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Research Article

Mathematical modeling and optimal control approaches for dengue

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Abstract

This research explores a continuous-time mathematical model that outlines the transmission dynamics of the dengue virus across different regions, involving both human and mosquito hosts. We propose an optimal strategy that includes awareness campaigns, safety measures, and health interventions in dengue-endemic areas, with the goal of reducing transmission between individuals and mosquitoes, thus lowering human infections and eliminating the virus in mosquito populations. Utilizing the discrete-time

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Pontryagin's maximum principle, we identify optimal control measures and employ an iterative approach to solve the optimal system. Numerical simulations are carried out using MATLAB, and a cost-effectiveness ratio is computed. Through an in-depth cost-effectiveness analysis, we highlight the effectiveness of strategies focused on protecting at-risk populations, preventing contact between infected humans and mosquitoes, and promoting the use of quarantine facilities as the most powerful methods for controlling the spread of the dengue virus.

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1 Introduction

Dengue is a disease caused by four closely related serotypes of the dengue virus ($DENV1-4$) and is transmitted through the bite of female Aedes aegypti mosquitoes [1, 14]. Symptoms typically manifest within 3 to 14 days following a bite from an infected mosquito. The World Health Organization reports that approximately 284.528 million cases occur worldwide each year, affecting individuals across nearly all age groups [13]. Early diagnosis is critical for providing appropriate medical treatment and for monitoring and managing outbreaks of dengue fever. Timely detection is essential to effectively control the spread of the disease. Diagnosing dengue can be done by identifying viral DNA, antigens, antibodies, or a combination thereof. The virus is commonly detectable in bodily fluids and certain tissues around five days after the onset of symptoms [5, 8, 21].

In India, control strategies are developed, including causative factors, early identification of cases and prompt treatment, vector control, personal protection against mosquito bites, environmental management, and community awareness [4, 17]. These strategies encourage people to keep their surroundings clean, ultimately leading to the removal of mosquito breeding sites. Public health initiatives and educational campaigns play a pivotal role in enhancing community participation in mosquito control efforts. Enhanced surveillance systems and diagnostic capabilities are also crucial components of these strategies, allowing for more accurate tracking of disease spread and better resource allocation (“Strategies for Effective Dengue Control,” by Singh. S et al.) [20].

Despite these efforts, there remains a significant gap in the research regarding the integration of various control strategies and their combined effects on dengue transmission dynamics. Most existing models tend to focus on individual control measures in isolation, failing to consider the synergistic effects of implementing multiple strategies simultaneously. Furthermore, the role of climate variables and socio-economic factors in influencing the effectiveness of these control measures has not been thoroughly explored [23]. Addressing these gaps is essential for developing a comprehensive approach to dengue control that can adapt to different environmental and social contexts.

To study different aspects of dengue transmission dynamics, several mathematical models have been proposed. In [7], a mathematical model of dengue disease was proposed, discussing vaccination and control strategies for the model. They focused on two types of controls: Mechanical control to eliminate mosquito breeding sites and chemical control to kill the dengue vector using insecticides and larvicides. In [2], another mathematical model for dengue fever was proposed, investigating the impact of available resources on the spread and control of dengue disease. Recently, Srivastav et al. [22] divided the infected population into undetected and detected groups and concluded that adequate management of quarantine/hospitalization may reduce the spread of the disease. Ghosh, Tiwari, and Chattopadhyay [12] indicates that health care organizations must agree on personal protection and mosquito control to achieve a rapid reduction in dengue cases. Esteva and Vargas [9] proposed an SIR-SI model by taking constant and variable human populations, discovering the existence of different equilibrium points, and discussing its stability properties . In [10], a cost-effective strategy is studied where vaccination and the use of insecticides on mosquitoes are considered as primary controls. Additionally, recent advancements in modeling have incorporated climate variables, highlighting the significant influence of weather patterns on mosquito breeding and disease transmission rates [23].

Information plays a crucial role in disease dynamics. In the presence of information, individuals take precautionary and preventive measures, leading to behavioral change in the population and making them less susceptible to the disease. Public awareness campaigns and dissemination of accurate information regarding dengue prevention and symptoms can significantly impact the disease's spread by encouraging timely medical intervention and reducing mosquito breeding sites [15]. In this paper, we propose a mathematical model of dengue in which we consider infected individuals to be divided into two subclasses: Known carriers and unknown carriers. We also consider that the disease is transmitted through the bite of a mosquito carrying the virus, as well as direct contact with infected individuals.

