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Research Article

Unconditionally stable high-order scheme for the fractional generalized Burgers–Huxley equation

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Abstract

This study presents a high-order numerical scheme for solving the generalized fractional-order Burgers–Huxley equation, a nonlinear evolutionary partial differential equation (PDE) combining integer-order spatial derivatives with temporal fractional derivatives. The model effectively captures key features of reaction–diffusion–convection systems and neural pulse propagation dynamics. To this end, we develop a fourth-order compact finite difference method for spatial discretization coupled with a Grünwald–Letnikov approximation for the fractional time derivative. The resulting nonlinear system is solved using an efficient iterative algorithm. A rigorous stability and convergence analysis shows that the proposed method is unconditionally stable and achieves $\mathcal{O}(\tau^3 + h^4)$ convergence, where τ and h represent temporal and spatial step sizes, respectively. Numerical experiments validate the theoretical results and demonstrate the

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scheme's accuracy and robustness across multiple test cases, confirming its effectiveness for this class of fractional PDEs.

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1 Introduction

Fractional calculus (FC) has established itself as a fundamental mathematical framework for modeling complex systems exhibiting memory effects and spatial nonlocality [5]. Fractional-order integrals and derivatives, which generalize their integer-order counterparts, provide a powerful framework for describing systems with memory effects and nonlocality [6, 2]. While FC offers powerful modeling capabilities, obtaining exact analytical solutions to fractional differential equations remains fundamentally challenging–indeed, often intractable. This persistent difficulty has motivated five decades of sustained research into numerical approximation techniques, particularly as FC applications have expanded across diverse domains including nonlinear wave phenomena, biological systems modeling, engineering dynamics, and transport processes. We introduce a novel discretization scheme specifically designed for boundary-value problems governed by the fractional generalized Burgers–Huxley (FGBH) equation:

$$\begin{cases} {}_0\mathcal{D}_t^{\alpha} w + \lambda\, w^{\theta} w_z - \sigma w_{zz} = \beta w(1-w^{\gamma})(w^{\gamma}-\eta) + H(z,t), & \text{(1a)} \\ w(z,0) = 0, \quad a \leqslant z \leqslant b, & \text{(1b)} \\ w(a,t) = \varphi_a(t), \quad w(b,t) = \varphi_b(t), \qquad 0 \leqslant t \leqslant T, & \text{(1c)} \end{cases}$$

where the function $w = w(z,t)$ is unknown, and parameters $\lambda > 0, \theta > 0, \beta \geqslant 0, \gamma > 0, 0 < \eta < 1$ are arbitrary constants. Moreover, w_z and w_{zz} denote the first- and second-order derivatives of function w with respect to z, respectively. The term ${}_0\mathcal{D}_t^{\alpha} w$ signifies the Caputo fractional derivative of order α of w defined by

$$ {}_0\mathcal{D}_t^{\alpha} w(z,t) := \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\partial w(z,\eta)}{\partial \eta} (t-\eta)^{-\alpha} d\eta, \tag{2}$$

where $0 < \alpha \leqslant 1$ and Γ denotes the Gamma function. For $\alpha = 1$, the FGBH equation reduces to the classical Burgers–Huxley equation (BHE), which can be modeled more accurately with its fractional version [11]. For $\lambda = 0, \theta = 1$ and $\sigma = 1$, (1) will be transformed into the fractional Huxley equation, which explains nerve pulse propagation and wall motion in liquid crystals [20]. For $\beta = 0, \theta = 1$ and, $\sigma = 1$, the presented equation (1) will be transformed to time-fractional Burgers' equation, which describes the far field of wave propagation in nonlinear systems [20]. Moreover, the time-fractional Fitzhugh–Nagoma equation will be derived when $\lambda = 0, \beta = \theta = \sigma = 1$ [3]. If $\theta = 1$, then the FGBH equation reduces to the time-fractional BHE [19]. The inhomogeneous FGBH equation (1) is utilized for modeling diffusion transport, nerve pulse propagation in nerve fibers, the interaction between reaction mechanisms, convection effects, and motion in liquid crystals [13]. Since (1) is a nonlinear partial differential equation (PDE), there is no effective method to derive its exact solution.

Recent years have witnessed significant progress in developing numerical methods for various versions of fractional BHE. Hammouch and Mekkaoui [8] developed the Adomian decomposition method for solving time-fractional BHEs. Alquran et al. [1] adapted the residual power series method to obtain traveling wave solutions of the FGBH equation. Tripathi [18] proposed an efficient technique for time-space fractional BHEs. Inc et al. [10] implemented Lie symmetry analysis for the time-fractional version of (1). Freihet and Zuriqat [4] applied the residual fractional power series method to solve time-fractional BHEs. Inc [9] designed a hybrid method combining line and group preserving schemes. Singh et al. [15] developed a shifted Legendre collocation method for space-time fractional BHEs. Majeed et al. [12] presented a cubic B-spline collocation method for the FGBH equation (1). Yang and Liu [21] established a finite difference method for time-fractional BHEs. Hajipour and Lakpour [7] developed a highly accurate method for variable-order fractional BHEs. To the best of our knowledge, existing numerical approaches for solving the general form of the FGBH equation (1) exhibit significant limitations in handling solutions with initial singularities and maintaining high-order accuracy for the general equation form.

The main goal of this manuscript is to propose a novel numerical scheme for solving the fractional generalized FGBH equation. The proposed method combines a fourth-order compact finite difference (FD) discretization in space with a third-order weighted shifted Grünwald–Letnikov (WSGL) approximation in time, resulting in a fully discrete system of nonlinear algebraic equations. We develop an efficient nonlinear solver to handle the resulting systems and provide rigorous theoretical analysis, proving the method's solvability, stability, and convergence. Our analysis establishes that the scheme achieves fourth-order spatial accuracy and third-order temporal accuracy, confirming its high precision. Numerical experiments on three benchmark test cases validate the theoretical convergence rates and demonstrate the method's computational robustness.

The paper is organized as follows. Section 2 introduces a highly accurate numerical technique for solving the FGBH equation. We present a detailed analysis of the proposed method, including its stability, solvability, and convergence properties. Section 3 demonstrates the applicability and accuracy of the method through several test examples. Section 4 concludes the paper, summarizing key findings and implications.

