natural mortality rate of people.
μ_M: Per capita natural mortality rate of malaria vectors (mosquitoes).
β_1: Transmission rate from exposed people to susceptible people.
β_2: Transmission rate from infected people to susceptible people.
β_3: Transmission rate from infected malaria vectors to susceptible people.
β_4: Transmission rate from infected malaria vectors to susceptible vectors.
α: Progression rate from exposed to infected people.
σ: Recovery rate of infected people.
δ: Disease-induced mortality rate due to complications.

4 Fractional derivation with optimal control problem

4.1 Control strategy formulation

To mitigate disease transmission, we introduce two control interventions: $u(t)$ for awareness and $v(t)$ for treatment. These controls are incorporated into the model to assess their effectiveness over a finite time horizon $[0, T]$. The objective is to reduce both the exposed and infected human populations while considering the cost associated with implementing the controls. Therefore

$$\begin{cases} D^\alpha S_P(t) = \Lambda_P - \mu_P S_P(t) - \beta_1(1-u(t))\dfrac{S_P(t)E_P(t)}{N} \\ \qquad\qquad - \beta_2(1-u(t))\dfrac{S_P(t)I_P(t)}{N} - \beta_3(1-u(t))\dfrac{S_P(t)I_M(t)}{N}, \\ D^\alpha E_P(t) = \beta_1(1-u(t))\dfrac{S_P(t)E_P(t)}{N} + \beta_2(1-u(t))\dfrac{S_P(t)I_P(t)}{N} \\ \qquad\qquad + \beta_3(1-u(t))\dfrac{S_P(t)I_M(t)}{N} - (\mu_P+\alpha)E_P(t), \\ D^\alpha I_P(t) = \alpha E_P(t) - \sigma(1-v(t))I_P - (\mu_P+\delta)I_P(t), \\ D^\alpha R_P(t) = \sigma(1-v(t))I_P(t) - \mu_P R_P(t), \\ D^\alpha S_M(t) = \Lambda_M - \mu_T S_M(t) - \beta_3\dfrac{S_M(t)I_M(t)}{N}, \\ D^\alpha I_M(t) = \beta_4\dfrac{S_M(t)I_M(t)}{N} - \mu_T\ I_M(t), \end{cases} \tag{2}$$

with $S_P(0) \geq 0$, $E_P(0) \geq 0$, $I_P(0) \geq 0$, $R_P(0) \geq 0$, $S_M(0) \geq 0$, and $I_M(0) \geq 0$ are the initial state. Hence

$$J(u,v) = \int_0^T \left[E_P(t) + I_P(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) \right] dt. \tag{3}$$

Here, $A > 0$ and $B > 0$ represent the weights associated with the costs of implementing the awareness program and treatment controls, respectively. These parameters reflect the relative importance of the control variables $u(t)$ and $v(t)$ at time t. We adopt a quadratic cost function on the controls to capture nonlinear effects that arise from high levels of intervention implementation. This cost function represents the financial resources required for treatment as well as for conducting the awareness campaigns. The variable T denotes the final time horizon of the control process.

In other words, our objective is to determine the optimal controls $u^*(t)$ and $v^*(t)$ such that

$$J(u^*, v^*) = \min_{(u,v)\in U} J(u,v) \tag{4}$$

subject to the controlled dynamic system described earlier.

The admissible control set is defined such that both $u(t)$ and $v(t)$ remain within $[0,1]$ throughout the interval defined by $U = \{(u,v)/, 0 \leq u_{\min} \leq u_i(t) \leq u_{\max} \leq 1, 0 \leq v_{\min} \leq v_i(t) \leq v_{\max} \leq 1, \text{ and } / \ t \in [0,T]\}$.