The proposed model is divided into eight compartments representing susceptible (S), exposed (E), asymptomatic infected (A), symptomatic infected (I), hospitalized (H), recovered (R), susceptible mosquitoes (S_a), and infected mosquitoes (I_a). This compartmental approach allows for a detailed analysis of the disease progression and the effectiveness of various control measures. By understanding the dynamics within each compartment, we can better design and implement strategies to mitigate the impact of dengue outbreaks and enhance public health responses [24]. Furthermore, the incorporation of optimal control strategies within this model can provide insights into the most efficient and cost-effective ways to allocate resources and implement interventions. These strategies can be tailored to specific regions and adapted to changing environmental and epidemiological conditions, thereby improving their overall effectiveness.

The potential implications of this research are far-reaching. By developing a more comprehensive understanding of dengue transmission dynamics and control measures, policymakers and public health officials can make more informed decisions that enhance the effectiveness of dengue prevention and control programs. The model can also be adapted to study other vector-borne diseases, thereby broadening its applicability and utility. Additionally, the insights gained from this research can inform the development of integrated vector management programs that combine multiple control strategies in a synergistic manner, ultimately leading to more sustainable and impactful public health outcomes [16, 18].

2 Model formulation

We analyze a mathematical model denoted by $SEAIS_mI_mHR$, which represents the dynamics of dengue virus transmission among the human population.

The model distinguishes between two main categories: the human population $N_h = S+E+A+I+H+R$ and the mosquito population $N_m = S_m+I_m$.

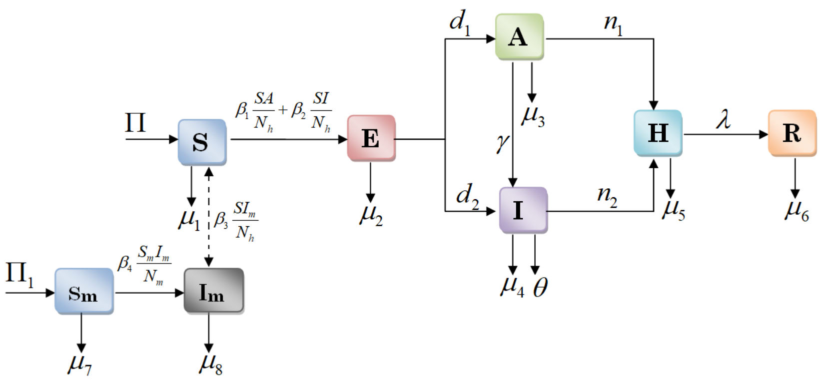

Figure 1: Description of the model.

Hence, we present the spread of dengue mathematical model is governed by the following system of differential equation:

$$\begin{cases} \dfrac{dS(t)}{dt} = \Pi - \beta_1 \dfrac{S(t)A(t)}{N_h} - \beta_2 \dfrac{S(t)I(t)}{N_h} - \beta_3 \dfrac{S(t)I_m(t)}{N_h} - \mu_1 S(t), \\ \dfrac{dE(t)}{dt} = \beta_1 \dfrac{S(t)A(t)}{N_h} + \beta_2 \dfrac{S(t)I(t)}{N_h} + \beta_3 \dfrac{S(t)I_m(t)}{N_h} - (d_1 + d_2 + \mu_2)E(t), \\ \dfrac{dA(t)}{dt} = d_1 E(t) - (n_1 + \gamma + \mu_3)A(t), \\ \dfrac{dI(t)}{dt} = d_2 E(t) + \gamma A(t) - (n_2 + \theta + \mu_4)I(t), \\ \dfrac{dH(t)}{dt} = n_1 A(t) + n_2 I(t) - (\lambda + \mu_5)H(t), \\ \dfrac{dR(t)}{dt} = \lambda H(t) - \mu_6 R(t), \\ \dfrac{dS_m(t)}{dt} = \Pi_1 - \beta_4 \dfrac{S_m(t)I_m(t)}{N_m} - \mu_7 S_m(t), \\ \dfrac{dI_m(t)}{dt} = \beta_4 \dfrac{S_m(t)I_m(t)}{N_m} - \beta_3 \dfrac{S(t)I_m(t)}{N_h} - \mu_8 I_m(t), \end{cases} \tag{1}$$

where $S(0) \geq 0,\ E(0) \geq 0,\ A(0) \geq 0,\ I(0) \geq 0,\ H(0) \geq 0,\ R(0) \geq 0,\ S_m(0) \geq 0,\ I_m(0) \geq 0$ are the initial rates.