2 Description of the numerical method

Here, we will attempt to design a high-order fully discrete method for the inhomogeneous FGBH equation (1). In the first step, consider a reformulation of (1) as

$$-\sigma w_{zz} + \lambda\, w^{\theta} w_z = \Psi, \quad a < z < b,\ 0 < t \leqslant T, \tag{3}$$

where $\Psi = f + H - {}_0\mathcal{D}_t^{\alpha} w$ and $f = \beta w(1 - w^{\gamma})(w^{\gamma} - \eta)$. Then, we recall the SGL difference formula [22]

$$\mathcal{A}_{\tau,p}^{\alpha} w(z,t) := \frac{1}{\tau^{\alpha}} \sum_{k=0}^{[\frac{\zeta}{\tau}]+p} \omega_k^{(\alpha)} w(z, t-(k-p)\tau), \tag{4}$$

in which $\tau > 0$, p is an integer, and the coefficients $\omega_k^{(\alpha)}$ are computed by a recursive formula

$$\omega_{k+1}^{(\alpha)} = (1 - \frac{1+\alpha}{1+k})\omega_k^{(\alpha)}, \qquad k = 0,1,2,\ldots, \tag{5}$$

where $\omega_0^{(\alpha)} = 1$. As shown in [17], if $w(z,0) = 0$, then ${}_0\mathcal{D}_t^{\alpha} w(z,t) = \mathcal{A}_{\tau,p}^{\alpha} w(z,t) + \mathcal{O}(\tau)$. In the next lemma of [17], a third-order WSGL formula is developed to approximate the Caputo fractional derivative of order α of w defined by (2).

Now, we discretize the solution domain $\Omega := [a,b] \times [0,T]$. For positive integers M and N, let $(z_i, t_n) := (a + ih, n\tau)$ with $0 \leqslant n \leqslant N+1, 0 \leqslant i \leqslant M+1$, be a uniform mesh on Ω of $h := (b-a)/(M+1)$ and $\tau := T/(N+1)$.

Lemma 1. Let $0 < \alpha < 1$ and $\frac{\partial^m w}{\partial t^m}|_{t=0} = 0$, for $m = 0,1,2,3$. Then we deduce

$${}_0\mathcal{D}_t^{\alpha} w(z_i, t_n) = \tau^{-\alpha} \sum_{k=0}^{n} g_k^{(\alpha)} w(z_i, t_n - k\tau) + \mathcal{O}(\tau^3), \quad \text{as} \quad \tau \to 0, \tag{6}$$

uniformly for $n \geqslant 1$, where

$$\begin{cases} g_0^{(\alpha)} = \rho_0 \omega_0^{(\alpha)}, \\ g_1^{(\alpha)} = \rho_0 \omega_1^{(\alpha)} + \rho_1 \omega_0^{(\alpha)}, \\ g_k^{(\alpha)} = \rho_0 \omega_k^{(\alpha)} + \rho_1 \omega_{k-1}^{(\alpha)} + \rho_2 w_{k-2}^{(\alpha)}, \quad k = 2, 3, \dots, \end{cases} \tag{7}$$

and $\rho_0 = 1 + \frac{17}{24}\alpha + \frac{1}{8}\alpha^2,\ \rho_1 = -\frac{11}{12}\alpha - \frac{1}{4}\alpha^2,\ \rho_2 = \frac{5}{24}\alpha + \frac{1}{8}\alpha^2.$

Lemma 2. [17] For $0 < \alpha < 1$, the coefficients $\{g_k^{(\alpha)}\}$ holds in

(1) $1 < g_0^{(\alpha)} < 1 + \frac{5}{6}\alpha, \quad g_1^{(\alpha)} < 0,\ g_2^{(\alpha)} > 0, \quad g_k^{(\alpha)} < 0, \quad$ for all $k \geqslant 3$,
(2) $\sum_{k=0}^{\infty} g_k^{(\alpha)} = 0$, and, $|g_k^{(\alpha)}| < \frac{10}{3}\alpha, \quad$ for all $k \geq 1$,
(3) $-\sum_{k=1}^{n} g_k^{(\alpha)} < g_0^{(\alpha)} + g_2^{(\alpha)}, \quad$ for all $n \geq 1$.

In the below, we formulate a fourth-order compact finite difference method for the space discretization of the nonlinear differential equation (3), which results in a system of time-dependent fractional equations. Then, to discretize the derived system in the temporal direction, we will apply the third-order WSGL method (6).

The spatial discretization of (3)

A semi-discrete form of the nonlinear differential equation (3) at z_i can be formulated as

$$\begin{aligned} (\ & -\frac{\sigma}{h^2} + \frac{\lambda}{4h}\theta w_i^{\theta-1}\delta_z w_i - \frac{\lambda^2}{12\sigma} w_i^{2\theta})\delta_z^2 w_i + (\frac{\lambda}{h} w_i^{\theta} - \frac{\lambda^2}{12\sigma} w_i^{2\theta-1}\delta_z w_i \\ & + \frac{\lambda}{12h}\theta(\theta-1) w_i^{\theta-2}(\delta_z w_i)^2)\delta_z w_i = (1 - \frac{\lambda h}{12\sigma} w_i^{\theta}\delta_z + \frac{1}{12}\delta_z^2)\Psi_i + R_i(h)\ , \end{aligned} \tag{8}$$

where $w_i := w(z_i, \cdot)$, $\Psi_i := \Psi(z_i, \cdot)$, and the difference operators δ_z and δ_z^2 are defined as

$$\delta_z w_i := \frac{1}{2}(w_{i+1} - w_{i-1}), \qquad \delta_z^2 w_i := w_{i+1} - 2w_i + w_{i-1}.$$

Lemma 3. Let the solution $w(z, \cdot)$ be a six-time continuously differentiable function with respect to z. Then the error term $R_i(h)$ of the formula (8) for (3) is of order four, that is, $R_i(h) = \mathcal{O}(h^4)$.

