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#### Research Article



# Comparative evaluation of large-scale many objective algorithms on complex optimization problems

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#### Abstract

In the field of optimization, there has been an enormous surge in interest in addressing large-scale many-objective problems. Numerous academicians and practitioners have contributed to evolutionary computation by developing a variety of optimization algorithms tailored to tackle computationally challenging optimization problems. Recently, various large-scale many-objective optimization algorithms (LSMaOAs) have been proposed to address complex large-scale many-objective optimization problems (LSMaOPs). These LSMaOAs have shown remarkable performance in addressing a variety of LSMaOPs. However, there is a pressing need to further investigate their performance in comparison to each other on

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different classes of LSMaOPs. In this study, we conduct a comparative investigation of three established LSMaOAs namely, LMEA, LMOCSO and S3CMAES over rigorous benchmarking on DTLZ, LSMOP, UF9-10, WFG test suites, encompassing problem sets with three to ten objectives and varying numbers of variables between 100 and 500. Additionally, we assess the algorithm's efficacy on a test suite specifically designed for large-scale multi/many-objective problems (100-1000 decision variables). In addition, we propose Hybrid-LMEA, a light hybrid that integrates decision-variable clustering with competitive learning to improve both convergence and diversity. The hybrid works especially well on high-dimensional large-scale many-objective optimization problems with better performance in 8 and 12 out of 27 test cases for IGD and GD, respectively. The outcomes of the experiments indicate the relative efficacy and effectiveness of the different algorithms in addressing large-scale many-objective problems. Researchers can leverage this comparative data to make informed decisions about which algorithms to employ for particular optimization problem domains.

**AMS subject classifications (2020):** Primary 68T05, 68W50; Secondary 90C59, 90C90, 65Y20.

**Keywords:** Optimization problems; Large-scale multi-objective optimization; Large-scale many-objective optimization

#### 1 Introduction

A large-scale many-objective problem (LSMaOP) refers to an optimization problem involving a significant number of decision variables (D > 100)and a considerable number of objectives (M > 3) [47, 35]. Such optimization problems frequently arise in real-world scenarios such as software package redesign [39], software module clustering [20], hybrid car controller design [13], and pickup and delivery logistics [52]. While several multi/many-objective optimization algorithms (MOA/MaOA) [18] have proven effective for addressing multi/many-objective optimization problems (MOPs/MaOPs) [40] with two or three objectives, they often struggle to strike a balance between convergence and diversity when faced with the extensive numbers of decision variables and objectives in LSMaOPs. The decision space, along with the

objective search space, grows exponentially, resulting in the curse of dimensionality [9].

Numerous researchers and practitioners have proposed various approaches in the literature to overcome scalability challenges imposed by LSMaOPs. These approaches include grouping decision variables [27], reducing the decision space [16], and introducing novel search techniques [43], all specifically developed to address the complexities inherent in LSMaOPs, such as conflicting objectives and complex interactions between decision variables [26]. Due to the poor scalability of conventional MOAs/MaOAs [24] on LSMaOPs, the first approach involves splitting decision variables into groups, possibly at random or using heuristic approaches, followed by optimizing each group individually. For instance, Ma et al. [30] proposed MOA/DVA, which divides decision variables into distance, position, and mixed variables based on convergence and diversity control properties, while Zhang et al. [50] suggested a large-scale many-objective evolutionary algorithm (LMEA), which partitions decision variables into groups based on convergence and diversity.

The second approach aims to address the high-dimensional decision space of LSMaOPs by reducing its volume. This can be achieved through dimensionality reduction [5] and problem transformation [53] strategies, as demonstrated by Zille et al. [54] with the weighted optimization framework (WOF), which focuses on grouping decision variables into subgroups and assigning weights to each subgroup. The third approach involves developing novel search techniques tailored to LSMaOPs, creating offspring within the actual decision space without reducing its dimensionality. These techniques may include probability models [43] and reproduction operators [43] designed to efficiently handle LSMaOPs and produce optimal results. For example, Chen et al. [10] proposed S3CMAES, a scalable small subpopulations-based covariance matrix adaptation evolution strategy utilizing probability-based methods. In addition to these approaches, other strategies [43] have been adopted in the literature to solve LSMaOPs. For instance, Zhang, Shen, and Yen [48] introduced LSMaODE, a multi-population-based large-scale manyobjective differential algorithm, while Ma et al. [29] presented LSMOEA/D, incorporating an adaptive localized decision variable strategy within a decomposition framework [32]. Xu et al. [46] have proposed multi-population based MOEA, that is,DVCOEA, which involves analyzing the decision variable contribution to objectives, here decision variables are categorized depending on their contribution objectives. Using these approaches, LSMaOPs may be solved by tweaking/using traditional evolutionary algorithms [4], reducing the difficulties of the high-dimensional search space [42].

Despite these advancements, there remains a gap in the literature concerning the performance of large-scale many-objective optimization algorithms (LSMaOAs) on LSMaOPs with a significant number of decision variables. Previous studies by Deb et al. [17] and Cheng et al. [12] have focused on LSMaOP benchmarks with a limited number of objectives and decision variables. Apart from these, various other LSMaOAs have been proposed to address complex LSMaOPs. These LSMaOAs have shown remarkable performance in addressing a variety of LSMaOPs. However, there is a pressing need to further research on the investigation of their performance in comparison to each other over different classes of LSMaOPs. Thus, our study aims to fill this gap by providing insights into the comparative performance of LSMaOAs on LSMaOPs with more than three objectives and varying decision variables between 100-2000.

In this paper, we conduct a detailed comparative analysis of three different algorithms based on strategies from current literature: LMEA [50], large-scale multi-objective optimization based on competitive swarm optimizer (LMOCSO) [17], and S3CMAES [10]. We evaluate the efficacy of these algorithms using benchmark test suites (DLTZ) [41] and LSMaOP test suites [31], employing the inverted generational distance (IGD) as our performance metric [55, 44]. In addition to this comparative evaluation, we introduce a new variant referred to as LMEA-hybrid, blending the decision variable grouping of LMEA with the competitive learning strategy of LMOCSO. The hybridization is designed to improve the balance of convergence and diversity in high-dimensional search spaces. The algorithm is empirically compared with LMEA, LMOCSO, and S3CMAES on benchmark sets DTLZ, LSMOP, UF, and WFG. The outcomes show that Hybrid-LMEA improves performance in respect to IGD, generation distance (GD), and HV significantly in all but a few test instances, particularly in problems involving over 500 decision variables.

The motivation behind the selection of algorithms for comparative analysis is guided by their different strategies to cope with large-scale many-objective optimization scenarios. LMEA is a population decomposition-based algorithm that divides decision variables into groups with similar convergence and diversity characteristics, a feature that greatly improves its performance against the curse of dimensionality. Conversely, LMOCSO employs swarm intelligence and competitive learning mechanisms to maintain a middle ground between convergence and diversity in high-dimensional space. Finally, S3CMAES uses a covariance matrix adaptation strategy with small subpopulations that allows it to explore complex search spaces effectively while remaining computationally inexpensive.

The three chosen algorithms are representative of different methodologies—an approach based on decomposition, one based on swarm intelligence, and an evolutionary strategy; thus, a diverse comparative analysis will be possible. Additionally, their adequate performance during previous studies makes them well-suited for evaluating LSMaOAs. To facilitate a relative comparison of the performance of various MOEAs, and understand their relative strengths and weaknesses, this study is centered around a set of such benchmark problems, intending to provide insights into which algorithm might be selected for many-objective optimization in large-scale dimensions.

The paper is further segmented into four different sections. In Section 2, the basic concepts of large-scale multi/many objective optimization (LSMOO) and Large-scale many-objective optimization (LSMaO) are discussed. Section 3 presents our experimental design. Section 4 demonstrates the results of experiments. Lastly in Section 5, the findings and recommendations for future research directions are laid out.

#### 2 Related work

To tackle LSMaOPs, several optimization approaches have been proposed by customizing existing multi-objective optimization techniques. In the following subsection, we outline the major approaches based on the strategies of large-scale multi-objective optimization and large-scale many-objective optimization.

#### 2.1 Large-scale multi-objective optimization

Over the last 20 years, MOEAs have shown their efficacy in dealing with MOPs. They are a useful method for handling MOPs since they are good at producing a large number of solutions in just one run. For this reason, numerous MOEAs have been proposed, namely SPEA2 [56], MOEA/D [49] and NSGA-II [15]. The majority of these algorithms are based on Paretobased techniques, which select solutions that have greater Pareto ranks by using the concept of Pareto dominance. Moreover, diversity-related criteria are utilized to encourage a broad range of solutions dispersed throughout the Pareto front (PF). While Pareto-based strategies including SPEA2 and NSGA-II have shown potential in addressing MOPs that have two or three objectives however are ineffective when confronted with MaOPs. The primary objective of evolutionary multi-objective algorithms is to strike an equilibrium between two opposing objectives: significant diversity and effective convergence. The substantial diversity involves dispersing solutions extensively along the PF whereas effective convergence refers to a narrowing of gaps between solutions and the PF.

Large-scale MOPs/MaOPs present a substantial problem in optimization due to the presence of over one hundred decision variables. Maintaining a careful balance between diversity and convergence within an evolutionary algorithm's search process gets increasingly difficult as the quantity of non-dominated solutions rises significantly as the search space expands. Over the past few years, several evolutionary algorithms focusing on decision variable analysis have been presented [21] to solve these challenges. MOEA/DVA, a unique approach focusing on analyzing decision variables, was developed by Ma et al. [30]. This is a revolutionary MOEA that was created explicitly to address the challenges given by large-scale MOPs. This method groups decision variables into three categories based on their dominance relationships such as diversity-related, convergence-related, and both convergence and diversity. Similarly, Antonio and Coello [2] have proposed a cooperative co-evolutionary framework where decision variables are partitioned into numerous co-evolved sub-components to address the complexities of large-

scale/High-Dimensional MOPs. This framework has proven to be successful in handling significant number of decision variables (up to 5000).