2.1 Fundamental characteristics of the model

2.1.1 Positivity of solutions.

Theorem 1. If $S(0) \geq 0,\ E(0) \geq 0,\ A(0) \geq 0,\ I(0) \geq 0,\ H(0) \geq 0,\ R(0) \geq 0,\ S_m(0) \geq 0,\ I_m(0) \geq 0$ are the initial rate $t \geq 0$ the solution of system are positive for all $t \geq 0$.

Proof. The first equation of system (1) implies that

$$\begin{aligned}\frac{dS(t)}{dt} &= \Pi - \left(\beta_1 \frac{A(t)}{N_h} + \beta_2 \frac{I(t)}{N_h} + \beta_3 \frac{I_m(t)}{N_h} + \mu_1\right) S(t)\\ &\geq -\left(\beta_1 \frac{A(t)}{N_h} + \beta_2 \frac{I(t)}{N_h} + \beta_3 \frac{I_m(t)}{N_h} + \mu_1\right) S(t),\end{aligned}$$

where

$$L(t) = \beta_1 \frac{A(t)}{N_h} + \beta_2 \frac{I(t)}{N_h} + \beta_3 \frac{I_m(t)}{N_h} + \mu_1,$$

$$\frac{dS(t)}{dt} + L(t)S(t) \ \geq \ 0.$$

By multiplying both sides of the last inequality by $\exp\left(\int_0^t L(s)ds\right)$, we get

$$\exp\left(\int_0^t L(s)ds\right)\frac{dS(t)}{dt} + L(t)\exp\left(\int_0^t L(s)ds\right)S(t) \ \geq \ 0,$$

$$\frac{d}{dt}\left(S(t)\exp\left(\int_0^t L(s)ds\right)\right) \ \geq \ 0.$$

By integrating this inequality over the interval from 0 to t, we obtain

$$\int_0^t \left(\frac{d}{ds}\left(S(t)\exp\left(\int_0^t L(s)ds\right)\right)\right) ds \geq \ 0.$$

Then

$$S(t) \geq S(0)\exp\left(-\int_0^t L(s)ds\right),$$

$$S(t) \geq 0.$$

Similarly, we show

$E(t) \geq 0\,,\ A(t) \geq 0\,,\ I(t) \geq 0\,,\ H(t) \geq 0\,,\ I_m(t) \geq 0, S_m(t) \geq 0,\ \textit{and}\ \ R(t) \geq 0.$

□

2.1.2 Boundedness of the solutions.

Theorem 2. The sets

$$\begin{cases} \Omega_h = \left\{ (S, E, I, A, H, R) \in \mathbb{R}^6_+ : 0 \leq S + E + I + A + H + R \leq \frac{\Pi}{\mu_h} \right\}, \\ \Omega_m = \left\{ (S_a, I_a) \in \mathbb{R}^2_+ : 0 \leq S_m + I_m \leq \frac{\Pi_1}{\mu_m} \right\}, \end{cases}$$

are positively invariant under system (1) with initial conditions
$S(0) \geq 0,\ \ E(0) \geq 0,\ \ A(0) \geq 0,\ \ I(0) \geq 0,\ \ H(0) \geq 0,\ I_m(0) \geq 0\,,\ S_m(0) \geq 0,$ and $R(0) \geq 0$.