Proof. Using the Taylor expansion of $w_{i\pm 1}$ at point z_i, the value of (3) can be calculated as

$$-\frac{\sigma}{h^2}\delta_z^2 w_i + \frac{\lambda}{h}\ w_i^{\theta}\delta_z w_i + \frac{h^2}{12}\zeta_i = \Psi_i - \frac{h^4}{360}(3\lambda w^{\theta} w_{zzzzz} - \sigma w_{zzzzzz})|_{z=\eta_i}. \tag{9}$$

where $\zeta = 2\lambda w^\theta w_{zzz} - \sigma w_{zzzz}$ and $z_{i-1} < \eta_i < z_{i+1}$. After calculating the derivatives of (3) with respect to z, we have

$$\begin{cases} \sigma w_{zzz} &= \lambda \left(w^\theta w_{zz} + \theta w^{\theta-1}(w_z)^2\right) - \Psi_z, \\ \sigma w_{zzzz} &= \lambda(w^\theta w_{zzz} + 3\theta w^{\theta-1} w_z w_{zz} + \theta(\theta-1)w^{\theta-2}(w_z)^3) - \Psi_{zz} \\ &= ((\lambda\ w^\theta)^2 + 3\lambda\theta w^{\theta-1} w_z) w_{zz} + (\theta\lambda^2\ w_z w^{\theta-1} \\ & \quad -\lambda\theta(\theta-1)w^{\theta-2}(w_z)^2) w_z - \lambda\ w^\theta \Psi_z - \Psi_{zz}. \end{cases}$$

This leads to

$$\zeta = \Psi_{zz} - \frac{\lambda}{\sigma}\ w^\theta \Psi_z + (\frac{\lambda^2}{\sigma} w^{2\theta} - 3\lambda\theta w^{\theta-1} w_z) w_{zz} \\ +(\frac{\lambda^2}{\sigma}\theta w^{\theta-1} - \lambda\theta(\theta-1)w^{\theta-2} w_z)(w_z)^2.$$

Using operators δ_z and δ_z^2, we can derive

$$\zeta_i = \zeta(z_i,\cdot) = \frac{1}{h^2}\delta_z^2\Psi_i - \frac{\lambda}{\sigma h} w_i^\theta \delta_z \Psi_i + (\frac{\lambda^2}{\sigma h^2} w_i^{2\theta} - \frac{3\lambda}{h^3}\theta w_i^{\theta-1}\delta_z w_i)\delta_z^2 w_i \\ +(\frac{\lambda^2}{\sigma h^2}\theta w_i^{\theta-1} - \frac{\lambda}{h^3}\theta(\theta-1)w_i^{\theta-2}\delta_z w_i)(\delta_z w_i)^2 + \mathcal{O}(h^2). \tag{10}$$

By substituting relation (10) into (9), we can conclude that $R_i(h) = \mathcal{O}(h^4)$. □

Full discretization of (3)

Here, to develop a high-order discretization technique for (1), the discrete forms (6) and (8) are implemented in temporal and spatial directions, respectively. Consider the following fully discrete method for solving the initial-boundary value problem (1) which determines approximations w_i^n to the values $w(z_i, t_n)$ from the relation

$$(-\frac{\sigma}{h^2} + \frac{\lambda}{4h}\theta(w_i^n)^{\theta-1}\delta_z w_i^n - \frac{\lambda^2}{12\sigma}(w_i^n)^{2\theta})\delta_z^2 w_i^n \\ +\Big(\frac{\lambda}{h}(w_i^n)^\theta - \frac{\lambda^2}{12\sigma}(w_i^n)^{2\theta-1}\delta_z w_i^n + \frac{\lambda}{12h}\theta(\theta-1)(w_i^n)^{\theta-2}(\delta_z w_i^n)^2\Big)\delta_z w_i^n \\ = (1 - \frac{\lambda h}{12\sigma}(w_i^n)^\theta \delta_z + \frac{1}{12}\delta_z^2)\Psi_i^n, \tag{11}$$

for $n = 1, 2, \ldots, N+1$ and $i = 1, 2, \ldots, M$, where

$$\Psi_i^n = f_i^n + H_i^n - \tau^{-\alpha} \sum_{k=0}^{n} g_k^{(\alpha)} w_i^{n-k}, \tag{12}$$

and $f_i^n := f(w_i^n, z_i, t_n),\ H_i^n := H(z_i, t_n)$. For calculating the value w_i^0 (for $i = 0, \dots, M+1$) and values w_0^n, w_{M+1}^n (for $n = 0, \dots, N+1$), we use the initial and boundary conditions (1b)–(1c) as follows:

$$w_i^0 = 0,\ 0 \leqslant i \leqslant M+1,\ \text{and}\ w_0^n = \varphi_a(t_n),\ w_{M+1}^n = \varphi_b(t_n),\ 0 \leqslant n \leqslant N+1.$$

Consider the matrix type of grid functions as $\mathbf{W}^n := [w_1^n, w_2^n, \dots, w_M^n]^T$ and $\mathbf{F}^n := f(\mathbf{W}^n),\ \mathbf{H}^n := [H_1^n, H_2^n, \dots, H_M^n]^T$. For $1 \leqslant n \leqslant N+1$, we present a matrix form of (11) as

$$\begin{aligned}
&\Big((-\frac{\sigma}{h^2}\mathbf{I} + \frac{\lambda}{4h}\theta(\mathbf{W}^n)^{\theta-1}\mathbf{J}_1\mathbf{W}^n - \frac{\lambda^2}{12\sigma}(\mathbf{W}^n)^{2\theta})\mathbf{J}_2 \\
&+ (\frac{\lambda}{h}(\mathbf{W}^n)^{\theta} - \frac{\lambda^2}{12\sigma}(\mathbf{W}^n)^{2\theta-1}\mathbf{J}_1\mathbf{W}^n + \frac{\lambda}{2h}\theta(\theta-1)(\mathbf{W}^n)^{\theta-2}(\mathbf{J}_1\mathbf{W}^n)^2)\mathbf{J}_1\Big)\mathbf{W}^n \\
&\quad = (\mathbf{I} - \frac{\lambda h}{12}(\mathbf{W}^n)^{\theta}\mathbf{J}_1 + \frac{1}{12}\mathbf{J}_2)\big(\mathbf{F}^n + \mathbf{H}^n - \tau^{-\alpha}\sum_{k=0}^{n} g_k^{(\alpha)}\mathbf{W}^{n-k}\big) + C^n,
\end{aligned} \tag{13}$$