Zille et al. [54] introduced a WOF. This method employs decision variable grouping and weighing to enhance the effectiveness of population-based algorithms for LSMOPs. This framework has been evaluated on NSGA-II and SMPSO algorithms with benchmark problems (WFG) having two and three objectives and with 1000 decision variables. He et al. [22] presented another generalized framework for LSMOO, that is, LSMOF. The primary objective of framework was to increase the multi-objective algorithm's computational efficiency on LSMOPs. The introduced framework LSMOF employs a twostage approach. The first stage of this approach involves problem reformulation which is responsible for generating an array of optimal solutions close to the PS and the second stage involves spreading a uniformly distributed approximate Pareto set, the LSMOF employed candidate solution spreading techniques and problem reformulation. Without employing any decision variable analysis method or grouping technique, LSMOF demonstrates effective performance and computational efficiency as compared to the current strategies in the literature. Deb et al. [17] put forwarded a competitive swarm optimizer (CSO) called LMOCSO for efficient search in LSMOPs. It involves particle updating strategy and competitive mechanism to enhance search efficiency. Hong et al. [23] proposed a model based on probabilistic prediction called LMOPPM. This model uses sampling and trend prediction models for effectively solving LSMOPs. Liu et al. [28] suggested an evolutionary algorithm based on dimensionality reduction and clustering for LSMOPs (dimensions up to 5000). The interdependence analysis has been employed to partition convergence variables into subgroups.

#### 2.2 Large-scale many-objective optimization

Zhang et al. [50] presented LMEA, a decision variable-based strategy for LSMaOPs. Using the convergence and diversity properties as a basis, this method groups decision variables. The statistical findings demonstrate how well LMEA handles large-scale MaOPs involving as many as 5000 decision

variables. Xu et al. [46]. presented a novel technique for LSMOO/LS-MaO termed DVCOEA. The primary goal of this is to increase convergence and diversity by classifying decision variables according to their objectives of contribution. The suggested methodology may find application in various machine learning techniques and evolutionary algorithms for estimating fitness.

Cao et al. [8] proposed distributed parallel PSO algorithms. This paper emphasizes that new PSO algorithms are required to handle LSMaOPs. This also highlights how crucial distributed parallel computing is to cutting down on operation time. Gu and Wang [19] introduced IFM-NSGA-III algorithms, which improve NSGAIII by using information feedback models for LSMOPs and LSMaOPs. The study highlights how important information feedback models are for MOEA optimization. Zhang et al. [51]proposed MOEA/D-IFM which includes information feedback models to enhance performance. The paper emphasizes using population information in optimizing algorithms. Wang et al. [45] proposed 1EA-IFM framework for LSMOPs. This framework balances diversity and convergence by retaining historical information and using fitness function values. Cao et al. [7] presented LS-MaOEA to overcome the drawback of RVEA [11] in handling problems which are large-scale evolutionary algorithm.

Table 1 showcases a range of many-objective and large-scale many-objective optimization approaches, highlighting how various methods have been developed by customizing existing optimization algorithms. Meanwhile, Table 2 provides brief descriptions of various comparative studies conducted by previous researchers.

#### 3 Experiment design

We empirically assess the efficacy of three algorithms, that is, LMEA [50], LMOCSO [12] and S3CMAES [10] by conducting experiments on a well-established set of benchmark problems, namely, the DTLZ test suites [41] and LSMOP test suite [31]. All the simulations were conducted using MATLAB R2023a environment on a Macbook Air powered by macOS, equipped with

Servers (ESs) deployment Software module clustering [38] turing Problem [36] Software package restrucing [1] charging schedules [34] Optimizing electric vehicle (EV) charging and dis-Food-energy-water nexus [33] Optimization covery [37] Software architecture re-Application Software module clusterof Edge five objective functions, more than 100 decision vari-Scalemore than ables tions More than six obnine objective func-More than four obfunctions and four objective Decision variables more than 480 5 objective functions range 3-7 and objective funcbetween Decision variables variables than 100 decision tions and more jective functions jective functions 100-991proach proach Clustering algorithms and Solution Technique Dimensionality reduction Search-based evolutionary algorithms Grid-based approach approaches Search-based optimization Dominance-based proach Clustering-based Preference-based approach, apapap-Coevolutionary Preference inspired algorithm coevolutionary PSOPSOPSOABC algorithmLCSA PSOAlgorithm algorithm Modified Modified LCSA rithm**PCMaLIA** GLMPSO LSM-PSO Customized PSO MaABC Algo-PICEAg-EV Problems FEWN LSMaOSPR MaODES LMSMCPs problems LSMaO-SAR MaSMCP

Table 1: Many-objective and large-scale many-objective Approaches

Table 2: Summary of comparative results of various algorithms

Algorithm	Test Suite	Decision Variables	Objectives	Performance Metric	Runs
LS-SMS-MOEA, LS-NSGAII, LS-MOEA/D, etc. [29]	LSMOP(1-9)	1000	3	IGD	20
LCSA, LMOCSO, S3CMAES, etc. [14, 25]	LSMOP(1-9), LS- MaOPFEWN	315	5	GD, IGD, HV	30
MOEA/D, NSGA-III, KnEA, etc. [50]	DLTZ(1-9)	100, 500, 1000	5, 10	IGD	20
MOEA/D, NSGA-III, KnEA, etc. [50]	WFG 3	100, 500, 1000	5, 10	IGD	20
MOEA/D, NSGA-III, KnEA, etc. [50]		100, 500, 1000	3	IGD	20
LMEA [50]	DLTZ(1-7)	2000, 5000	5, 10	HV	20
MOEA/D, NSGA-III, KnEA, etc. [50]	LSMOP(1-9)	500	22	IGD	20
DVCOEA/ DVCOEA(NO-CO) DLTZ(1-7) [46]	DLTZ(1-7)	100, 500	22	IGD	20
DVCOEA/DVCOEA(NO-CO) [46]	LSMOP(1-9)	112	3	IGD	20
DVCOEA, DVCOEA(NO-CO) [46]	$\mathrm{UF}(3-8)$	100, 500	2	IGD	20
DVCOEA, DVCOEA(NO-CO) [46]	$\mathrm{UF}(9\text{-}10)$	100, 500	3	IGD	20
30 [46]	$\mathrm{UF}(1-7)$	100, 500	2	IGD	20
DVCOEA, MOEA/DVA, LMEA, CPSO [46]	$\mathrm{UF}(8\text{-}10)$	100, 500	3	IGD	20
DVCOEA, MOEA/DVA, DLTZ(1-7) LMEA, CPSO [46]	DLTZ(1-7)	100, 500	22	IGD	20
DVCOEA, MOEA/DVA, LMEA, CPSO [46]	LSMOP(1-9)	514	22	IGD	20
LSMaODE, CCGD3, MOEA/DVA, LMEA, LMCSO, S3CMAES [48]	LSMOP(1-9)	300, 500, 1000	8	IGD	30

an M1 Apple chip, 8 GB memory, 7 core Graphics processing unit, 8 core Central processing unit and 256 GB Solid state drive.

#### 3.1 Test problems

Previous researchers and practitioners have developed various test suites to assess the performance of optimization problems. These test suites encompass a range of complexities and difficulties. In this work, to compare the optimization approaches, we use the following two test suites: 1) DTLZ -Riquelme, von Lücken, and Baran [41] introduced test suite (DTLZ). The problems of this suite are flexible to meet wide range of objectives. The PF is represented by the initial M-1 decision variables, whereas the remaining variables are related to the convergence characteristic. In particular, the DTLZ suite has several distinguishing features. DTLZ1 and DTLZ3 include many local PFs. In DTLZ4, Pareto optimum solutions are not evenly distributed, and in DTLZ5 and DTLZ6, PF curves are degenerated. In DTLZ7, PF is broken, whereas DTLZ8 and DTLZ9 consist of constrained test problems. The DTLZ test suite makes a significant contribution by proposing a general design paradigm for creating test problems that are extended to accommodate a broad range of objectives and decision variables. 2) LSMOP- The test suite comprises nine problems tailored for LSMOO and LSMaO. Across these problems, the Pareto Front (PF) characteristics vary: LSMOP1-4 features linear PFs, LSMOP5-8 exhibits nonlinear PFs, and LSMOP9 presents a disconnected PF. 3) WFG [12]- Huband et al. proposed the WFG test suite in 2006. This test suite contains several many-objective test cases, which are characterised by their specific features. The PF of WFG test suite test cases have a different type of shapes. Therefore, the WFG test suite would accurately reflect the performance of convergence and diversity algorithms. 4) UF [12]- This test suite contains ten unconstrained test problems UF1-10. In our study, we have considered only two problems UF9 and UF10 for comparative analysis.

#### 3.2 Algorithms in comparison

The selection of the LMEA, LMOCSO, and S3CMAES algorithms for conducting the comparative study was based on several factors: 1) relevance to the problem domain,2) diversity in approaches, and 3) scalability and performance. By selecting LMEA, LMOCSO, and S3CMAES for our comparative study, we aim to provide valuable insights into their relative strengths and weaknesses in addressing the challenges posed by large-scale many-objective optimization problems.