Proof. By the definition, we have

$$N_h = S + E + A + I + H + R;$$

hence

$$\frac{dN_h}{dt} = \Pi - \theta I(t) - \mu_h N_h,$$

$$\frac{dN_h}{dt} = \Pi - \theta I(t) - \mu_h N_h \leq \Pi - \mu_h N_h,$$

$$\frac{dN_h}{dt} \leq \Pi - \mu_h N_h,$$

$$N_h(t) \leq \frac{\Pi}{\mu_h} + N_h(0)e^{-\mu_h t}.$$

If $t \to \infty$, then

$$\lim_{t\to\infty} \sup N(t) = \frac{\Pi}{\mu_h}.$$

This indicates that the region Ω_h is a positively invariant set for system (1).
Also,

$$N_h(t) \leq \frac{\Pi}{\mu_h}.$$

Afterward, it can be demonstrated that

$$N_m(t) \leq \frac{\Pi_1}{\mu_m}.$$

□

2.2 Existence of solutions.

The system (1) can be expressed in the following form: $\Phi(X) = AX + \mathrm{M}(X)$. Then

$$X = \begin{pmatrix} S(t) \\ E(t) \\ A(t) \\ I(t) \\ H(t) \\ R(t) \\ S_m(t) \\ I_m(t) \end{pmatrix}, \qquad \mathrm{M}(X) = \begin{pmatrix} \frac{dS(t)}{dt} \\ \frac{dE(t)}{dt} \\ \frac{dA(t)}{dt} \\ \frac{dI(t)}{dt} \\ \frac{dH(t)}{dt} \\ \frac{dR(t)}{dt} \\ \frac{dS_m(t)}{dt} \\ \frac{dI_m(t)}{dt} \end{pmatrix},$$

where

$$A = \begin{pmatrix} -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & d_1 & A_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & d_2 & \gamma & A_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & n_1 & n_2 & -(\lambda+\mu_5) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & -\mu_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\mu_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_8 \end{pmatrix},$$

$A_1 = -(d_1 + d_2 + \mu_2)$, $A_2 = -(n_1 + \gamma + \mu_3)$, and $A_3 = -(n_2 + \theta + \mu_4)$, and

$$\mathrm{M}(X)=\begin{pmatrix} \Pi-\beta_1\dfrac{S(t)A(t)}{N_h}-\beta_2\dfrac{S(t)I(t)}{N_h}-\beta_3\dfrac{S(t)I_m(t)}{N_h}\\ \beta_1\dfrac{S(t)A(t)}{N_h}+\beta_2\dfrac{S(t)I(t)}{N_h}+\beta_3\dfrac{S(t)I_m(t)}{N_h}\\ 0\\ 0\\ 0\\ 0\\ \Pi_1-\beta_4\dfrac{S_m(t)I_m(t)}{N_m}\\ \beta_4\dfrac{S_m(t)I_m(t)}{N_m}-\beta_3\dfrac{S(t)I_m(t)}{N_h} \end{pmatrix}.$$

Assuming that X_1 and X_2 are solutions to the system of equations presented in (1), it follows that

$$\begin{aligned}|B(Y_1)-B(Y_2)| \leq & 2\left|\beta_1\frac{S_1A_1}{N_h}+\beta_2\frac{S_1I_1}{N_h}+\beta_3\frac{S_1I_{m1}}{N_h}\right.\\ &\left.-\beta_1\frac{S_2A_2}{N_h}-\beta_2\frac{S_2I_2}{N_h}-\beta_3\frac{S_2I_{m2}}{N_h}\right|\\ &+2\left|\beta_4\frac{S_{m1}I_{m1}}{N_m}-\beta_3\frac{S_{m2}I_{m2}}{N_m}\right|\\ =&2\left|\frac{\beta_1S_1}{N_h}(A_1-A_2)+\frac{\beta_1A_2}{N_h}(S_1-S_2)+\frac{\beta_2S_1}{N_h}(I_1-I_2)\right.\\ &\left.+\frac{\beta_2I_2}{N_h}(S_1-S_2)+\frac{\beta_3S_1}{N_h}(I_{m1}-I_{m2})+\frac{\beta_3I_{m2}}{N_h}(S_1-S_2)\right|\\ &+2\left|\frac{\beta_4S_{m1}}{N_m}(I_{m1}-I_{m2})+\frac{\beta_4I_{m2}}{N_m}(S_{m1}-S_{m2})\right|\\ \leq&2\left(\frac{\beta_1S_1}{N_h}|A_1-A_2|+\frac{\beta_1A_2}{N_h}|S_1-S_2|+\frac{\beta_2S_1}{N_h}|I_1-I_2|\right.\\ &\left.+\frac{\beta_2I_2}{N_h}|S_1-S_2|+\frac{\beta_3S_1}{N_h}|I_{m1}-I_{m2}|+\frac{\beta_3I_{m2}}{N_h}|S_1-S_2|\right)\\ &+2\left(\frac{\beta_4S_{m1}}{N_m}|I_{m1}-I_{m2}|+\frac{\beta_4I_{m2}}{N_m}|S_{m1}-S_{m2}|\right),\end{aligned}$$