where $\mathbf{J}_1 := \mathbf{tridiag}(-\frac{1}{2}, 0, \frac{1}{2})$ and $\mathbf{J}_2 := \mathbf{tridiag}(1, -2, 1)$. Moreover, $\mathbf{I}$ denotes an identity matrix, $\mathbf{W}^0 = [0, 0, \dots, 0]^T$, and vectors C^n can be determined by using the boundary conditions. The implicit numerical method (13) forms a system of nonlinear algebraic equations, for every $1 \leqslant n \leqslant N+1$. Therefore, a simple and fast nonlinear solver is needed to solve these nonlinear systems. For every time level n, we propose a linearizing version for (13) as follows:

$$\begin{aligned}
&\Big((-\frac{\sigma}{h^2}\mathbf{I} + \frac{\lambda}{4h}\theta(\mathbf{U}^{[s]})^{\theta-1}\mathbf{J}_1\mathbf{U}^{[s]} - \frac{\lambda^2}{12\sigma}(\mathbf{U}^{[s]})^{2\theta})\mathbf{J}_2 + (\frac{\lambda}{h}(\mathbf{U}^{[s]})^{\theta} \\
&-\frac{\lambda^2}{12\sigma}(\mathbf{U}^{[s]})^{2\theta-1}\mathbf{J}_1\mathbf{U}^{[s]} + \frac{\lambda}{2h}\theta(\theta-1)(\mathbf{U}^{[s]})^{\theta-2}(\mathbf{J}_1\mathbf{U}^{[s]})^2)\mathbf{J}_1\Big)\mathbf{U}^{[s+1]} \\
&\quad = (\mathbf{I} - \frac{\lambda h}{12}(\mathbf{U}^{[s]})^{\theta}\mathbf{J}_1 + \frac{1}{12}\mathbf{J}_2)\Big(\mathbf{F}(\mathbf{U}^{[s]}, \mathbf{U}^{[s+1]}) - \tau^{-\alpha} g_0^{(\alpha)}\mathbf{U}^{[s+1]} \\
&\qquad + \mathbf{H}^n - \tau^{-\alpha}\sum_{k=1}^{n} g_k^{(\alpha)}\mathbf{W}^{n-k}\Big) + C^n,
\end{aligned} \tag{14}$$

where

$$\begin{aligned}
\mathbf{F}(\mathbf{U}^{[s]}, \mathbf{U}^{[s+1]}) &= \beta(1 - (\mathbf{U}^{[s]})^{\gamma})((\mathbf{U}^{[s]})^{\gamma} - \eta)\mathbf{U}^{[s+1]} \\
&\quad + \beta\gamma(\mathbf{U}^{[s]})^{\gamma}(2(\mathbf{U}^{[s]})^{\gamma} - \eta - 1)(\mathbf{U}^{[s+1]} - \mathbf{U}^{[s]}).
\end{aligned}$$

The symbol $\mathbf{U}^{[s]}$ represents the approximate solution which is found in the s-th iteration of the method (14). The number of iterations of this method can be determined such that the condition $\|\mathbf{U}^{[s+1]} - \mathbf{U}^{[s]}\| < \varepsilon$ is attained.

2.1 The analysis of solvability, stability, and convergence of the present method

The analysis of the solvability, stability, and convergence of the present iterative method (13) for the FGBH equation (1) will be stated by following the lemma and theorems. For the matrix $\mathbf{A}$, let $\|\mathbf{A}\|$ be its L_2 norm. As it is concluded in [16], the eigenvalues of $\mathbf{J}_1$ and $\mathbf{J}_2$ are, respectively, derived as

$$\lambda_s^{\mathbf{J}_1} := \mathbf{i}\cos(\frac{s\pi}{M+1}), \quad \text{and} \quad \lambda_s^{\mathbf{J}_2} := -4\sin^2(\frac{s\pi}{2(M+1)}), \tag{15}$$

where $s = 1, 2, \ldots, M$, and $\mathbf{i} = \sqrt{-1}$.

Lemma 4 (Solvability)**.** For every integer number n, the system of equations (13) has a unique solution.

Proof. Let $q = \max |w|$. Using system (11), we can reformulate (13) as follows:

$$A\ \mathbf{W}^n = -\sum_{k=1}^{n} B_k\ \mathbf{W}^{n-k} + B_0\ (\mathbf{F}^n + \mathbf{H}^n) + C^n, \tag{16}$$

where $B_0 = \mathbf{I} - \frac{\lambda h}{12\sigma} q^\theta \mathbf{J}_1 + \frac{1}{12}\mathbf{J}_2$, $A = \tau^{-\alpha} g_0^{(\alpha)} B_0 - (\frac{\sigma}{h^2} + \frac{\lambda^2}{12\sigma} q^{2\theta})\mathbf{J}_2 + \frac{\lambda}{h} q^\theta \mathbf{J}_1$, and $B_k = \tau^{-\alpha} g_k^{(\alpha)} B_0$ for $k = 1, 2, \ldots$. The M-dimensional matrix A has the tridiagonal form $A = \mathbf{tridiag}(b, d, c)$, where its lower-main-upper diagonal elements are the following expressions, respectively:

$$\begin{cases} b &= -\frac{\sigma}{h^2} - \frac{\lambda^2}{12} q^{2\theta} - \frac{\lambda}{2h} q^\theta + (\frac{1}{12} - \frac{\lambda h}{24} q^\theta)\tau^{-\alpha} g_0^{(\alpha)}, \\ d &= \frac{2\sigma}{h^2} + \frac{\lambda^2}{6} q^{2\theta} + \frac{5}{6}\tau^{-\alpha} g_0^{(\alpha)}, \\ c &= -\frac{\sigma}{h^2} - \frac{\lambda^2}{12} q^{2\theta} + \frac{\lambda}{2h} q^\theta + (\frac{1}{12} + \frac{\lambda h}{24} q^\theta)\tau^{-\alpha} g_0^{(\alpha)}, \end{cases}$$

Using Lemma 2, we obtain $1 < g_0^{(\alpha)} < \frac{11}{6}$. Therefore, for sufficiently small step sizes h and τ, we can derive

$$|b| + |c| \leq \frac{2\sigma}{h^2} + \frac{\lambda^2}{6} q^2 + \frac{1}{6}\tau^{-\alpha} g_0^{(\alpha)} < |d|.$$

This shows that the matrix A is diagonally dominant and thus nonsingular. Consequently, the system (13) has a unique solution. $\square$

To establish the unconditional stability and convergence of the proposed method, we begin by recalling from [14, Lemma 1.4.2], which is stated as follows.