- LMEA- A novel method that addresses the challenges of LSMaOPs is the clustering of a decision variable based evolutionary algorithm (LMEA). This method provides a decision variable clustering technique that classifies variables based on their properties of convergence and diversity. This method makes use of a fast nondominated sorting technique called T-ENS to enhance computation performance. According to empirical research, LMEA is quite useful for handling large-scale MaOPs, particularly those with up to 5000 decision variables.
- LMOCSO- An enhanced version of CSO, that is, large-scale Multi-Objective Competitive Swarm Optimizer (LMOCSO) is developed for LSMOPs. It incorporates a competitive mechanism and a unique particle updating approach to enhance search efficiency. This approach is adopted to overcome the limitations of traditional MOEAs when dealing with large-scale MOPs. Through experimental evaluations, this method has demonstrated its superiority over existing MOEAs. Its potential to handle the complexity of optimization problems would become more apparent with more improvements.
- S3CMAES-The S3CMAES is designed primarily to address MOPs having large-scale decision variables. Its primary purpose consists of approximate a collection of solutions that are Pareto-optimal, using a novel technique employing tiny subpopulations rather than a standard whole-population strategy. In this novel approach, each subpopulation is dedicated to exploring and improving a specific solution, leveraging a small population for this purpose. An important feature of S3CMAES

is the inclusion of a diversity enhancement approach. This approach is used to choose new solutions from those generated by convergent subpopulations.

• LMEA-hybrid- To tackle the scalability issues inherent in MaOPs, we introduce a hybrid Learning-based multi-objective evolutionary algorithm (LMEA-hybrid). The proposed approach combines the virtues of decision space decomposition and sophisticated selection mechanisms to improve both convergence and diversity when solving large-scale MaOPs. The algorithm starts with the initialization of a random population and partitioning the decision variables into the groups related to convergence and diversity. It is done through the LMEA's variable clustering strategy to maintain the problem structure and better direct the process of creating offspring. Once clustering is done, the population is assessed and the Tchebycheff-based environmental selection (T-ENS) is utilized to assign the fitness values. T-ENS balances convergence and diversity and acts as a sorting mechanism for the whole optimization process. In each iteration, two parents are chosen randomly, and a fitness-based tournament selection is employed to decide upon a winner and a loser. The offspring is produced by taking advantage of decision variable group knowledge: convergence-related variables are copied from the winner to preserve convergence pressure, and diversity-related variables are perturbed to search the space more widely. The offspring is assessed and re-placed back into the population via an environmental selection procedure that hybridizes T-ENS with angle-penalized distance (APD). This hybrid selection method guarantees that only individuals of good quality and well-distributed are maintained across generations. The algorithm proceeds until the number of function evaluations reaches the maximum. The last nondominated solutions in the population are retrieved to estimate the PF. In general, Hybrid-LMEA efficiently optimizes convergence and diversity by leveraging both structural properties in the decision space and advanced objective-space selection methods, making it applicable to large-scale many-objective cases. The Pseudocode of LMEA-hybrid is provided in Algorithm 1.

#### Algorithm 1 Hybrid-LMEA

```
1: P \leftarrow \text{InitializePopulation}(N)
 2: [C_{group}, D_{group}] \leftarrow \text{ClusterDecisionVariables}(P)
                                                                    ▶ LMEA clustering
 3: EvaluatePopulation(P)
 4: Fitness \leftarrow TENS(P)
                                                                        ▶ T-ENS sorting
 5: evalCount \leftarrow N
    while evalCount < MaxEval do
 6:
 7:
         [parent_1, parent_2] \leftarrow \text{SelectRandomPair}(P)
        if Fitness(parent_1) < Fitness(parent_2) then
 8:
             Winner \leftarrow parent_1
 9:
             Loser \leftarrow parent_2
10:
        else
11:
12:
             Winner \leftarrow parent_2
13:
            Loser \leftarrow parent_1
14:
        end if
        Offspring \leftarrow GenerateOffspring(Winner, Loser, C_{group}, D_{group})
15:
        Evaluate (Offspring)
16:
        P \leftarrow \text{EnvironmentalSelection}(P \cup \{Offspring\}, \text{TENS}, \text{APD})
17:
        evalCount \leftarrow evalCount + 1
18:
19: end while
20: return PF \leftarrow \text{ExtractNonDominatedSolutions}(P)
```

#### 3.3 Parameter setting

To attain the best performance, the suggested parameter values have been used. The experimental parameter settings are described below.

- Size of population: To guarantee the experiment's fairness, some of the experiment's settings were set consistently. The population size of every algorithm is fixed at 100.
- Runs and stopping criteria: The stopping criteria for all algorithm involves reaching permitted no. of evaluations. The optimum no. of evaluations for every test instance having 100, 300, or 500 decision variables is fixed to 10,000. To acquire statistical findings, each algorithm was subjected to twenty independent runs.
- Mutation and crossover: For DTLZ test suite problems all three algorithms being compared utilizes simulated binary crossover (SBX) [14]

approach to produce offspring. In addition, in all algorithms, a polynomial mutation is employed to every test problem. The parameters for the both crossover (nc) and mutation (nm) distribution indices are equal to twenty. The crossover probability (pc) is equal to 1.0 and mutation probability (pm) is equal to 1/K, where K represents the amount of decision variables. As recommended in [25], the control parameters F is equal to 0.5 and CR is equal to 1.0 in the differential evolution (DE) [3] technique.

- Other parameters: In the reference to LMEA, we establish the following parameters: In decision variable clustering, nSel (number of solutions) is equal to 2, nPer (no. of perturbations for solution) is equal to 4. Furthermore, nCor is set to six, which signifies the number of solutions used in the analysis of decision variable interaction. The penalty parameter (α) of APD in the case of LMOCSO is fixed to 2.
- Metric used for evaluating performance: IGD [41], GD, and HV analyzes the effectiveness of all three algorithms that are being compared.
   IGD considers convergence as well as diversity. A lower IGD value, Low GD value and higher HV value suggests that the solution set obtained is of higher quality.
- Significance Test: The Wilcoxon signed-rank test is used to determine the statistical significance of differences at a 5 % level. This test is used to compare the results of three competing algorithms. With a significance level of 0.05, it uses a two-tailed test to assess if S3CMAES performs better ("+"), similarly ("="), or worse ("-") than the comparison algorithms on each test problem.

#### 4 Results and discussions

In this section, we present the performance results and their implications of LMEA, LMOCSO, and S3CMAES on benchmark MaOPs and LSMOPs.

Table 3: IGD values (Mean and Standard deviation) of three algorithms on DTLZ (1-7)

Problem	M	Q	LMEA	LMOCSO	S3CMAES
	က	100	2.8056e+3 (1.07e+2)-	$4.6583e + 2 \; (1.15e + 2) +$	2.4024e+3 (1.64e+2)
DTLZ1		300	8.9264e+3 (2.42e+2)+	$1.4490e + 3 \; (1.98e + 2) +$	1.1277e+4 (1.53e+3)
		200	1.5172e+4 (2.62e+2)+	$2.3005\mathrm{e}{+3}\ (3.53\mathrm{e}{+2})+$	2.0124e+4 (2.69e+3)
	က	100	6.5683e+0 (2.21e-1)+	$6.3459 \mathrm{e} ext{-}1 \; (1.05 \mathrm{e} ext{-}1) +$	7.9829e+0 (5.12e-1)
DTLZ2		300	2.1774e+1 (4.39e-1)+	$1.8936\mathrm{e}{+0} \; (2.19\mathrm{e}{-1}) +$	2.5233e+1 (1.37e+0)
		200	3.7663e+1 (6.57e-1)+	$3.5089\mathrm{e}{+0} \ (4.65\mathrm{e}{-1}){+}$	4.1922e+1 (1.81e+0)
	3	100	8.7444e+3 (2.96e+2)-	$1.4619\mathrm{e}{+3}\; (2.02\mathrm{e}{+2})+$	8.2134e+3 (3.18e+2)
DTLZ3		300	2.9070e+4 (6.50e+2)+	$4.1622\mathrm{e}{+3}\; (6.87\mathrm{e}{+2})+$	3.1935e+4 (1.34e+3)
		200	5.0050e+4 (4.85e+2)+	$6.2391\mathrm{e}{+3}\; (1.34\mathrm{e}{+3})+$	5.3996e+4 (1.63e+3)
	3	100	6.9133e+0 (2.33e-1)+	$1.191\mathrm{e}{+0} \; (3.12\mathrm{e}{-1})+$	8.4777e+0 (3.98e-1)
DTLZ4		300	2.2244e+1 (4.83e-1)+	$5.1170\mathrm{e}{+0} \; (2.39\mathrm{e}{+0}){+}$	2.5002e+1 (1.01e+0)
		200	3.7661e+1 (5.94e-1)+	1.0649e + 1 (5.17e + 0) +	4.3203e+1 (1.33e+0)
	3	100	6.4890e+0 (2.43e-1)+	$5.1211e ext{-}1\ (1.14e ext{-}1) +$	8.1563e+0 (3.72e-1)
DTLZ5		300	2.1754e+1 (5.59e-1)+	$2.0570\mathrm{e}{+0} \; (3.13\mathrm{e}{-1}) \; +$	2.5061e+1 (1.31e+0)
		200	3.7625e+1 (5.55e-1)+	$3.5189\mathrm{e}{+0} \; (6.32\mathrm{e}{-1}){+}$	4.1749e+1 (1.33e+0)
	3	100	8.7049e+1 (3.82e-1)-	$3.3804\mathrm{e}{+1}~(5.72\mathrm{e}{+0})+$	7.9943e+1 (1.30e+0)
DTLZ6		300	2.6715e+2 (4.57e-1)+	$1.3167\mathrm{e}{+2} \; (5.41\mathrm{e}{+0}) +$	2.7156e + 2 (1.50e + 0)
		200	4.4843e+2 (8.45e-1)+	$2.2919\mathrm{e}{+2} \ (8.35\mathrm{e}{+0})+$	4.5261e+2 (1.97e+0)
	3	100	1.0803e+1 (4.73e-1)-	$6.9905\mathrm{e}{+0} \ (9.38\mathrm{e}{-1}){+}$	9.9503e+0 (6.29e-1)
DTLZ7		300	1.1243e+1 (2.87e-1)+	$1.0059\mathrm{e}{+1}\; (3.52\mathrm{e}{-1})+$	1.3865e+1 (8.06e-1)
		200	1.1348e+1 (2.93e-1)+	$1.0525\mathrm{e}{+1}\; (2.67\mathrm{e}{-1})+$	1.3494e+1 (1.03e+0)
≈ / - /+			17/4/0	21/0/0	