where

$$|S_1|\leq\frac{\Pi}{N_h},\quad |A_2|\leq\frac{\Pi}{N_h},\quad |S_{m1}|\leq\frac{\Pi_1}{N_m},\ and\quad |I_{m2}|\leq\frac{\Pi_1}{N_m},$$

$$|B(Y_1)-B(Y_2)| \leq\frac{2\Pi}{N_h}\left(\beta_1|A_1-A_2|+\beta_1|S_1-S_2|+\beta_2|I_1-I_2|\right.$$

$$+\beta_2 |S_1 - S_2| + \beta_3 |I_{m1} - I_{m2}| + \beta_4 |S_1 - S_2|)$$
$$+\frac{2\Pi_1}{N_m}\left(\beta_4 |I_{m1} - I_{m2}| + \beta_4 |S_{m1} - S_{m2}|\right).$$

By factoring out the common element, we obtain the following expression:

$$\begin{aligned}|B(Y_1) - B(Y_2)| \leq & \frac{2\beta_1\Pi}{N_h}|A_1 - A_2| + \frac{2\Pi}{N_h}(\beta_1 + \beta_2 + \beta_4)|S_1 - S_2| \\ & + \frac{2\beta_2\Pi}{N_h}|I_1 - I_2| \\ & + \left(\frac{2\beta_3\Pi}{N_h} + \frac{2\beta_4\Pi_1}{N_m}\right)|I_{m1} - I_{m2}| + \frac{2\beta_4\Pi_1}{N_m}|S_{m1} - S_{m2}| \\ \leq & M|Y_1 - Y_2|,\end{aligned}$$

$$M = \max\left(\frac{2\beta_1\Pi}{N_h}, \frac{2\Pi}{N_h}(\beta_1 + \beta_2 + \beta_4), \frac{2\beta_2\Pi}{N_h}, \frac{2\beta_3\Pi}{N_h} + \frac{2\beta_4\Pi_1}{N_m}, \frac{2\beta_4\Pi_1}{N_m}, \|\mathrm{A}\|\right).$$

Thus, we can conclude that the function B is uniformly Lipschitz continuous, ensuring that small changes in the input lead to proportionally small changes in the output, maintaining stability within the system. Additionally, by considering the constraints $S(t) \geq 0$, $E(t) \geq 0$, $A(t) \geq 0$, $I(t) \geq 0$, $H(t) \geq 0$, $R(t) \geq 0$, $S_m(t) \geq 0$, and $I_m(t) \geq 0$ in $\mathbb{R}_+$, we can ensure that all state variables remain nonnegative throughout the system's evolution. This nonnegativity is crucial for the physical realism of the model, as negative values for these variables would not make sense in the context of the problem. Therefore, we can assert the existence of a solution to the system, which respects these nonnegativity conditions. This solution not only validates the model but also aligns with the initial conditions and the constraints imposed on the system. This comprehensive approach confirms that the system's behavior is accurately captured under the specified initial conditions and system constraints [3].

3 The controlled mathematical model

In response to the spread of dengue fever, it is crucial to implement effective control measures by enhancing public awareness and maintaining vigilance against the disease to curb its transmission. To address this need, our study introduces two carefully crafted control strategies designed to combat the spread of dengue fever. We present two distinct control strategies, each targeting a specific aspect of the epidemic's spread dynamics.

In the subsequent sections, we delve into each control method in detail, explaining their mechanisms and assessing their potential impact on the containment and mitigation of dengue fever. These strategies are founded on thorough research and aim to provide a comprehensive approach to disease control.

The first control strategy, represented as $u(t)$, aims to minimize direct contact between the public and asymptomatic carriers of the dengue virus. This is especially critical for those who may not be aware of the associated risks. This strategy is executed through educational campaigns and community outreach programs designed to improve public understanding of dengue fever. By doing so, we seek to promote a higher level of caution and preventive behavior among the population, thereby reducing the probability of disease transmission. These educational initiatives include distributing informational materials, organizing workshops, and leveraging media platforms to raise awareness.