Lemma 5 (Gronwall's inequality)**.** Let k_n be a nonnegative sequence. Suppose ϕ_n satisfies the inequality:

$$\phi_0 \leq s_0, \qquad \phi_n \leq s_0 + \sum_{i=0}^{n-1} k_i \phi_i, \quad n \geq 1,$$

where $s_0 \geq 0$. Then ϕ_n satisfies the bound

$$\phi_n \leq s_0 \exp(\sum_{i=0}^{n-1} k_i), \quad n \geq 1.$$

Theorem 1 (Unconditionally stability)**.** The iterative method (13) is unconditionally stable to solve the FGBH equation (1).

Proof. For $q = \max |w|$, the method (13) reduces to (16). Let $\tilde{\mathbf{W}}^0 = [\phi_1, \phi_2, \ldots, \phi_M]^T$ be a perturbation of the initial value $\mathbf{W}^0 = [0, 0, \ldots, 0]^T$, and let $\tilde{\mathbf{W}}^n$ hold in below

$$\begin{cases} \tilde{\mathbf{W}}^0 = \Phi^0, \\ A\tilde{\mathbf{W}}^n = -\sum_{k=1}^{n} B_k \tilde{\mathbf{W}}^{n-k} + B_0(\tilde{\mathbf{F}}^n + \mathbf{H}^n) + C^n, \end{cases} \tag{17}$$

in which $\Phi^0 := [\phi_1, \phi_2, \ldots, \phi_M]^T$ and $\tilde{\mathbf{F}}^n = f(\tilde{\mathbf{W}}^n)$. Now, we define $\mathbf{E}^n := \tilde{\mathbf{W}}^n - \mathbf{W}^n - \Phi^0$; subtract (17) from (16), a roundoff error equation is concluded as

$$A\mathbf{E}^n = -\sum_{k=1}^{n} B_k \mathbf{E}^{n-k} - (\sum_{k=1}^{n} B_k + A)\Phi^0 + B_0(\tilde{\mathbf{F}}^n - \mathbf{F}^n), \quad n \geq 1, \tag{18}$$

where $\mathbf{E}^0 := [0, \ldots, 0]^T$. Applying (15), the eigenvalues of tridiagonal matrices B_0 and A are, respectively, obtained as

$$\begin{cases} \lambda_s^{B_0} = (1 - \frac{1}{3}\sin^2(\frac{s\pi}{2(M+1)})) - \mathbf{i}\frac{\lambda h}{12} q^\theta \cos(\frac{s\pi}{M+1}), \\ \lambda_s^{A} = \tau^{-\alpha} g_0^{(\alpha)} \lambda_s^{B_0} + (\frac{4\sigma}{h^2} + \frac{\lambda^2}{3} q^{2\theta}) \sin^2(\frac{s\pi}{2(M+1)}) + \mathbf{i}\frac{\lambda}{h} q^\theta \cos(\frac{s\pi}{M+1}). \end{cases} \tag{19}$$

Consequently $\|(A^n)^{-1}B_0\| \leqslant \frac{1}{g_0^{(\alpha)}}\tau^\alpha$. Using Lemma 2, we can achieve that

$$\|(A^n)^{-1}B_k\| = \tau^{-\alpha}\|(A^n)^{-1}B_0\||g_k^{(\alpha)}| \leqslant \frac{|g_k^{(\alpha)}|}{g_0^{(\alpha)}} \leqslant |g_k^{(\alpha)}|. \tag{20}$$

Moreover, this lemma leads to

$$\sum_{k=1}^{n}|g_k^{(\alpha)}| = g_2^{(\alpha)} - \sum_{k=1}^{n} g_k^{(\alpha)} \leqslant g_0^{(\alpha)} + 2g_2^{(\alpha)} \leqslant \frac{17}{2}, \qquad \text{for all } n \geq 1, \tag{21}$$

when $0 < \alpha \leqslant 1$. Since f is a Lipschitz continuous function with respect to w, then there exists a positive constant $L := \max|\frac{\partial f}{\partial w}|$ such that

$$|f(\tilde{w}) - f(w)| \leqslant L|\tilde{w} - w|.$$

Now, by applying (20)–(21) to the difference equations (18) and using the properties of $\omega_k^{(\alpha)}$ yields

$$\begin{aligned}
|1-\tau^\alpha L|\|\mathbf{E}^n\| &\leqslant \sum_{k=1}^{n}\|(A^n)^{-1}B_k\|\|\mathbf{E}^{n-k}\| \\
&\quad +(\sum_{k=1}^{n}\|(A^n)^{-1}B_k\| + (1+T^\alpha L))\|\Phi^0\|, \\
&\leqslant \sum_{k=1}^{n}|g_k^{(\alpha)}|\|\mathbf{E}^{n-k}\| + \|\Phi^0\|\sum_{k=1}^{n}|g_k^{(\alpha)}| + (1+T^\alpha L)\|\Phi^0\| \\
&\leqslant \sum_{k=1}^{n}|g_k^{(\alpha)}|\|\mathbf{E}^{n-k}\| + \frac{17}{2}\|\Phi^0\| + (1+T^\alpha L)\|\Phi^0\| \\
&\leqslant \sum_{k=0}^{n-1}|g_{n-k}^{(\alpha)}|\ \|\mathbf{E}^k\| + (\frac{19}{2} + T^\alpha L)\|\Phi^0\|.
\end{aligned} \tag{22}$$

Let $\tau^\alpha L \neq 1$. Equation (22) leads to

$$\|\mathbf{E}^n\| \leqslant \frac{1}{|1-\tau^\alpha L|}\sum_{k=0}^{n-1}|g_{n-k}^{(\alpha)}|\|\mathbf{E}^k\| + \frac{19+2T^\alpha L}{|2-2\tau^\alpha L|}\|\Phi^0\|, \qquad n = 1,\ldots,N+1.$$

Using Gronwall's Lemma 5, we have

$$\|\mathbf{E}^n\| \leqslant C_1\|\Phi^0\|, \qquad n = 1,\ldots,N+1, \tag{23}$$

where $C_1 = \frac{19+2T^\alpha L}{|2-2\tau^\alpha L|}\exp(\frac{17}{|2-2\tau^\alpha L|})$. Therefore, the unconditional stability of the iterative method (13) is established, as required. $\square$

Theorem 2 (Convergence analysis)**.** Let $w \in \mathcal{C}_{x,t}^{6,4}(\Omega)$ be the solution of the FGBH equation (1) satisfying in $\frac{\partial^k w}{\partial t^k}|_{t=0} = 0$ for $k = 0,1,2,3$; then solution of the iterative method (13) converges to w as $h, \tau \to 0$. Furthermore, its convergence rate is of order $\mathcal{O}(\tau^3 + h^4)$.