The main point in fractional optimal control problems is to find the optimal control (u,v) that minimizes the following objective function:

$$J"(u,v) = \int_0^T \left[E_P(t) + I_P(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) \right] dt \tag{5}$$

subject to the constraint
$D^\alpha S_P(t) = g_1(S_P, E_P, I_P, R_P, S_M, I_M, u, v)$,
$D^\alpha E_P(t) = g_2(S_P, E_P, I_P, R_P, S_M, I_M, u, v)$,
$D^\alpha I_P(t) = g_3(S_P, E_P, I_P, R_P, S_M, I_M, u, v)$,
$D^\alpha R_P(t) = g_4(S_P, E_P, I_P, R_P, S_M, I_M, u, v)$,
$D^\alpha S_M(t) = g_4(S_P, E_P, I_P, R_P, S_M, I_M, u, v)$,
$D^\alpha I_M(t) = g_5(S_P, E_P, I_P, R_P, S_M, I_M, u, v)$, where $g_j(1 <= j <= 6)$ represent the right side of the system (2), and where the initial conditions are positives. The following expression defines a modified objective function. To find the optimal control, we follow the traditional approach and define a modified performance index as

$$J(u,v) = \int_0^T \left[H^*(S_P, E_P, I_P, R_P, S_M, I_M, u, v, t)\right] dt$$
$$- \int_0^T \left[\sum_{i=1}^{6} \Lambda_i(t)\, g_i(S_P, E_P, I_P, R_P, S_M, I_M, u, v, t)\right] dt$$

In order to derive the necessary condition for the optimal control, we apply pontryagin's maximum principle. The idea is introducing the adjoint function to attach the system of differential equations to the objective functional resulting in the formation of a function called the Hamiltonian. This principle converts into a problem of minimizing Hamiltonian $H_j(t)$ at time t defined by $H(S_P, E_P, I_P, R_P, S_M, I_M, u, v, t)$:

$$\begin{aligned} &H(S_P, E_P, I_P, R_P, S_M, I_M, u, v, t) \\ &= E_P(t) + I_P(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) \\ &\quad + \sum_{i=1}^{6} \Lambda_i(t)\, g_i(S_P, E_P, I_P, R_P, S_M, I_M, u, v, t). \end{aligned}$$

From (5), we can derive the following equalities:

$$\begin{aligned} D^\alpha S_P(t) &= -\tfrac{\partial H^*(t)}{\partial \Lambda_1}(t), \quad D^\alpha E_P(t) = -\tfrac{\partial H^*(t)}{\partial \Lambda_2}(t), \\ D^\alpha I_P(t) &= -\tfrac{\partial H^*(t)}{\partial \Lambda_3}(t), \quad D^\alpha R_P(t) = -\tfrac{\partial H^*(t)}{\partial \Lambda_4}(t), \\ D^\alpha S_M(t) &= -\tfrac{\partial H^*(t)}{\partial \Lambda_5}(t), \quad D^\alpha I_M(t) = -\tfrac{\partial H^*(t)}{\partial \Lambda_6}(t), \end{aligned} \tag{6}$$

with the transversality conditions at time T:

$$\Lambda_1(T) = 0,\ \Lambda_2(T) = -1,\ \Lambda_3(T) = -1,\ \Lambda_4(T) = 0,\ \Lambda_5(T) = 0 \text{ and } \Lambda_6(T) = 0. \tag{7}$$

Equations (6) and (7) describe the necessary conditions in terms of a Hamiltonian for the fractional optimal control problem defined previously.

These conditions result in a set of fractional differential equations, in terms of the variables state $S_P, E_P, I_P, R_P, S_M, I_M$, controls u, v, w and Lagrange multiplier Λ_i, to be solved analytically or numerically or even both.

4.2 Characterization of optimal control

Here we apply Pontryagin's principle [23, 16, 13] to solve the optimal control problem given above. This principle converts into a problem of minimizing a Hamiltonian $H(t)$ at time t defined by

$$\begin{aligned} &H(S_P, E_P, I_P, R_P, S_M, I_M, u, v, t) \\ &= E_P(t) + I_P(t) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) \\ &\quad + \sum_{i=1}^{6} \Lambda_i(t)\, f_i(S_P, E_P, I_P, R_P, S_M, I_M, u, v, t) \end{aligned}$$

with g_i is the right side of the differential equation of the ith state variable of the system at time t.