The second control strategy, designated as $v(t)$, focuses on protecting individuals from infection by advocating proactive health measures. This strategy highlights the importance of avoiding contact with infected mosquitoes, the primary vectors of dengue fever. It also promotes the widespread adoption of available vaccinations to build community immunity. Moreover, this strategy encourages symptomatic individuals to seek immediate medical care and follow recommended treatment protocols. The objective is to alleviate the disease burden by ensuring timely and appropriate care for infected individuals, thereby preventing further transmission. This approach encompasses public health announcements, vaccination campaigns, and support for healthcare facilities to effectively manage and treat dengue cases.

By incorporating these control measures into our mathematical model, we aim to establish a robust framework for predicting and managing the spread of dengue fever. The combined impact of these strategies is expected to significantly reduce the incidence of the disease, leading to improved health outcomes for affected populations. Hence, the resulting controlled mathematical system is described as follows:

$$
\begin{cases}
\dfrac{dS(t)}{dt} =& \Pi - \beta_1 \dfrac{S(t)A(t)}{N_h}(1-u(t)) - \beta_2 \dfrac{S(t)I(t)}{N_h}(1-u(t)) \\
& - \beta_3 \dfrac{S(t)I_m(t)}{N_h}(1-v(t)) - \mu_1 S(t), \\
\dfrac{dE(t)}{dt} =& \beta_1 \dfrac{S(t)A(t)}{N_h}(1-u(t)) + \beta_2 \dfrac{S(t)I(t)}{N_h}(1-u(t)) \\
& + \beta_3 \dfrac{S(t)I_m(t)}{N_h}(1-v(t)) - (d_1+d_2+\mu_2)E(t), \\
\dfrac{dA(t)}{dt} =& d_1 E(t) - (n_1+\gamma+\mu_3)A(t), \\
\dfrac{dI(t)}{dt} =& d_2 E(t) + \gamma A(t) - (n_2+\theta+\mu_4)I(t), \\
\dfrac{dH(t)}{dt} =& n_1 A(t) + n_2 I(t) - (\lambda+\mu_5)H(t), \\
\dfrac{dR(t)}{dt} =& \lambda H(t) - \mu_6 R(t), \\
\dfrac{dS_m(t)}{dt} =& \Pi_1 - \beta_4 \dfrac{S_m(t)I_m(t)}{N_m} - \mu_7 S_m(t), \\
\dfrac{dI_m(t)}{dt} =& \beta_4 \dfrac{S_m(t)I_m(t)}{N_m} - \beta_3 \dfrac{S(t)I_m(t)}{N_h}(1-v(t)) - \mu_8 I_m(t).
\end{cases} \tag{2}
$$

4 The challenge of achieving optimal control

The primary challenge in this optimization problem is to minimize the objective functional, which aims to achieve an optimal balance between the application of control measures and their associated costs over a specified time period. The objective functional is defined as

$$
\begin{aligned}
J(u,v) =& I(T) + A(T) + I_m(T) \\
& + \int_0^T \left(I(t) + A(t) + I_m(t) + \frac{e_1}{2}u^2(t) + \frac{e_2}{2}v^2(t) \right) dt.
\end{aligned} \tag{3}
$$

In this context, $I(T)$, $A(T)$, and $I_m(T)$ represent the number of symptomatic and asymptomatic individuals, as well as mosquito virus carriers at the final time T, respectively. The integral term accounts for the total cost over the time interval $[0,T]$, combining the number of symptomatic individuals $I(t)$, asymptomatic individuals $A(t)$, as well as virus carriers from mosquitoes $I_m(t)$, and the quadratic costs associated with the control measures $u(t)$ and $v(t)$.

The coefficients $e_1 \geq 0$ and $e_2 \geq 0$ are cost factors that are carefully chosen to reflect the relative importance and expense of applying the control functions $u(t)$ and $v(t)$ at any given time t. These coefficients ensure that the costs associated with the control measures are properly weighted in the objective functional. The final time $t_f = T$ marks the endpoint of the period under consideration for the optimization.