Proof. Define $e_i^n := w(z_i, t_n) - w_i^n$, where $0 \leqslant n \leqslant N+1,\ 0 \leqslant i \leqslant M+1$. Since f is a Lipschitz continuous function with respect to w, then there exists a positive constant $L = \max|\frac{\partial f}{\partial w}|$ such that $|f(w_i^n) - f(w(z_i,t_n))| \leqslant L|e_i^n|$. Then subtract (8) from (16), we obtain the error equation

$$A\mathbf{e}^n = -\sum_{k=1}^{n} B_k \mathbf{e}^{n-k} + B_0(f(\mathbf{w}^n) - f(\mathbf{W}^n)) + \mathbf{R}^n(h,\tau), \quad n \geq 1, \tag{24}$$

where $\mathbf{w}^n = [w(z_1,t_n),\dots,w(z_M,t_n)]^T$, $\mathbf{e}^0 := [0,\dots,0]^T$, $\mathbf{e}^n := [e_1^n,\dots,e_M^n]^T$ and $\mathbf{R}^n := [R_1^n,\dots,R_M^n]^T$. From Lemmas 1 and 3, there exists a constant C_2 such that $\|\mathbf{R}^n(h,\tau)\| \leqslant C_2(\tau^3 + h^4)$. Let $\tau^\alpha L \neq 1$. Applying Lemma 2 and (19)–(21) deduces

$$\begin{aligned} |1-\tau^\alpha L|\|\mathbf{e}^n\| &= \sum_{k=1}^{n} \|(A)^{-1}B_k\|\|\mathbf{e}^{n-k}\| + C_2(\tau^3+h^4) \\ &\leqslant \sum_{k=0}^{n-1} |g_{n-k}^{(\alpha)}|\|\mathbf{e}^{n-k}\| + C_2(\tau^3+h^4), \qquad n \geq 1. \end{aligned} \tag{25}$$

Using Gronwall's Lemma 5, we can derive

$$\|\mathbf{e}^n\| \leqslant \frac{C_2}{|1-\tau^\alpha L|}\exp\left(\frac{17}{|2-2\tau^\alpha L|}\right)(\tau^3+h^4), \qquad n \geq 1. \tag{26}$$

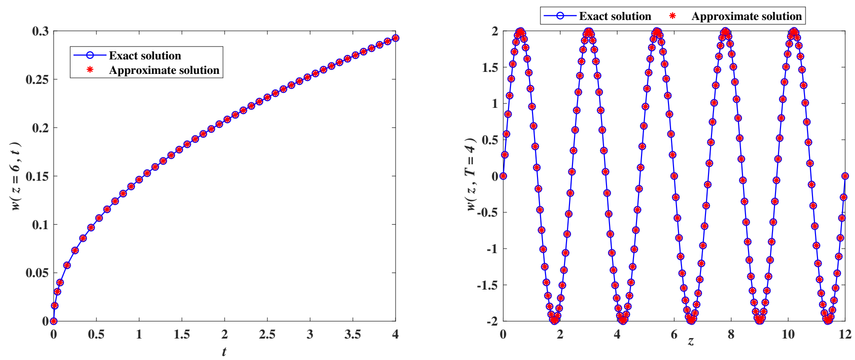
□

As Theorem 2 establishes, the proposed iterative method (13) has fourth- and third-order accuracy in spatial an temporal directions, respectively.

3 Numerical results

In this section, numerical results of the iterative method (14) to solve three types of the inhomogeneous FGBH equation are presented. The provided numerical results verify the theoretical results of Theorems 1 and 2.

All numerical simulations were conducted in MATLAB on a workstation equipped with an Intel ® Core™ i7-4702MQ CPU (2.20 GHz), 8 GB of RAM, running Windows 8 operating system. The convergence rate of the present iterative method is evaluated by



(a) Plots of solutions versus t. (b) Plots of solutions versus z.

Figure 1: Comparison of the exact and approximate solutions to Example 1 after 20 iterations are made when $\mu = 0.5, \alpha = 0.5$ and $\Omega = [0, 12] \times [0, 4]$ is divided into 214×1280 cells. The MAE of the present method is 1.5×10^{-2}.

Table 1: Temporal convergence rate of the method (14) after $r = 20$ iterations to solve Example 1, the exact solution $w(z,t) = t^3 \sqrt[5]{t}\ \sin(\frac{5\pi}{6}z)$, $M \times N$ cells and $M \approx \sqrt[4]{N^3}$.

	$\alpha = 0.25$		$\alpha = 0.5$		$\alpha = 0.75$	
N	MAE	Rate	MAE	Rate	MAE	Rate
20	3.6532e-05	-	3.8011e-05	-	5.2457e-05	-
80	6.7354e-07	2.88	6.9027e-07	2.89	1.0004e-06	2.86
320	1.0717e-08	2.99	1.0965e-08	2.99	1.5900e-08	2.99
1280	1.6567e-10	3.00	1.7067e-10	3.00	2.4598e-10	3.00

$$\text{Rate} := \log_{(\frac{m_2}{m_1})} \left(\frac{\varepsilon_1}{\varepsilon_2}\right),$$

where ε_1 and ε_2 are the maximum absolute errors (MAEs) of the present method using m_1 and m_2 cells in the time/space direction, respectively.