Theorem 1. The optimal controls (u^*, v^*) and the solutions S_P^*, E_P^*, I_P^*, R_P^*, S_M^* and I_M^* of the corresponding co-state system (2) are given by the adjoint variables $\Lambda_1(t)$, $\lambda_2(t)$,$\Lambda_3(t)$, $\Lambda_4(t)$, $\Lambda_5(t)$ and $\Lambda_6(t)$ satisfying

$$\begin{aligned} D^\alpha \Lambda_1(t) =& -\Lambda_1(t)\left[-\mu_P - \beta_1(1-u(t))(\frac{E_P(t)}{N} - \beta_2\frac{I_P(t)}{N} - \beta_3\frac{I_M(t)}{N})\right] \\ &-\Lambda_2(t)\left(\beta_1(1-u(t))(\frac{E_P(t)}{N} - \beta_2\frac{I_P(t)}{N} - \beta_3\frac{I_M(t)}{N})\right) \\ D^\alpha \Lambda_2(t) =& -1 - \Lambda_1(t)\left[-\beta_1(1-u(t))\frac{S_P(t)}{N}\right] - \Lambda_2(t)\left[\beta_1(1-u(t))\frac{S_P(t)}{N} - (\mu_P + \alpha)\right] \\ &-\Lambda_3(t)\alpha + \Lambda_4(t)\beta_3\frac{S_M(t)}{N} - \Lambda_5(t)\beta_3\frac{S_M(t)}{N}, \\ D^\alpha \Lambda_3(t) =& -1 - \Lambda_1(t)\left[-\beta_2(1-u(t))\frac{S_P(t)}{N}\right] - \Lambda_2(t)\left[\beta_2(1-u(t))\frac{S_P(t)}{N} - (\mu_P + \alpha\right]) \\ &-\Lambda_3(t)\left[(\mu_P - (1-v(t))\sigma) - \Lambda_4(t)((1-v(t))\sigma)\right], \\ D^\alpha \Lambda_4(t) =& \Lambda_4(t)\mu_P, \\ D^\alpha \Lambda_5(t) =& \Lambda_5(t)(\mu_M\ I_M(t) + \beta_4\frac{E_P(t)}{N}) - \Lambda_6(t)(\beta_4\frac{I_M(t)}{N}, \\ D^\alpha \Lambda_6(t) =& \Lambda_1(t)(\beta_3\frac{S_P(t)}{N}\ {}_4\frac{E_P(t)}{N}) - \Lambda_6(t)(\mu_M\ + \beta_4\frac{S_M(t)}{N}. \end{aligned}$$

The transversality conditions at the terminal time T are given by

$$\Lambda_1(T) = 0,\ \Lambda_2(T) = -1,\ \Lambda_3(T) = -1,\ \Lambda_4(T) = 0,\ \Lambda_5(T) = 0. \text{ and } \Lambda_6(T) = 0 \tag{8}$$

Moreover, for $t \in [0,T]$, the optimal controls $u^*(t)$ and $v^*(t)$ are expressed as

$$u^* = \min\left(u_{\max}, \max\left(u_{\min}, (\frac{(\Lambda_1-\Lambda_2)}{A} \times \beta_1 \frac{S_P^* E_P^*}{N}) + \beta_2 \frac{S_P^* I_P^*}{N}) + \beta_3 \frac{S_P^* I_M^*}{N}(t)\right)\right) \tag{9}$$

$$v^* = \min\left(v_{\max}, \max\left(v_{\min}, \frac{\Lambda_3(t)-\Lambda_4(t))}{B} \times I_P^*)(t)\right)\right). \tag{10}$$

Proof. Here we apply Pontryagin's principle [23] to solve the optimal control problem given below.