The goal of our optimization problem is to find the optimal control functions u^* and v^* that minimize the objective functional $J(u, v)$. Mathematically, this is expressed as

$$J(u^*, v^*) = \min_{(u,v)\in V_{ad}} J(u, v), \tag{4}$$

where U_{ad} denotes the set of admissible controls. These controls must satisfy certain constraints to be considered valid within the optimization framework.

The set of admissible controls U_{ad} is defined by the following constraints:

$$\begin{aligned} U_{ad} = \{&(u, v)/0 \leq u_{\min} \leq u(t) \leq u_{\max} \leq 1 \\ &\text{and} 0 \leq v_{\min} \leq v(t) \leq v_{\max} \leq 1 \text{for} t \in [0, t_f]\}\,. \end{aligned}$$

These constraints ensure that the control functions $v(t)$ and $w(t)$ remain within feasible and realistic bounds throughout the entire time period. The lower and upper bounds $v_{\min}$, $v_{\max}$, $w_{\min}$, and $w_{\max}$ are determined based on practical considerations and limitations of the control measures.

By carefully selecting and applying these control functions within the defined constraints, we aim to achieve an optimal reduction in the number of symptomatic and asymptomatic individuals, as well as virus carriers from mosquitoes while managing the costs associated with the control measures. This approach ensures that the implemented control strategies are both effective and economically viable.

4.1 The optimal control: Existence

Theorem 3. We address the task of optimizing the regulation of the system (2). There exists an optimal control pair $(u^*(t), v^*(t)) \in U_{ad}$ such that

$$J(u^*(t), v^*(t)) = \min_{(u,v)\in U_{ad}} J(u(t), v(t)).$$

This is valid under the following conditions:

1. The set of admissible control actions U_{ad} and their corresponding state variables must be nonempty.
2. The right-hand side of the differential equations governing the system is bounded by a linear function involving both the state and control variables.
3. The set U_{ad} of control actions is both convex and closed.
4. The integral $N(S, E, A, I, H, R, S_m, I_m, u, v)$ in the objective function is required to be convex over the set of permissible controls U_{ad}.

In more detail, consider the problem of determining an optimal control strategy for regulating the given system (2). An optimal control $(u^*(t), v^*(t))$ can be identified within the set U_{ad}, ensuring that the cost function $J(u^*(t), v^*(t))$ achieves its minimum value over all admissible controls $u(t)$ and $v(t)$ within U_{ad}. The conditions that need to be satisfied include the nonemptiness of the control action set and state variables, a bounded right-hand side of the system in relation to the state and control variables, the convexity and closure of the control set, and the convexity of the integral in the objective function over the admissible controls.

Proof. **Condition 1.**

To verify the nonemptiness of the control actions set and associated state variables, we reference a simplified existence result from Boyce, DiPrima, and Mead [6]. Consider the system governed by
$\dot{Z}_i = Z_{K_i}(t, Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8)$ for $i \in \{1, \dots, 8\}$,
where $(Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8) = (S, E, A, I, H, R, S_m, I_m)$.
Here, $Z_S, Z_E, Z_A, Z_I, Z_H, Z_R, Z_{S_m}$ and Z_{I_m} represent the right-hand side of equation (4). Assume that the control functions are defined as $u(t) = c_1$ and $v(t) = c_2$ for some constants c_1, c_2. Given that all parameters are constants and that $K_1, K_2, K_3, K_4, K_5, K_6, K_7$ and K_8 are continuous functions, it follows that $Y_S, Y_E, Y_A, Y_I, Y_H, Y_R, Y_{S_m}$, and Y_{I_m} are also continuous. Moreover, the partial derivatives $\frac{\partial Y_{K_i}}{\partial K_i}$ for $i \in \{1, \dots, 8\}$ are continuous. Therefore,

there exists a unique solution $(S, E, A, I, H, R, S_m, I_m)$ satisfying the initial conditions. Consequently, the set of control actions and their corresponding state variables is nonempty, thereby satisfying the first condition.

Condition 2.