Problem DTLZ7 DTLZ1 DTLZ6 DTLZ5 DTLZ4 DTLZ2 DTLZ3 Z ٥٦ 100 100 300 100 300 100 500300 500300 100 500 300 U 4.9434e+4 (5.22e+2)+2.8728e+4(3.18e+2)+1.2393e+4(3.41e+2)+8.6843e+3 (2.72e+2)-7.3092e+3 (1.88e+2)+2.1501e+1 (5.56e-1)+3.7147e+1 (6.80e-1)+3.7182e+1 (7.26e-1)+2.1670e+1 (5.15e-1)+6.6572e+0 (2.43e-1)+4.4612e+2 (7.75e-1)+2.6512e + 2 (8.75e - 1) +3.7145e+1 (5.99e-1)+6.1582e+0 (2.99e-1)+2.1744e+1 (5.98e-1)+6.8204e+0 (2.08e-1)+2.3502e+3 (9.23e+1)-8.5176e+1 (2.93e-1)-1.7922e+1 (5.37e-1)-.9066e+1 (4.42e-1)+..9050e+1 (5.67e-1)+LMEA 4.5784e+2 (1.00e+2)+9.2129e+0 (4.86e+0)+2.2844e+2 (8.98e+0)+1.3220e+23.7580e+1 (2.20e+0)+8.6185e+0 (1.78e+0)+4.4336e + 32.2234e+3 (3.57e+2)+1.3851e+3 (1.85e+2) +7.0644e+3 (8.91e+2)+1.5013e + 33.3173e+0 (4.60e-1)+3.3942e+0 (7.90e-1)+1.9727e+0 (3.52e-1)+4.4682e+0 (8.52e-1)+1.4514e + 0 (2.47e - 1) +2.0493e+0 (2.82e-1)+ $1.7968\mathrm{e}{+1}~(1.22\mathrm{e}{+0})+$  $1.6877\mathrm{e}{+1}~(2.12\mathrm{e}{+0}){+}$ 9.4997e-1 (7.61e-2)+7.1284e-1 (2.31e-1)+LMOCSO 21/0/0(2.23e+2)+(8.30e+0)+(6.63e+2)+2.6912e+2 (1.53e+0)2.5092e+1 (1.55e+0)3.1929e+4 (1.31e+3)8.2648e+3 (3.59e+2)4.2061e+1 (1.79e+0)2.1072e+3 (1.10e+2)4.5008e+2 (1.67e+0)4.1757e+1 (1.76e+0)2.5080e+1 (1.38e+0)5.2833e+4 (1.46e+3)2.4789e+1 (9.80e-1)1.8099e+4 (3.89e+3)2.2818e+1 (1.27e+0)7.8989e+1 (1.28e+0)4.2167e+1 (9.33e-1)8.1818e+0 (3.87e-1)1.1644e+4 (2.00e+3)2.3008e+1 (9.58e-1)7.9740e+0 (5.01e-1)7.9336e+0 (6.02e-1)1.6081e+1 (9.61e-1)S3CMAES

Table 4: IGD values (Mean and standard dDeviation) of three algorithms on DTLZ (1-7)

Table 5: IGD values (Mean and Standard Deviation) of three algorithms on DTLZ (1-7)

Problem DTLZ1	M 10	100 300	LMEA 2.0209e+3 (1.68e+2)- 6.4660e+3 (3.57e+2)+	$\frac{\text{LMOCSO}}{4.7954\text{e}+2\ (2.64\text{e}+2)+}\\ 1.1081\text{e}+3\ (4.27\text{e}+2)+$	S3CMAES 1.5317e+3 (9.58e+1) 1.0343e+4 (1.71e+3)
DTLZ2	10	300 500 500	1.0131e+4 (2.44e+3)+6.4863e+0 (2.72e-1)+2.1846e+1 (6.00e-1)+3.7295e+1 (5.68e-1)+	1.8623e+3 (6.58e+2)+ $1.2657e+0 (1.88e-1)+$ $2.6807e+0 (1.18e+0)+$ $4.6228e+0 (2.05e+0)+$	1.8370e+4 (3.20e+3) 7.6738e+0 (3.62e-1) 2.4682e+1 (1.43e+0) 4.1881e+1 (1.78e+0)
DTLZ3	10	100 300 500	8.0562e+3 (3.08e+2)= 2.8608e+4 (3.56e+2)+ 4.9083e+4 (5.49e+2)+	1.4040e+3 (2.26e+2)+ $4.3359e+3 (7.42e+2)+ $ $6.9921e+3 (1.48e+3)+$	7.9046e+3 (4.80e+2) 3.1742e+4 (9.98e+2) 5.3422e+4 (1.58e+3)
DTLZ4	10	100 300 500	6.7155e+0 (1.81e-1)+ 2.1871e+1 (4.69e-1)+ 3.7280e+1 (6.66e-1)+	$1.7600e+0 (1.22e-1)+ \\ 4.2560e+0 (8.62e-1)+ \\ 6.9600e+0 (1.43e+0) +$	7.2796e+0 (3.82e-1) 2.4822e+1 (1.22e+0) 4.1852e+1 (1.64e+0)
DTLZ5	10	100 300 500	$\begin{array}{c} 5.9466e+0 \; (3.66e+1) + \\ 2.1430e+1 \; (5.64e+1) + \\ 3.6438e+1 \; (7.19e+1) + \end{array}$	$7.1356e-1 \; (5.15e-2)+ \ 1.8793e+0 \; (1.05e+0)+ \ 3.8951e+0 \; (1.51e+0)+$	$\begin{array}{c} 7.6499e+0 \; (3.97e-1) \\ 2.4832e+1 \; (1.15e+0) \\ 4.1086e+1 \; (1.48e+0) \end{array}$
DTLZ6	10	100 300 500	8.0928e+1 (3.25e-1)- 2.6064e+2 (8.53e-1)+ 4.4133e+2 (7.52e-1)+	$3.8122e+1 \ (1.96e+0)+ \ 1.3104e+2 \ (4.47e+0)+ \ 2.2564e+2 \ (6.79e+0)+$	7.5570e+1 (1.23e+0) 2.6520e+2 (1.48e+0) 4.4615e+2 (1.78e+0)
DTLZ7	10	100 300 500	3.7228e+1 (1.72e+0)- 4.0189e+1 (8.21e+1)+ 3.9829e+1 (8.05e+1)+	$egin{array}{l} 6.7123e+0 & (3.45e+0)+ \ 2.6920e+1 & (3.19e+0)+ \ 3.0637e+1 & (3.35e+0)+ \end{array}$	3.1854e+1 (1.53e+0) 4.6188e+1 (1.91e+0) 4.6048e+1 (2.16e+0)
≈/-/+			17/3/1	21/0/0	

### 4.1 Performance comparison of LMEA, LMOCSO, and S3CMAES on benchmark MaOPs

Tables 3, 4, and 5 display the IGD values (mean and standard deviation) of the three compared algorithms on the seven DLTZ problems having decision variables (100,300,500), as well as number of objectives (3,5,10) acquired via 20 independent runs. The superior result on every test instance is highlighted in bold. The Wilcoxon rank sum test has been employed at a significance level of 0.05. The symbols "+", "−", and "≈" signify that the result is statistically better, significantly worse and significantly similar to that obtained by S3CMAES, respectively. We can infer the following observations from the Tables 3–5. This can be observed that IGD values generated by LMOCSO, LMEA on every test problem are consistently improve as the amount of decision variables grows from 100 to 500, that indicates a potential scalability of LMOCSO and LMEA. LMOSCO surpasses the other two algorithms in solving DLTZ1-7 whereas LMEA coming second. LMOCSO is the best in all instances while S3CMAES have inferior performance. LMEA performs fairly better in DLTZ2, DLTZ4 and DLTZ5. Low IGD values indicate higher algorithm performance.

The performance of the three algorithms is visually compare with respect to diversity and convergence, the coordinates of output solution acquired by these three algorithms on ten objective DLTZ6 are presented in Figure 1. From Figure 1b, we can deduce that the output solution of the LMOCSO algorithm outperforms the other two compared algorithms with respect to diversity and convergence. In LMEA, solutions at objective ten fails to converge.

### 4.2 Performance comparison of LMEA, LMOCSO and S3CMAES on LSMOP test suite

To assess the effectiveness of LMEA, LMOCSO, and S3CMAES on difficult Large-scale MaOPs, experiments are conducted on the LSMOP suite. The IGD values of three algorithms are presented in Table 6 having six objectives

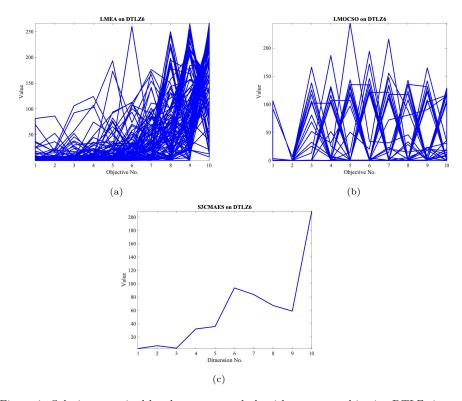


Figure 1: Solutions acquired by three compared algorithms on ten-objective DTLZ six problems with 300 decision variables (a) LMEA on DTLZ6. (b) LMOCSO on DTLZ6. (c) S3CMAES on DTLZ6.

and (100, 300, and 500) decision variables acquired via twenty independent runs. From Table 6, the subsequent observations can be drawn. LMEA having 500 decision variables is performing better than two algorithms on LSMOP2, LSMOP3, and LSMOP7-8. LMOCSO completely outperforms the other two algorithms on LSMOP1, LSMOP4, LSMOP5-6, and LSMOP9. In the case of LSMaOPs, LMOCSO outperforms the other two algorithms on 23 out of 27 test instances.