Example 1. Consider the FGBH equation (1) as follows [12, 21]:

$$ {}_0\mathcal{D}_t^{\alpha} w + 0.01 w w_z - w_{zz} = w(1-w)(w-0.5) + H, \quad \text{in} \quad \Omega, \tag{27}$$

where the governing function $H = H(z,t)$ and all initial-boundary conditions are derived from the exact analytical solution: $w(z,t) = t^{\mu} \sin(\frac{5\pi}{6}z)$.

Here, we examine the present iterative method (14) for Example 1 when $\mu = \frac{16}{5}, \frac{1}{2}$ and $\mu = 3$. For different values $\alpha = 0.25, 0.5, 0.75$ and $\mu = \frac{16}{5}$, the MAE and the temporal/spatial convergence rate of this iterative method after 20 iterations are presented in Tables 1–2. The domain $\Omega = [0,1] \times [0,1]$ is divided into $M \times N$ cells, where M and N are two positive integers

Table 2: Spatial convergence rate of the present iterative method (14) after $r = 20$ iterations to solve Example 1, the exact solution $w(z,t) = t^3 \sqrt[5]{t}\ \sin(\frac{5\pi}{6} z)$, $M \times N$ cells and $N \approx \sqrt[3]{M^4}$.

	$\alpha = 0.25$		$\alpha = 0.5$		$\alpha = 0.75$	
M	MAE	Rate	MAE	Rate	MAE	Rate
4	1.4012e-03	-	1.4133e-03	-	1.8852e-03	-
16	5.4371e-06	4.00	5.5444e-06	4.00	7.4116e-06	4.00
64	2.1284e-08	4.00	2.1752e-08	4.00	2.8852e-08	4.00
256	8.3054e-11	4.00	8.4690e-11	4.00	1.1196e-10	4.00

Table 3: Comparison of MAEs obtained by the B-spline collocation method [12], compact difference scheme [21], and the present iterative method to solve Example 1 in $\Omega = [0,1] \times [0,1]$ with the exact solution $w(z,t) = t^3 \sin(\frac{5\pi}{6} z)$.

α	BSC method [12] with ($M = 100,\ N = 3125$)	CD scheme [21] with ($M = 64, N = 1024$)	Present method with ($M = 64, N = 1024$)
0.3	-	6.5363e-06	1.9546e-08
0.5	3.0000e-06	-	1.8805e-08
0.6	-	7.0496e-06	1.6886e-08

satisfying $N^3 = M^4$. The obtained numerical results in Table 1 illustrate that this new method is of order $\mathcal{O}(\tau^3)$ in time direction. Moreover, Tables 1 and 2 show that the present method is unconditionally stable. These results confirm the statements of Theorems 1 and 2.

For $\mu = 0.5$, comparison of the exact and approximate solutions to Example 1 in the large domain $[0,12] \times [0,4]$ is also made in Figure 1 when $\alpha = 0.5$ and 214×1280 cells are selected. Part (a) shows solutions at $z = 6$ versus $t \in [0,4]$, and part (b) demonstrates them at $t = 4$ versus $z \in [0,12]$. To derive these plots, the proposed iterative method converged after 20 iterations and 91.02 seconds, and its maximum absolute error is 1.2232×10^{-2}. This figure illustrates that the present method is unconditionally stable and is not affected by round-off errors in the large computational domain $\Omega = [0,12] \times [0,4]$.

Finally, the MAEs obtained by the B-spline collocation (BSC) method [12], compact difference (CD) scheme [21], and the present iterative method for solving Example 1 with $\mu = 3$ are compared in Table 3. This table shows that the present iterative method (13) is more accurate and effective than the methods given in [12, 21].

Example 2. Consider the following inhomogeneous FGBH equation:

$$ {}_0\mathcal{D}_t^{\alpha} w + \frac{\pi}{2} w^2 w_z - 20 w_{zz} = 50 w(1 - w^3)(w^3 - \frac{2}{3}) + g, \quad (z,t) \in \Omega, \tag{28}$$

where the function $g := g(z,t)$ is calculated using the exact solution $w(z,t) = \frac{1}{8} t^{2+4\alpha}\ (z - z^2)$.

Table 4: Temporal convergence rate of the iterative technique (14) after $r = 20$ iterations to solve Example 2 at $T = 2$ with $M \times N$ cells and $M \approx \sqrt[4]{0.5N^3}$.

	$\alpha = 0.25$		$\alpha = 0.5$		$\alpha = 0.75$		
N	MAE	Rate	MAE	Rate	MAE	Rate	CPU time
10	1.1433e-05	-	5.7396e-05	-	3.3925e-04	-	0.04
20	3.0962e-06	1.88	1.4212e-05	2.01	7.2916e-05	2.22	0.06
40	4.9555e-07	2.64	2.1440e-06	2.73	1.0575e-05	2.79	0.10
80	8.4224e-08	2.56	3.5233e-07	2.60	1.5525e-06	2.77	0.20
160	1.1150e-08	2.92	4.6702e-08	2.92	2.2482e-07	2.79	0.50
320	1.5055e-09	2.87	6.0369e-09	2.95	3.2025e-08	2.81	1.78
640	1.8980e-10	2.99	7.5944e-10	2.99	4.5482e-09	2.82	6.08

Table 5: Spatial convergence rate of the iterative technique (14) after $r = 30$ iterations to solve Example 2 at $T = 2$ with $M \times N$ cells and $N \approx \sqrt[3]{2M^4}$.

	$\alpha = 0.25$		$\alpha = 0.5$		$\alpha = 0.75$		
M	MAE	Rate	MAE	Rate	MAE	Rate	CPU time
4	1.1558e-05	-	6.0110e-05	-	3.6824e-04	-	0.05
8	1.1263e-06	3.36	5.0731e-06	3.57	2.5876e-05	3.83	0.06
16	8.4422e-08	3.74	3.5816e-07	3.83	1.8329e-06	3.82	0.14
32	5.7420e-09	3.89	2.3813e-08	3.91	1.3064e-07	3.81	0.47
64	3.7460e-10	3.94	1.6436e-09	3.86	9.3027e-09	3.82	2.74
128	2.3957e-11	3.97	1.1208e-10	3.87	6.4064e-10	3.86	22.12
256	1.4975e-12	4.00	7.0836e-12	3.98	4.5029e-11	3.83	227.05

The MAE, temporal/spatial convergence rate, and elapsed CPU time (in second) of the proposed iterative technique (14) after $r = 20$ iterations for solving the inhomogeneous FGBH equation (28) are presented in Tables 4 and 5. The domain $\Omega = [0, 1]\times[0, 2]$ is divided into $M\times N$ cells which M and N are two positive integers satisfying $M^4 = 2N^3$. These tables demonstrate that the convergence rate of this method is $\mathcal{O}(\tau^3 + h^4)$ for different values of α. Moreover, the present method is not affected by round-off errors. Also, it is unconditionally stable when the number of cells is increased. Consequently, these numerical results follows analytical results of Theorems 1 and 2.