So, we define the Hamiltonian H as follows:

$$\begin{aligned}
H^*(t) =& E_P(T) + I_P(T) + \frac{A}{2}u^2(t) + \frac{B}{2}v^2(t) \\
&+ \Lambda_1(t)[\Lambda_P - \mu_P S_P(t) - \beta_1(1-u(t))\frac{S_P(t)E_P(t)}{N} \\
&- \beta_2(1-u(t))\frac{S_P(t)I_P(t)}{N} - \beta_3(1-u(t))\frac{S_P(t)I_M(t)}{N}] \\
&+ \Lambda_2(t)[\beta_1(1-u(t))\frac{S_P(t)E_P(t)}{N} + \beta_2(1-u(t))\frac{S_P(t)I_P(t)}{N} \\
&+ \beta_3(1-u(t))\frac{S_P(t)I_M(t)}{N} - (\mu_P+\alpha)E_P(t)] \\
&+ \Lambda_3(t)\left[\alpha E_P(t) - \sigma(1-v(t))I_P - (\mu_P+\delta)I_P(t)\right] \\
&+ \Lambda_4(t)\left[\sigma(1-v(t))I_P(t) - \mu_P R_P(t)\right] \\
&+ \Lambda_5(t)\left[\Lambda_T - \mu_T S_M(t) - \beta_3\frac{S_M(t)I_M(t)}{N}\right] \\
&+ \Lambda_6(t)\left[\beta_4\frac{S_M(t)I_M(t)}{N} - \mu_T I_M(t)\right]
\end{aligned}$$

For $t \in [0,T]$, the adjoint equations and transversality conditions can be obtained by using Pontryagin's maximum principle [23, 16, 13, 18] such that

$$\begin{aligned}
D^\alpha \Lambda_1(t) &= -\frac{\partial H(t)}{\partial S_P(t)}, \\
D^\alpha \Lambda_2(t) &= -\frac{\partial H(t)}{\partial E_P(t)}, \\
D^\alpha \Lambda_3(t) &= -\frac{\partial H(t)}{\partial I_P(t)}, \\
D^\alpha \Lambda_4(t) &= -\frac{\partial H(t)}{\partial R_P(t)},
\end{aligned}$$

$$D^{\alpha}\Lambda_5(t) = -\frac{\partial H(t)}{\partial S_M(t)},$$
$$D^{\alpha}\Lambda_6(t) = -\frac{\partial H(t)}{\partial I_M(t)},$$

with the transversality conditions at time T: $\Lambda_1(T) = 0,\ \Lambda_2(T) = -1,\ \Lambda_3(T) = -1,\ \Lambda_4(T) = 0;$ $\Lambda_5(T) = 0$ and $\Lambda_6(T) = 0$.

For, $t \in [0, T]$, the optimal controls u and v can be solved from the optimality condition

$$\begin{aligned}
-\frac{\partial H(t)}{\partial u(t)} = 0,\ \Rightarrow -Au(t) - (\Lambda_2(t) - \Lambda_1(t))\beta_1\frac{S_P^*(t)E_P^*(t)}{N}) + \beta_2\frac{S_P^*(t)I_P^*(t)}{N}) \\
+\beta_3\frac{S_P^*(t)I_M^*(t)}{N} = 0, \\
-\frac{\partial H(t)}{\partial v(t)} = 0, \Rightarrow -Bv(t) - (\Lambda_4(t) - \Lambda_3(t))I_P(t) = 0,
\end{aligned}$$

We have

$$u(t) = \frac{(\Lambda_1(t) - \Lambda_2(t))}{A} \times \beta_1\frac{S_P(t)E_P(t)}{N}) + \beta_2\frac{S_P(t)I_P(t)}{N}) + \beta_3\frac{S_P(t)I_M(t)}{N},$$

$$v(t) = \frac{\Lambda_3(t) - \Lambda_4(t))}{B} \times I_P(t).$$

By the bounds in U of the controls, it is easy to obtain that u^* and v^* are given by (9), (10) in the form of system (6). □

5 Simulation results

In this section, we present the results obtained by numerically solving the optimality system. In our control problem, we have initial conditions for the state variables and terminal conditions for the adjoints. That is, the optimality system is a two-point boundary value problem with separated boundary conditions at times step $i = 0$ and $i = T$. We solve the optimality system by an iterative method with forward solving of the state system followed by backward solving of the adjoint system. We start with an initial guess for the controls at the first iteration and then before the next iteration, we update the controls by using the characterization. We continue until convergence of successive iterates is achieved. A code is written and compiled in MATLAB by using the following Euler method and using the following data in Table 1.