Table 7 presents the GD values of all three compared algorithms. LMEA performs better on 14 out of 27 test instances, while LMOCSO performs better on 9 out of 27 test instances. S3CMAES performs better on 4 out of 27 test instances. Therefore, LMEA performs superior to the other two algorithms in terms of GD values, while LMOCSO performs second best.

LMEA outperforms LSMOP1-2 with 500 decision variables and LSMOP4,7,8 with 300 and 500 decision variables. LMEA significantly performs better on LSMOP5,8. LMOCSO performs better than other algorithms on LSMOP1-2 with 300 decision variables. LMOCSO performs significantly better on LSMOP3,6 with 100, 300, and 500 decision variables and on LSMOP 9 with 100 decision variables.

Table 8 shows the HV values of three algorithms. Out of 27 LSMOP test instances, LMEA performs better in 19 instances. In comparison, LMOCSO performs better in 8 out of 27 instances. LMOCSO outperforms the other two algorithms on LSMOP1-2 having 100 and 300 decision variables, also LSMOP4 with 100, 300 and 500 decision variables and LSMOP5 with 100 decision variables. With regards to HV performance on the majority of LSMOP issues, it can be concluded that LMEA and LMOCSO are the best algorithms.

To confirm the computational efficiency of LMEA, LMOCSO and S3CMAES, Table 9 presents the average runtime(s) of the three compared algorithms on all runs for the test instances with 100, 300, and 500 decision variables, respectively. It can be noted that LMOCSO is computationally more efficient than the LMEA and S3CMAES on mostly all the test instances except LSMOP9 with 100 decision variables, where S3CMAES outperforms the other two algorithms.

The performance of the LMEA, LMOCSO, and S3CMAES is compared visually in terms of both convergence and diversity, and the output solution acquired by the three algorithms on 10-objective LSMOP6 is presented in Figure 2. From Figure 2a, we can deduce that the output solution set of the LMEA algorithm is much better than the other two compared algorithms with regard to both convergence and diversity. S3CMAES is performing worst in terms of convergence. The output solution of three algorithms on 10-objective LSMOP7 is presented in Figure 3. From Figure 3c, we can observe that S3CMAES is performing drastically worse than the other two algorithms with regard to convergence. From Figure 3a we can conclude that LMEA is performing more effectively.

The output solution of three algorithms on 10-objective LSMOP8 is presented in Figure 4. From Figure 4a, solutions of LMEA fail to converge at

objective 10. From Figure 4b, the solution set in the case of LMOCSO is not diverse.

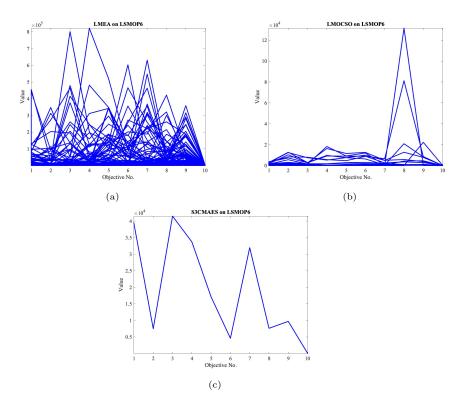


Figure 2: Solutions acquired by three compared algorithms on 10-objective LSMOP6 problems with 1000 decision variables (a) LMEA on LSMOP6. (b) LMOCSO on LSMOP6. (c) S3CMAES on LSMOP6.

## 4.3 Performance comparison of LMEA, LMOCSO and S3CMAES on UF and WFG test suites

In this section, we have compared the performance analysis of LMEA, LMOCSO and S3CMAES on UF and WFG test suites. The two problems UF9, UF10 and five problems WFG1-WFG5 have been considered from UF and WFG test suites respectively. Table 10 displays the statistical results of the IGD values of three algorithms with three objectives and 100, 300,

Table 6: IGD values (Mean and standard deviation) of three algorithms on LSMOP (1-9) using Wilcoxon signed-Rank test.

9.4298e + 2(3.38e + 2)	1.5070e+2 (6.13e+1) +	4.2396e + 2(2.17e + 1) +	500		
8.8449e + 2(3.87e + 2)	1.1058e + 2 (5.15e + 1) +	4.0461e + 2(2.14e + 1) +	300	6	LSMOP9
4.3889e + 2(7.85e + 1)	4.7217e+1 (2.21e+1)+	3.5670e + 2(3.19e + 1) +	100	)	
3.2233e + 1(1.91e + 1)	1.2575e+1 (1.24e+0)+	1.1879e+1 (7.97e-1)+	500		
3.0094e + 1(1.67e + 1)	3.7021e+0 (5.83e-1) +	1.1069e + 1(1.35e + 0) +	300	6	LSMOP8
1.6548e + 1(5.45e + 0)	2.3124e+0 (6.34e-1) +	1.0310e + 1(1.53e + 0) +	100		
2.8951e + 5(2.82e + 5)	2.9089e + 4(8.93e + 3) +	2.8057e+4 (4.39e+3) +	500		
3.1050e + 5(3.35e + 5)	1.7644e + 3 (1.15e + 3) +	2.5816e + 4(4.81e + 3) +	300	6	LSMOP7
1.9017e + 4(8.13e + 3)	3.1630e+1 (2.78e+1) +	$1.6657e + 4(4.20e + 3) \approx$	100		
2.4187e + 5(2.78e + 5)	$7.3031\mathrm{e}{+1}\; (3.20\mathrm{e}{+2})\; +$	1.8879e + 3(1.53e + 3) +	500		
2.0585e + 5(2.41e + 5)	$1.6835\mathrm{e}{+0} \ (6.56\mathrm{e}{-2}) \ +$	1.8804e + 3(1.84e + 3) +	300	6	LSMOP6
2.6136e + 0(3.26e - 1)	$1.6894\mathrm{e}{+0} \; (2.19\mathrm{e}{-1}) \; +$	4.3009e + 1(5.91e + 1) -	100		
6.1219e + 1(3.92e + 1)	1.1221e+1 (6.08e+0) +	1.6260e + 1(7.47e - 1) +	500		
4.6207e + 1(3.61e + 1)	$4.4084e+0 \ (9.32e-1) +$	1.5523e + 1(1.31e + 0) +	300	6	LSMOP5
1.9173e + 1(4.15e - 1)	$2.1034\mathrm{e}{+0} \ (5.56\mathrm{e}{-1}) \ +$	1.2874e + 1(2.03e + 0) +	100		
1.0240e + 0(4.33e - 1)	3.9979e-1 (7.76e-2) +	4.1160e - 1(2.53e - 2) +	500		
1.6182e + 0(8.10e - 1)	3.8595e-1 (4.89e-2) +	4.9135e - 1(3.24e - 2) +	300	6	LSMOP4
9.6423e - 1(1.82e - 1)	$5.5492\mathrm{e-1}\;(4.99\mathrm{e-2})\;+$	7.8327e - 1(8.90e - 2) +	100		
2.3369e + 5(3.74e + 5)	1.2884e + 4(7.44e + 3) +	$9.1783e+3 \ (4.15e+3) +  $	500		
2.2229e + 5(3.51e + 5)	5.5185e+1 (9.98e+1) +	8.9438e + 3(4.77e + 3) +	300	6	LSMOP3
6.9334e + 2(1.34e + 3)	$1.5982\mathrm{e}{+1}\ (2.25\mathrm{e}{+1})\ +$	5.1876e + 3(2.21e + 3) -	100		
9.9936e - 1(3.57e - 1)	3.4882e - 1(1.60e - 2) +	$3.2267 \mathrm{e} ext{}1 \; (2.41 \mathrm{e} ext{}2)$	500		
1.1558e + 0(5.50e - 1)	2.8391e-1 (2.77e-2) +	3.8340e - 1(2.62e - 2) +	300	6	LSMOP2
5.6671e - 1(5.79e - 2)	$4.9476\mathrm{e} ext{}1~(2.38\mathrm{e} ext{}2)~+$	6.5253e - 1(3.89e - 2) -	100		
5.7233e + 1(3.93e + 1)	1.0365e+1 (1.03e+0)+	1.0371e + 1(6.96e - 1) +	500		
3.4866e + 1(2.53e + 1)	$1.9560\mathrm{e}{+0}~(4.79\mathrm{e}{-1})~+$	9.7993e + 0(8.53e - 1) +	300	6	LSMOP1
9.3382e + 0(6.24e + 0)	1.1162e + 0(2.20e - 1) +	$7.8505e + 0(1.18e + 0) \approx$	100		
S3CMAES	LMOCSO	LMEA	D	Z	Problem
				1	

Table 7: GD values (Mean and standard deviation) of three algorithms on LSMOP (1-9) using Wilcoxon signed- Rank test.