Example 3. Consider the inhomogeneous FGBH equation (1) as

$$ {}_0\mathcal{D}_t^{\alpha} w + w w_z - w_{zz} = \; w(1 - w^2)(w^2 - 0.75) + g, \tag{29} $$

where $(z, t) \in (-1, 1) \times (0, 1]$ and the function $g = g(z, t)$ is calculated using the exact solution $w(z, t) = (1 - z^6) \exp(1 - z^2)t^{\alpha+\sqrt{5}\cosh(z)}$.

For solving the inhomogeneous FGBH equation (29) with $\alpha = 0.25, 0.5, 0.75$, the present iterative method converges after $r = 20$ iterations. Numerical results are provided in Tables 6 and

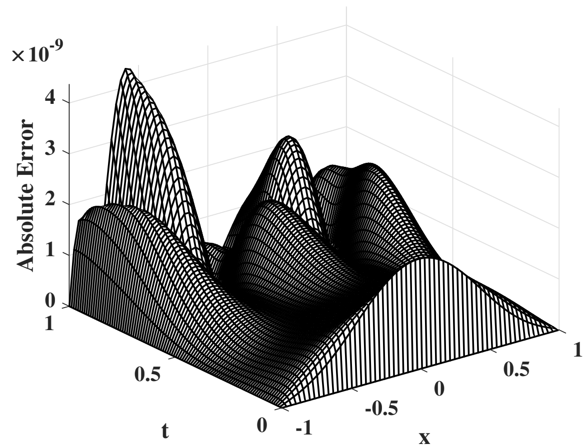


Figure 2: The absolute error of the present iterative technique after $r = 20$ iterations to solve Example 3 with $\alpha = 0.75$ and 200×1000 cells.

Table 6: Temporal convergence rate of the iterative technique (14) after $r = 20$ iterations to solve the Example 3 with $M \times N$ cells and $M \approx \sqrt[4]{N^3}$.

	$\alpha = 0.25$		$\alpha = 0.5$		$\alpha = 0.75$		
N	MAE	Rate	MAE	Rate	MAE	Rate	CPU time
10	4.9633e-03	-	4.8430e-03	-	4.6422e-03	-	0.03
20	5.0011e-04	3.31	6.055e-04	3.00	4.8041e-04	3.27	0.04
40	7.0557e-05	2.83	7.5800e-05	3.00	6.7283e-05	2.84	0.06
80	8.4545e-06	3.06	9.4508e-06	3.00	8.2037e-06	3.04	0.14
160	1.1169e-06	2.92	.1802e-06	3.00	1.0726e-06	2.94	0.53
320	1.3667e-07	3.03	.4804e-07	3.00	1.3192e-07	3.02	2.11
640	1.7022e-08	3.00	1.6760e-08	3.14	1.6374e-08	3.01	12.80

7. The numerical results in these tables confirm that the iterative technique (14) is numerically stable (insensitive to roundoff errors) while maintaining the predicted third-order (temporal) and fourth-order (spatial) convergence rates. The scheme accurately captures the solution's initial singularity, thereby validating Theorems 1 and 2. Moreover, Figure 2 shows the absolute error of the proposed technique after $r = 20$ iterations on a 200×1000 grid for solving Example 3 with $\alpha = 0.75$. The figure indicates that the method is robust against round-off errors.

Table 7: Spatial convergence rate of the iterative technique (14) after $r = 20$ iterations to solve Example 3 with $M \times N$ cells and $N \approx \sqrt[3]{M^4}$.

	$\alpha = 0.25$		$\alpha = 0.5$		$\alpha = 0.75$		
M	MAE	Rate	MAE	Rate	MAE	Rate	CPU time
8	1.0773e-03	-	1.0731e-03	-	1.0536e-03	-	0.03
16	7.0565e-05	3.92	6.9298e-05	3.95	6.7362e-05	3.97	0.06
32	4.3136e-06	4.03	4.2591e-06	4.02	4.1816e-06	4.01	0.20
64	2.7281e-07	3.98	2.6864e-07	3.99	2.6242e-07	3.99	1.46
128	1.7022e-08	4.00	1.6762e-08	4.00	1.6380e-08	4.00	12.52
256	1.0646e-09	4.00	1.0446e-09	4.00	1.0206e-09	4.00	139.05

4 Conclusion

In this study, we developed an efficient high-order numerical scheme for solving the fractional generalized Burgers–Huxley (FGBH) equation, a nonlinear partial differential equation that models various physical phenomena involving diffusion, reaction, and convection mechanisms under fractional dynamics. The proposed approach combines a compact finite difference method for spatial discretization with the weighted and shifted Grünwald–Letnikov (WSGL) scheme for approximating temporal fractional derivatives. This hybrid framework transforms the FGBH equation into a nonlinear algebraic system, which is efficiently solved using a tailored iterative algorithm. Theoretical analysis establishes the unconditional stability and convergence of the method, with an error bound of order $\mathcal{O}(\tau^3 + h^4)$. Comprehensive numerical experiments, including three representative test cases, validate the accuracy and robustness of the method. The results exhibit excellent agreement with theoretical convergence rates, demonstrate resilience in capturing singularities, and confirm computational efficiency across various parameter settings. These findings support the theoretical results established in Theorems 1 and 2 and highlight the practical applicability of the scheme. This study not only advances numerical treatment for fractional PDEs but also provides a computational tool suitable for practical applications in physics, biology, and engineering systems governed by memory and nonlocal effects. Future work will focus on extending the methodology to more complex settings, including systems with variable-order fractional derivatives, higher-dimensional versions of the FGBH equation, and adaptive implementations for capturing intricate solution structures.

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Conflicts of Interest

The author declares no conflicts of interest.

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