Table 1: Parameter values used in numerical simulation

Paramter	value in d^{-1}
Λ_P	0.02
Λ_M	0.2
μ_P	0.1
μ_T	0.8
β_1	0.4
β_2	0.2
β_3	0.1
β_4	0.1
α	0.2
σ	0.2
δ	0.3

Since control and state functions are on different scales, the weight constant value is chosen as follows: $A = 100$, $B = 100$.

5.1 Strategy without control

In this strategy, we applied no control.

The data presented in Figure 2 and Table 1 indicate a substantial increase in both malaria infections and related fatalities, highlighting a worsening outbreak. This escalation can be attributed to ineffective containment measures, including delays in case identification and isolation, as well as the absence of widespread vaccination and preventive interventions. Consequently, the disease continues to spread rapidly, particularly in regions with limited healthcare infrastructure. Additionally, human behavioral factors, such as close physical contact and insufficient awareness of transmission routes, further facilitate viral dissemination. The prolonged infectious period also increases the risk of undetected secondary transmission.

These findings emphasize the necessity for immediate and effective public health interventions. Targeted strategies, such as mass vaccination campaigns, enhanced surveillance, and public awareness initiatives, are essential to mitigate further transmission. Implementing timely and evidence-based measures can significantly reduce infection rates, minimize mortality, and limit the overall impact of the outbreak. Failure to adopt such interventions may result in continued disease propagation.

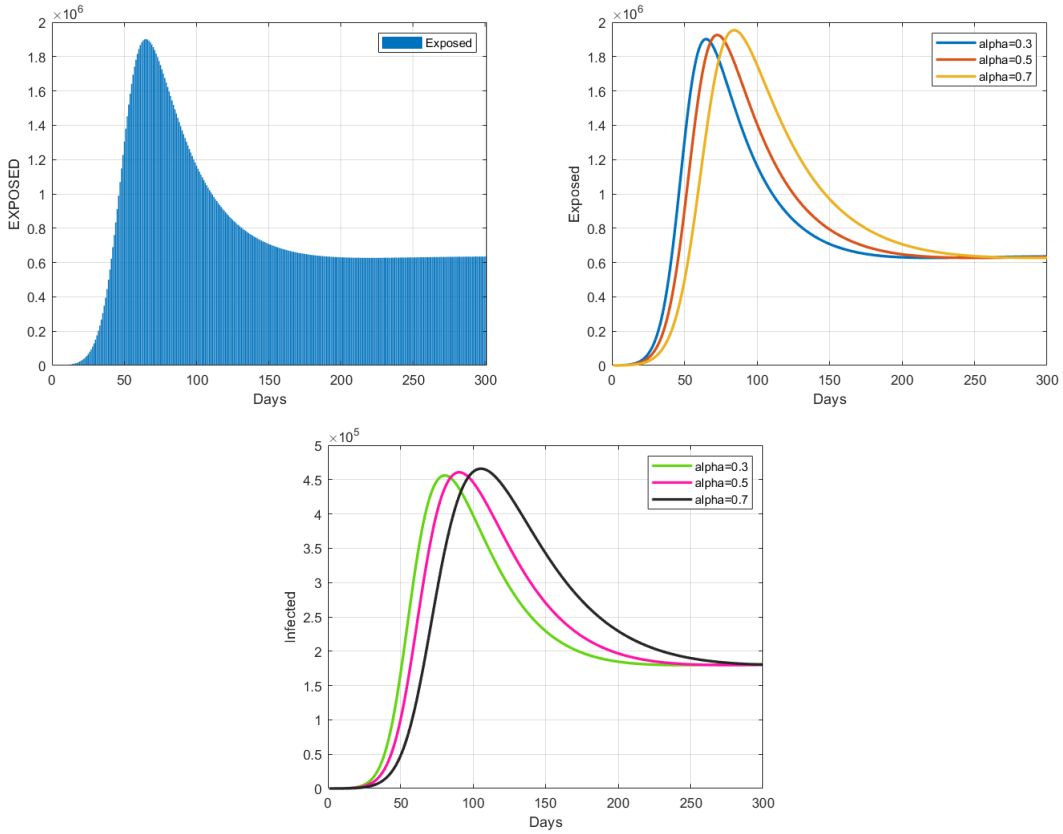

Figure 2: The evolution of the number of exposed and infected human without control

5.2 Strategy 1: Awareness program: Education

In this strategy, we used only the optimal control $u(t)$.