Problem	Z	D	LMEA	LMOCSO	S3CMAES
		100	4.9528e + 0(6.55e - 1) -	3.0575e + 0(8.21e - 1) -	1.7881e+0 (1.62e+0)
LSMOP1	9	300	$5.1742e + 0(5.11e - 1) \approx$	$4.5261\mathrm{e} + 0(1.54\mathrm{e} + 0) +$	8.5532e + 0(6.30e + 0)
		200	$5.2389e+0 \ (3.86e-1) \ +$	$7.7224e + 0(1.19e + 0) \approx$	1.4128e + 1(9.81e + 0)
		100	2.0471e - 1(1.08e - 2) -	1.1158e - 1(8.90e - 3) -	8.1107e-2 (4.86e-3)
LSMOP2	9	300	$7.4545e - 2(3.94e - 3) \approx$	4.6465e-2 (7.63e-3)+	1.3081e - 1(9.84e - 2)
		200	4.2939e-2 (2.93e-3)+	$6.7061e - 2(8.24e - 3) \approx$	7.8811e - 2(5.31e - 2)
		100	4.4162e + 4(8.99e + 3) -	4.0139e+2 (8.10e+2)+	7.5513e + 3(2.21e + 3)
LSMOP3	9	300	7.2507e + 4(8.24e + 3) -	3.9390e + 3 (6.40e + 3) +	5.5572e + 4(8.77e + 4)
		200	7.5216e + 4(7.48e + 3) -	$3.7786\mathrm{e}{+4} (5.28\mathrm{e}{+4}) pprox$	5.8423e + 4(9.36e + 4)
		100	3.8524e - 1(2.57e - 2) -	3.1893e - 1(5.87e - 2) -	1.1344e-1 (8.29e-2)
LSMOP4	9	300	$1.3473 \mathrm{e} ext{-}1 \; (1.00 \mathrm{e} ext{-}2) pprox$	$1.3485e - 1(2.61e - 2) \approx$	2.4017e - 1(1.69e - 1)
		200	7.6567e-2 $(6.30 \mathrm{e}{ ext{-}3}) pprox$	1.3315e - 1(4.38e - 2) -	1.0934e - 1(7.95e - 2)
		100	$1.1538\mathrm{e}{+1} (1.85\mathrm{e}{+0}) pprox$	$1.6819e + 1(7.42e + 0) \approx$	1.3671e + 1(6.48e + 0)
LSMOP5	9	300	$8.4363\mathrm{e}{+0}(8.73\mathrm{e}{-1}){pprox}$	$1.2601e + 1(4.34e + 0) \approx$	1.1438e + 1(9.03e + 0)
		200	$8.0116\mathrm{e}{+0(6.60\mathrm{e}{-1})}{+}$	$1.2601e + 1(4.34e + 0) \approx$	1.5188e + 1(9.81e + 0)
		100	2.6669e + 4(5.36e + 3) +	$1.2090\mathrm{e}{+1}(2.47\mathrm{e}{+1})+$	8.5007e + 4(5.97e + 4)
$_{ m LSMOP6}$	9	300	$5.0053e + 4(8.07e + 3) \approx$	$3.1980\mathrm{e}{+4} (4.01\mathrm{e}{+4}) pprox$	5.1463e + 4(6.03e + 4)
		200	$4.8144e + 4(8.84e + 3) \approx$	$4.6665\mathrm{e}{+4} (3.76\mathrm{e}{+4}) \approx$	6.0468e + 4(6.96e + 4)
		100	3.6251e + 4(2.15e + 4) -	$1.1068e + 4(2.00e + 4) \approx$	9.1504e + 3(7.38e + 3)
LSMOP7	9	300	$5.1071\mathrm{e}{+4} (2.03\mathrm{e}{+4}) \approx$	$5.3012e + 4(5.64e + 4) \approx$	7.7625e + 4(8.38e + 4)
		200	$4.2243\mathrm{e}{+4} (1.25\mathrm{e}{+4}) \approx$	$5.6367e + 4(2.23e + 4) \approx$	7.2377e + 4(7.04e + 4)
		100	$7.0013\mathrm{e}{+0} \; (1.38\mathrm{e}{+0}) pprox$	$8.9649e+0 (3.51e+0) \approx$	7.1583e + 0(3.95e + 0)
LSMOP8	9	300	$4.4857\mathrm{e}{+0(3.33\mathrm{e}{-1)}}{+}$	9.0603e + 0(2.34e + 0) -	7.4020e + 0(4.18e + 0)
		200	$4.1446e+0 \ (4.83e-1)+$	$7.6076e + 0(6.95e - 1) \approx$	7.9355e + 0(4.78e + 0)
		100	1.2912e + 2(1.19e + 1) -	4.8371e + 1(1.78e + 1) +	6.5185e + 1(1.01e + 1)
LSMOP9	9	300	1.1175e+2 (7.51e+0) +	1.5240e + 2(6.36e + 1) +	2.2024e + 2(9.66e + 1)
		500	1.0975e+2 (6.53e+0) +	$1.9926e + 2(7.90e + 1) \approx$	2.3486e + 2(8.45e + 1)
≈/-/+			8/8/11	7/5/15	

Table 8: HV values (Mean and standard deviation) of three algorithms on LSMOP (1-9) using Wilcoxon signed-Rank test.

	8/0/19	5/1/21			+/-/≈
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}~(0.00\mathrm{e}{+0}){pprox}$	500		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}\;(0.00\mathrm{e}{+0})pprox$	300	6	LSMOP9
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0 \approx$	$0.0000\mathrm{e}{+0}\;(0.00\mathrm{e}{+0}) pprox$	100		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}\;(0.00\mathrm{e}{+0})pprox$	500		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}\;(0.00\mathrm{e}{+0})pprox$	300	6	LSMOP8
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}\;(0.00\mathrm{e}{+0})pprox$	100		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}\;(0.00\mathrm{e}{+0})pprox$	500		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}\;(0.00\mathrm{e}{+0})pprox$	300	6	LSMOP7
(0+00000e+000000e+000000000000000000000	$0.0000e + 0(0.00e + 0 \approx$	$0.0000\mathrm{e}{+0}\;(0.00\mathrm{e}{+0})pprox$	100		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}(0.00\mathrm{e}{+0}){pprox}$	500		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}(0.00\mathrm{e}{+0}~){pprox}$	300	6	LSMOP6
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}\;(0.00\mathrm{e}{+0})pprox$	100		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0} \; (0.00\mathrm{e}{+0}){pprox}$	500		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}\;(0.00\mathrm{e}{+0})pprox$	300	6	LSMOP5
0.0000e + 0(0.00e + 0)	$1.7955 \mathrm{e} ext{-}4 (8.03 \mathrm{e} ext{-}4) pprox$	$0.0000e + 0(0.00e + 0) \approx$	100		
1.0102e - 1(7.89e - 2)	$oxed{6.4191e-1 (1.25e-1) +}$	6.2718e - 1(3.16e - 2) +	500		
4.3841e - 2(6.35e - 2)	7.4661e-1 (8.03e-2) +	5.1515e - 1(2.95e - 2) +	300	6	LSMOP4
7.6731e - 2(1.20e - 1)	$5.4001e ext{-}1\ (9.23e ext{-}2)\ +$	1.5055e - 1(5.86e - 2) +	100		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}~(0.00\mathrm{e}{+0}){pprox}$	500		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}\;(0.00\mathrm{e}{+0})pprox$	300	6	LSMOP3
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}~(0.00\mathrm{e}{+0}){pprox}$	100		
9.1660e - 2(9.43e - 2)	6.7722e - 1(2.89e - 2) +	7.5768e-1 (2.21e-2)+	500		
7.5139e - 2(8.32e - 2)	$8.3408e-1 \ (3.92e-2) \ +$	6.5017e - 1(2.59e - 2) +	300	6	LSMOP2
4.6599e - 1(8.46e - 2)	5.1213e-1 (3.87e-2) +	2.6831e - 1(3.84e - 2) -	100		
0.0000e + 0(0.00e + 0)	$0.0000e + 0(0.00e + 0) \approx$	$0.0000\mathrm{e}{+0}(0.00\mathrm{e}{+0})pprox$	500		
0.0000e + 0(0.00e + 0)	4.1879e-4(1.49e-3) +	$0.0000e + 0(0.00e + 0) \approx$	300	6	LSMOP1
(0+90000)0+9000000	$oxed{2.9069e-2\ (4.14e-2)} +$	$\approx (0 + 900.00) + 90000000$	100		
S3CMAES	${f LMOCSO}$	$\mathbf{LMEA}$	D	Μ	$\operatorname{Problem}$

Table 9: Run Time values (Mean and Standard Deviation) of three algorithms on LSMOP (1-9) using Wilcoxon signed-Rank test.