Figure 3 illustrates a significant reduction in the number of exposed and infected individuals following the implementation of sensitization campaigns aimed at at-risk populations. These campaigns focus on educating individuals about the risks associated with close contact with infected persons, thereby reducing the transmission rate of the malaria disease.

The numerical results indicate a clear distinction between scenarios with and without control measures. In the absence of control interventions, the number of exposed individuals ranges from 50 to 3.46×10^5 (for $\alpha = 0.3$) after 300 days. However, with control measures in place, this range decreases from 300 to 3.59×10^5 (for $\alpha = 0.5$), demonstrating the effectiveness of awareness programs in mitigating exposure. Similarly, the number of infected individuals without control

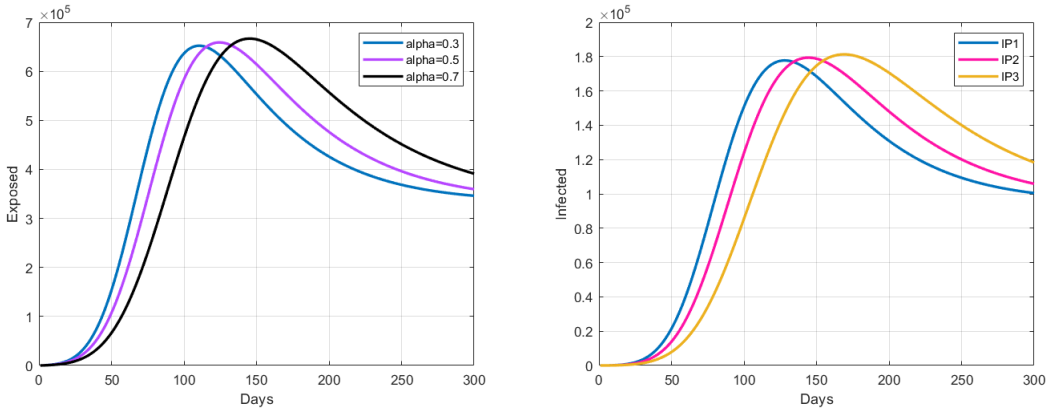

Figure 3: The evolution of the number of exposed and infected human without and with control $u(t)$

measures varies between 50 to 9.06×10^4 (for $\alpha = 0.7$). With control measures, this range is reduced to 10 to 8.14×10^4 (for $\alpha = 0.7$), indicating a decline in infection rates due to intervention strategies. These results provide strong evidence that sensitization campaigns play a crucial role in minimizing both exposure and infection rates, thereby improving disease control efforts.

In addition to the effectiveness of control measures, the study highlights the significance of fractional derivatives in modeling epidemiological dynamics. Fractional-order models incorporate memory effects, allowing for a more accurate representation of disease transmission over time. As the fractional derivative order α approaches 0.7, the memory effect within the system diminishes, resembling classical integer-order models. However, fractional derivatives provide a more generalized framework, capturing the long-term dependencies and non-local interactions inherent in real-world disease dynamics.

Figure 3 further demonstrates that when $\alpha = 0.7$, the memory effect within the system is maximized, leading to a prolonged period of reduced infection rates. This phenomenon underscores the importance of fractional calculus in epidemiological modeling, as it enables the incorporation of past infection dynamics into predictive models. By accounting for memory effects, fractional-order models offer a more comprehensive understanding of disease progression, enhancing the accuracy of predictions and the effectiveness of intervention strategies.

5.3 Strategy 2: Education and treatment

In This strategy, we use only the optimal control $v(t)$.

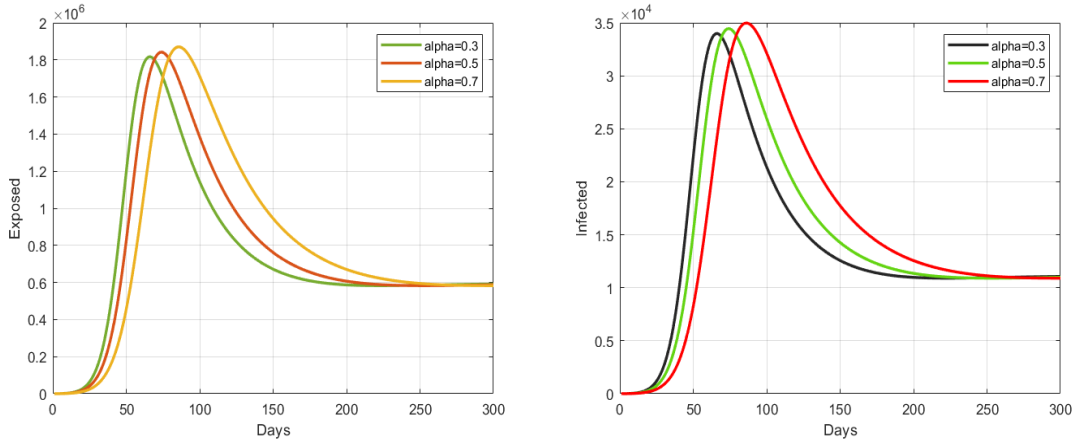

Figure 4: The evolution of the number of infected and exposed people with control $v(t)$

In Figure 4, we observe a significant reduction in the number of individuals infected with the malaria disease after 150 days. So we observe after a high increasing the number of exposed and infected were decreasing in to accepted value.

Number of Exposed Individuals:

Without control measures, the number of exposed individuals ranges from 150 to 1.85×10^6 (for $\alpha = 0.7$). With control measures, this range decreases to 50 (for $\alpha = 0.5$) to 2.61×10^6 (for $\alpha = 0.5$).

Number of Infected Individuals:

Without control measures, the number of infected individuals ranges from 10 to 4.32×10^4 (for $\alpha = 0.3$). With control measures, this range decreases to 100 to 6.04×10^4 (for $\alpha = 0.4$).

The implementation of control strategies, such as vaccination, antiviral treatments, and quarantine measures, has led to these favorable outcomes. These interventions have proven effective in reducing the spread of the malaria disease, leading to a significant decrease in the number of infected individuals. The results underscore the importance of timely and coordinated public health measures in managing and mitigating the impact of the outbreak.

6 Conclusion

In conclusion, Using fractal fractional-order derivative, we have successfully established theoretical and computational analysis for a malaria disease mathematical model, a fractional mathematical model, that describes the transmission of malaria in mosquito-to-human, which includes two controls. We also proposed three controls which, respectively, represent education and treat-

ment. We also studied the optimal control, led to reduce the spread of the malaria disease, by reducing the number of pigs infected and the number of ticks infected. We applied the results of the control theory, and we managed to obtain the characterization of the optimal control. The numerical simulation of the obtained results showing the effectiveness of the proposed control strategies. The entire investigation of this work revealed that control of the dynamic spread rate is vital for stopping the transmission of the spreading malaria disease.

These findings emphasize two key aspects: (1) the success of sensitization campaigns in reducing the spread of malaria and (2) the critical role of fractional derivatives in capturing the extended dynamics of disease outbreaks. Implementing awareness programs alongside advanced mathematical modeling approaches can significantly improve public health responses and mitigate future outbreaks. Ultimately, this research contributes to the broader understanding of disease transmission dynamics and offers an innovative approach to optimizing control strategies, which is essential for improving the management of current and future malaria outbreaks. Finally, we will develop this work in spatio temporels by studying the spread this disease in time and place.

Availability of data and materials

The disciplinary data used to support the findings of this study have been deposited in the Network Repository (http://www.networkrepository.com).

Competing interests

All the authors have no competing interests.

Authors' contributions

All authors contributed equally to the writing of this manuscript. All authors read and approved the final version

Funding

This research received no external funding.

Conflicts of Interest

The authors have no financial or proprietary interests in any material discussed in this article.

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