Problem	M	D	LMEA	LMOCSO	S3CMAES
		100	3.2656e + 1 (1.13e + 2) -	4.7273e-1 $(7.90e-2) +$	2.9315e + 0 (2.24e - 1)
LSMOP1	9	300	8.5448e + 1 (4.33e + 1) -	8.6804e-1 (1.34e-1) +	1.7116e + 1 (1.51e - 1)
		200	8.2375e + 2 (2.33e + 3) -	5.5474e-2  (4.94e-2) +	4.7838e + 1 (1.57e + 0)
		100	8.0957e + 0 (3.61e + 0) -	6.3782e-1  (5.55e-2) +	2.8009e + 0 (5.68e - 2)
LSMOP2	9	300	6.3424e + 1 (4.14e + 1) -	7.6440e-1 (4.81e-2) +	1.9163e + 1 (1.42e - 1)
		200	4.5228e + 2 (1.26e + 3) -	8.5914e-2 (1.02e-1) +	5.3438e + 1 (3.15e + 0)
		100	8.5731e + 0 (2.99e + 0) -	1.7761e+0 (8.12e-1) +	2.7852e + 0 (6.46e - 2)
LSMOP3	9	300	9.5423e + 1 (4.09e + 1) -	1.3840e+0  (4.57e-1) +	1.8732e + 1 (1.21e - 1)
		200	3.0195e + 2(2.47e + 2) -	6.7954e-2  (4.85e-2) +	5.2643e + 1 (1.92e + 0)
		100	8.7249e + 0 (4.40e + 0) -	6.4083e-1  (2.03e-2) +	2.8146e + 0 (1.17e - 1)
LSMOP4	9	300	9.9880e + 1 (4.28e + 1) -	8.1007e-1 (5.90e-2) +	1.8770e + 1 (2.12e - 1)
		200	5.6169e + 2 (9.16e + 2) -	1.4838e-1 (1.45e-1) +	5.2188e + 1 (4.37e - 1)
		100	7.6427e + 0 (6.46e + 0) -	7.0459e-1  (9.46e-2) +	2.6329e + 0 (1.15e - 1)
LSMOP5	9	300	7.9805e + 1 (2.86e + 1) -	7.0803e-1  (4.88e-2) +	1.8190e + 1 (2.53e - 1)
		200	1.3344e + 2 (1.18e + 2) -	1.2870e-1  (9.68e-2) +	5.0431e + 1 (7.99e - 1)
		100	$4.5128e + 0(2.48e + 0) \approx$	$9.6853e-1 \ (2.03e-1) +$	3.0962e + 0(1.10e - 1)
LSMOP6	9	300	4.6114e + 1(2.46e + 1) -	$8.5650 \mathrm{e}\text{-}1 \; (1.47 \mathrm{e}\text{-}1) \; +$	2.0664e + 1(2.09e + 1)
		200	$6.6066e + 2(2.42e + 3) \approx$	2.2104e-1 $(1.40$ e-1) +	5.7946e + 1(1.05e + 0)
		100	8.5955e + 0(2.63e + 0) -	1.9460e+0 (1.00e+0) +	2.9440e + 0(9.10e - 2)
LSMOP7	9	300	7.2473e + 1(4.23e + 1) -	$1.4504e+0 \ (4.83e-1) +$	2.0520e + 1(2.96e - 1)
		200	2.0263e + 2(1.20e + 2) -	$5.0781e-2 \; (1.73e-2) +$	5.6966e + 1(4.94e - 1)
		100	8.8211e + 0(2.42e + 0) -	$6.9920e-1 \ (1.27e-1) +$	2.8547e + 0(7.95e - 2)
LSMOP8	9	300	7.8453e + 1(3.62e + 1) -	8.3320e-1 $(9.83$ e-2 $) +$	2.1785e + 1(4.12e - 1)
		200	5.7518e + 2(1.11e + 3) -	$7.6261e-2 \; (3.79e-2) +   $	5.6005e + 1(3.54e - 1)
		100	6.8140e + 0(4.24e + 0) -	3.6036e+0 (4.28e-1) -	2.6631e+0 (7.04e-2)
LSMOP9	9	300	8.1672e + 1(5.39e + 1) -	$4.2458e+0 \ (3.68e-1) +$	1.8807e + 1(3.62e - 1)
		500	2.6924e + 2(1.41e + 2) -	$4.3651\mathrm{e}{+0} \; (4.67\mathrm{e}{-1}) \; + \;$	5.2610e + 1(6.81e - 1)
≈ / - /+			0/25/2	26/1/0	

Table 10: IGD values (Mean and standard ddeviation) of three algorithms on UF9,10 and WFG (1-9) using Wilcoxon signed-Rank test.

	21 / 0 / 0	15 / 5 / 1			+/-/≈
3.4122e + 0(2.24e - 1)	2.7839e-1 (8.80e-3) +	8.8048e - 1(2.87e - 2) +	500		
3.1492e + 0(3.59e - 1)	$2.7397 \mathrm{e} ext{}1 \; (1.05 \mathrm{e} ext{}2) \; +$	8.8290e - 1(2.24e - 2) +	300	ಬ	WFG5
6.5978e - 1(1.99e - 2)	2.7060e-1 (1.16e-2) +	8.8359e - 1(3.15e - 2) -	100		
3.0926e + 0(5.54e - 1)	$3.5734 e ext{-}1 \ (1.19 e ext{-}2) \ +$	8.7826e - 1(1.07e - 1) +	500		
3.4322e + 0(4.16e - 1)	3.5481e-1 (1.06e-2) +	8.6043e - 1(8.15e - 2) +	300	ယ	WFG4
6.5867e - 1(2.39e - 2)	3.4440e-1 (1.16e-2) +	8.8413e - 1(7.30e - 2) -	100		
2.4104e + 0(4.38e - 1)	5.9368e-1 (1.25e-2) +	8.8828e - 1(6.01e - 3) +	500		
2.3970e + 0(4.62e - 1)	$5.8650 e ext{}1 \ (2.28 e ext{}2) \ +$	8.8114e - 1(8.36e - 3) +	300	ಬ	WFG3
8.4965e - 1(3.47e - 2)	5.3772e-1 (2.89e-2) +	8.5789e - 1(1.04e - 2) =	100		
4.0737e + 0(1.08e + 0)	5.1609e-1 (4.68e-2) +	1.0731e + 0(8.60e - 2) +	500		
4.1776e + 0(9.06e - 1)	$5.0352 \mathrm{e} ext{}1 \; (3.72 \mathrm{e} ext{}2) \; + \;$	1.0423e + 0(1.22e - 1) +	300	သ	WFG2
1.1275e + 0(1.03e + 0)	$4.6531e-1 \ (3.14e-2) \ +$	1.1216e + 0(1.38e - 1) +	100		
2.7337e + 0(4.60e - 2)	1.8972e+0 (2.12e-1) +	2.3966e + 0(5.69e - 2) +	500		
2.7204e + 0(5.07e - 2)	1.9678e + 0 (1.86e - 1) +	2.4084e + 0(6.20e - 2) +	300	ယ	WFG1
1.9070e + 0(3.46e - 2)	$oxed{1.8117\mathrm{e}{+0}\ (1.56\mathrm{e}{-1})} +$	2.4084e + 0(6.41e - 2) -	100		
2.7901e + 1(6.56e + 0)	1.0587e+1 (1.91e+0) +	2.0747e + 1(4.20e - 1) +	500		
2.9497e + 1(8.98e + 0)	$8.4764 \mathrm{e}{+0} \; (1.79 \mathrm{e}{+0}) \; +$	2.0183e + 1(6.79e - 1) +	300	ယ	UF10
9.7564e + 0(1.68e + 0)	$\mid$ 4.7046e+0 (9.49e-1) +	1.8980e + 1(9.27e - 1) -	100		
7.3970e + 0(2.55e + 0)	1.8838e+0 (3.87e-1) +	4.4402e + 0(1.04e - 1) +	500		
7.1931e + 0(2.40e + 0)	1.7067e+0 (3.76e-1) +	4.3633e + 0(1.15e - 1) +	300	ယ	UF9
2.0081e + 0(4.32e - 1)	$8.5927e-1 \ (9.65e-2) \ +$	4.0297e + 0(1.87e - 1) -	100		
S3CMAES	LMOCSO	LMEA	D	Z	$\mathbf{Problem}$

Table 11: IGD values (Mean and standard deviation) of three algorithms on LSMOP (1-9) using Wilcoxon signed-Rank test.

S3CMAES	3.7324e+1 (4.88e+1)	7.2293e-1 (1.93e-1)	3.4912e+5 (2.40e+5)	8.8056e-1 (2.12e-1)	5.7533e+1 (4.40e+1)	1.4742e+5 (2.75e+5)	1.3955e+5 (1.13e+5)	2.9253e+1 (1.48e+1)	8.3041e+2 (2.62e+2)	
$\Gamma$ MOCSO	2.1477e+0 (2.88e-1) +	3.0383e-1 (3.27e-2) +	$1000 \mid 8.4019e+3 (6.98e+3) + \mid 7.8985e+3 (4.22e+3) + \mid 3.4912e+5 (2.40e+5)$	3.3057e-1 (3.09e-2) +	$1000 \mid 1.6723e+1 \ (1.10e+0) + \mid 1.6423e+1 \ (7.28e-1) + \mid 5.7533e+1 \ (4.40e+1)$	$1000 \mid 2.6925e+3 (2.18e+3) + \mid 1.3219e+3 (1.75e+3) + \mid 1.4742e+5 (2.75e+5)$	$1000 \mid 2.7767e+4 (5.22e+3) + \mid 2.6676e+4 (3.01e+3) + \mid 1.3955e+5 (1.13e+5)$	1000   1.1996e+1 (1.19e+0) +   1.157e+1 (3.88e-1) +   2.9253e+1 (1.48e+1)	LSMOP9   6   1000   4.5064e+2 (1.27e+1) +   $4.3014e+2$ ( $1.59e+1$ ) +   $8.3041e+2$ ( $2.62e+2$ )	0/0/6
$\mathbf{LMEA}$	1.0244e+1 (7.71e-1) +	3.0604e-1 (2.39e-2) +	8.4019e+3 (6.98e+3) +	3.8051e-1 (3.25e-2) +	1.6723e+1 (1.10e+0) +	2.6925e+3 (2.18e+3) +	2.7767e+4 (5.22e+3) +	1.1996e+1 (1.19e+0) +	4.5064e+2 (1.27e+1) +	0/0/6
Ω	1000	1000	1000	1000	1000	1000	1000	1000	1000	
Σ	9	9	9	9	9	9	9	9	9	
Problem   M   D	LSMOP1	LSMOP2 6	LSMOP3	LSMOP4	LSMOP5	LSMOP6	LSMOP7	LSMOP8	SMOP9	≈ / - /+

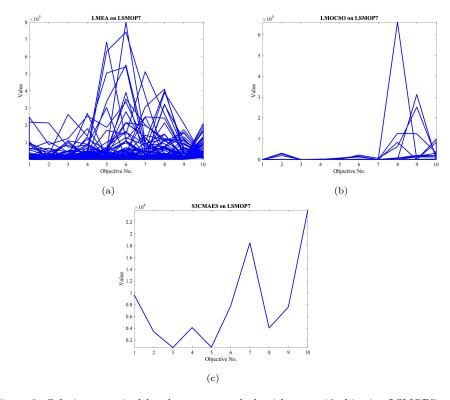


Figure 3: Solutions acquired by three compared algorithms on 10-objective LSMOP7 problems with 1000 decision variables (a) LMEA on LSMOP7. (b) LMOCSO on LSMOP7. (c) S3CMAES on LSMOP7.

and 500 decision variables obtained during 20 separate runs. From Table 10, it can be observed that LMOCSO performs significantly better than the other two algorithms. Whereas LMEA performs significantly better than S3CMAES on 15 test instances.

### 4.4 Performance comparison of LMEA, LMOCSO and S3CMAES with decision variables 1000

In this section, the performance of LMEA, LMOCSO, S3CMAES has been compared by challenging them with 1000 decision variables. Table 11 presents IGD values obtained by the three algorithms on LSMOP1-LSMOP9 with

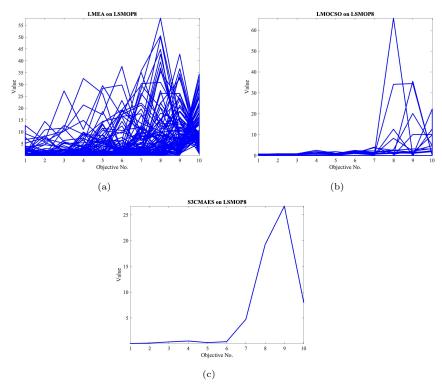


Figure 4: Solutions acquired by three compared algorithms on 10-objective LSMOP8 problems with 1000 decision variables (a) LMEA on LSMOP8. (b) LMOCSO on LSMOP8. (c) S3CMAES on LSMOP8.

six objectives and 1000 decision variables, via 20 runs. It can be seen that LMOSCO still performs the best on LSMOPs, which confirms its effectiveness in solving LSMaOPs.

### 4.5 Performance comparison of LMEA, LMOCSO, S3CMAES and LMEA-hybrid with decision vriables varying between 100-500

In this section, we have compared the performance of LMEA, LMOCSO, S3CMAES with the proposed algorithm LMEA-hybrid. Table 12 shows IGD values obtained by four algorithm on LSMOP1-9 with six objectives. It

can been observed that LMOCSO performs best on 100 and 300 decision variables (LSMOP1,3,4,5,6,7,9), while LMEA-hybrid performs better in case of 500 decision variables (LSMOP1,2,3,4,6,7,8). Table 13 shows GD values of four algorithms on LSMOP1-9. It can be observed that LMOCSO performs better on LSMOP(3,6), whereas LMEA-hybrid is performing best on 12 test instances out of 27.

#### 5 Conclusion

With an increase in the number of objectives, Pareto dominance becomes less efficient, such that most solutions end up being nondominated. This calls for algorithms that can efficiently solve LSMaOPs with a large number of decision variables and objectives.

To overcome this difficulty, we performed a thorough comparative study of three cutting-edge LSMaOEAs—LMEA, LMOCSO, and S3CMAES—over decision variable ranges of 100 to 1000, with uniform objective counts. Their performance was measured by four important metrics: IGD, GD, HV, and runtime. The results showed that LMOCSO performs consistently well for the majority of test cases, showing scalability and robustness on benchmark suites like DTLZ, LSMOP, WFG, and UF.

As part of this study, we also made a new variant—Hybrid-LMEA—that combines decision variable clustering with competitive learning dynamics. The hybrid model maintains a balance of the convergence-diversity tradeoff by retaining structure during exploitation and encouraging exploration by diversity-aware selection. It clearly demonstrated performance benefits on LSMOPs with more than 500 decision variables, where it improves both IGD and GD metrics consistently.

In future research, we intend to compare the performance of Hybrid-LMEA on actual optimization problems, for example, software module clustering, in order to further investigate its potential utility.

Table 12: IGD values (Mean and standard deviation) of four algorithms on LSMOP (1-9) using Wilcoxon signed-rank test.

_	_			_			_			_			_			_			_			_			_			
${f LMEA\_hybrid}$	7.1491e + 0(1.51e + 0)	7.4620e + 0(7.95e - 1)	7.3197e+0 (6.28e-1)	5.8248e - 1(3.36e - 2)	2.7352e-1 (7.03e-3)	$2.3563e-1 \ (7.04e-3)$	3.0315e + 3(1.68e + 3)	6.2121e + 3(2.65e + 3)	6.7433e + 3 (2.93e + 3)	9.2827e - 1(1.43e - 1)	4.6084e - 1(3.48e - 2)	3.6192e-1(2.41e-2)	3.3727e + 0(2.09e + 0)	1.1137e + 1(2.57e + 0)	1.1688e + 1(1.55e + 0)	2.4560e + 0(2.33e - 1)	1.6849e + 0(7.88e - 2)	1.5111e+0 (3.01e-2)	1.6939e + 2(1.61e + 2)	1.9055e + 3(1.25e + 3)	8.1598e+3 (4.93e+3)	$2.1625e+0 \ (1.12e+0)$	7.0942e + 0(2.38e + 0)	$8.1381e+0 \ (1.14e+0)$	1.0407e + 2(1.31e + 1)	7.0942e + 0(2.38e + 0)	1.6877e + 2(2.14e + 1)	
S3CMAES	9.3382e + 0(6.24e + 0) -	3.4866e + 1(2.53e + 1) -	5.7233e + 1(3.93e + 1) -	$5.6671e - 1(5.79e - 2) \approx$	1.1558e + 0(5.50e - 1) -	9.9936e - 1(3.57e - 1) -	6.9334e + 2(1.34e + 3) +	2.2229e + 5(3.51e + 5) -	2.3369e + 5(3.74e + 5) -	$9.6423e - 1(1.82e - 1) \approx$	1.6182e + 0(8.10e - 1) -	1.0240e + 0(4.33e - 1) -	1.9173e + 1(4.15e - 1) -	4.6207e + 1(3.61e + 1) -	6.1219e + 1(3.92e + 1) -	$2.6136e + 0(3.26e - 1) \approx$	2.0585e + 5(2.41e + 5) -	2.4187e + 5(2.78e + 5) -	1.9017e + 4(8.13e + 3) -	3.1050e + 5(3.35e + 5) -	2.8951e + 5(2.82e + 5) -	1.6548e + 1(5.45e + 0) -	3.0094e + 1(1.67e + 1) -	3.2233e + 1(1.91e + 1) -	4.3889e + 2(7.85e + 1) -	8.8449e + 2(3.87e + 2) -	9.4298e + 2(3.38e + 2) -	1 / 23 / 3
LMOCSO	1.1162e+0 (2.20e-1) +	$1.9560e+0 \ (4.79e-1) +$	1.0365e + 1(1.03e + 0) -	4.9476e-1 (2.38e-2) +	$2.8391e - 1(2.77e - 2) \approx$	3.4882e - 1(1.60e - 2) -	1.5982e+1 (2.25e+1) +	5.5185e+1 (9.98e+1) +	1.2884e + 4(7.44e + 3) -	5.5492e-1 (4.99e-2) +	3.8595e-1 (4.89e-2) +	$3.9979e - 1(7.76e - 2) \approx$	$2.1034\mathrm{e}{+0} \; (5.56\mathrm{e}{-1}) pprox$	$4.4084e+0 \ (9.32e-1) +$	$1.1221\mathrm{e}{+1}\; (6.08\mathrm{e}{+0}) pprox$	1.6894e+0 (2.19e-1) +	$1.6835\mathrm{e}{+0}\; (6.56\mathrm{e}{-2}) pprox$	7.3031e + 1(3.20e + 2) -	3.1630e+1 (2.78e+1) +	$1.7644\mathrm{e}{+3} \; (1.15\mathrm{e}{+3}){pprox}$	2.9089e + 4(8.93e + 3) -	$2.3124e + 0(6.34e - 1) \approx$	3.7021e+0 (5.83e-1) +	1.2575e + 1(1.24e + 0) -	4.7217e+1 (2.21e+1)+	1.1058e+2 (5.15e+1) +	$1.5070\mathrm{e}{+2} \; (6.13\mathrm{e}{+1}) \; + \;  $	14 / 6 / 7
$\mathbf{LMEA}$	$7.8505e + 0(1.18e + 0) \approx$	9.7993e + 0(8.53e - 1) -	1.0371e + 1(6.96e - 1) -	6.5253e - 1(3.89e - 2) -	3.8340e - 1(2.62e - 2) -	3.2267e - 1(2.41e - 2) -	5.1876e + 3(2.21e + 3) -	$8.9438e + 3(4.77e + 3) \approx$	$9.1783e + 3(4.15e + 3) \approx$	7.8327e - 1(8.90e - 2) +	4.9135e - 1(3.24e - 2) -	4.1160e - 1(2.53e - 2) -	1.2874e + 1(2.03e + 0) -	1.5523e + 1(1.31e + 0) -	1.6260e + 1(7.47e - 1) -	4.3009e + 1(5.91e + 1) -	1.8804e + 3(1.84e + 3) -	1.8879e + 3(1.53e + 3) -	1.6657e + 4(4.20e + 3) -	2.5816e + 4(4.81e + 3) -	2.8057e + 4(4.39e + 3) -	1.0310e + 1(1.53e + 0) -	1.1069e + 1(1.35e + 0) -	1.1879e + 1(7.97e - 1) -	3.5670e + 2(3.19e + 1) -	4.0461e + 2(2.14e + 1) -	4.2396e + 2(2.17e + 1) -	1 / 23 / 3
Q	100	300	200	100	300	200	100	300	200	100	300	200	100	300	200	100	300	200	100	300	200	100	300	200	100	300	500	
M		9			9			9			9			9			9			9			9			9		≈
$\mathbf{Problem}$		LSMOP1			LSMOP2			LSMOP3			LSMOP4			LSMOP5			LSMOP6			LSMOP7			LSMOP8			LSMOP9		-/+

Table 13: GD values (Mean and standard dDeviation) of three algorithms on LSMOP (1-9) using Wilcoxon signed- Rank test.

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