By the definition, the set of admissible controls U_{ad} is closed. Consider any controls $u, v \in U_{ad}$ and $\tau \in [0,1]$. We need to show that $0 \leq \tau u + (1-\tau)v \leq 1$. Indeed, we observe that $\tau u \leq \tau$ and $(1-\tau)v \leq (1-\tau)$, which implies that $\tau u + (1-\tau)v \leq \tau + (1-\tau) = 1$. Therefore, we have $0 \leq \tau u + (1-\tau)v \leq 1$ for all $u, v \in U_{ad}$ and $\tau \in [0,1]$. This demonstrates that U_{ad} is both closed and convex, thereby satisfying the second condition.

Condition 3.

The right-hand sides of the equations from system (2) are continuous and remain bounded by the sum of the bounded state variables and control inputs. Furthermore, they can be represented by a linear function in terms of u and v, with coefficients that depend on both the state variables and time.

Condition 4.

The integral within the objective function $I(t) + I_a(t) + A(t) + \frac{e_1}{2}u^2(t) + \frac{e_2}{2}v^2(t)$ is inherently convex over the set U_{ad}. Indeed, there exist constants Θ_1 and Θ_2, with $\kappa > 1$, such that the integral in the objective functional satisfies

$$N(S, E, A, I, H, R, S_m, I_m, u, v) \geq \Theta_1 + \Theta_2(|u|^2 + |v|^2)^{\frac{\kappa}{2}},$$

where

$$I(t) + I_a(t) + A(t) + \frac{e_1}{2}u^2(t) + \frac{e_2}{2}v^2(t) \geq \Theta_1 + \Theta_2(|u|^2 + |v|^2)^{\frac{\kappa}{2}}.$$

Given that the state variables are bounded, let

$$\Theta = 3 \inf_{t\in[0,T]}(I(t) + I_a(t) + A(t)), \quad \Theta = \inf\left(\frac{e_1}{2}, \frac{e_2}{2}\right), \quad \text{and} \quad \kappa = 2.$$

Therefore, based on the results presented by Fleming and Rishel in [11], we can assert the existence of an optimal control. □

4.2 The optimal control: Characterization

In this section, we explore the application of Pontryagin's maximum principle as delineated in [19]. The primary strategy involves the introduction of the adjoint variable, which serves to connect the system of differential equations with the objective function. This connection is instrumental in formulating a functional referred to as the Hamiltonian.

The essence of Pontryagin's principle is to transform the challenge of identifying an optimal control for the objective function, under specified initial conditions, into the task of determining a control strategy that maximizes the Hamiltonian function at every moment throughout the defined time interval.

At a specific time t, we formulate the Hamiltonian, denoted by H, in the following manner:

$$\begin{aligned} H =& A(T) + I(T) + I_m(T) + \frac{e_1}{2}u^2(t) + \frac{e_2}{2}v^2(t) \\ &+ \sum_{i=1}^{8} \xi_i(t).q_i(S, E, A, I, H, R, S_m, I_m). \end{aligned}$$

Within the framework of our analysis, each state variable i in the system of differential equations (see (2)) is represented by q_i, which forms the right-hand side of these equations. The associated adjoint functions, denoted as ξ_i, play a crucial role. They are intricately linked with their respective state variables. The primary purpose of these adjoint functions is to encapsulate the impact of variations in state variables on the objective cost functional. This linkage is fundamental in optimizing the system behavior under study.

Theorem 4. We postulate that the control pair (u^*, v^*) belongs to the admissible set U_{ad}, serving as the optimal control variables within this framework. Correspondingly, the optimal paths for the state variables are designated by S^*, E^*, A^*, I^*, H^*, R^*, S_m^*, and I_m^*. Associated with each of these trajectories are the adjoint functions ξ_1, ξ_2, ξ_3, ξ_4, ξ_5, ξ_6, ξ_7, and ξ_8, each fulfilling specific conditions that align with the dynamics of the system under consideration:

$$
\begin{aligned}
\xi_1{}' &= -\frac{\partial H}{\partial S} = \left(\beta_1 \frac{A}{N_h} + \beta_2 \frac{I}{N_h}\right)(1-u(t))\,(\xi_1-\xi_2) \\
&\quad + \beta_3 \frac{I_m}{N_h}(1-v(t))\,(\xi_1-\xi_2+\xi_8) + \xi_1\mu_1, \\
\xi_2{}' &= -\frac{\partial H}{\partial E} = d_1(\xi_1-\xi_3) + d_2(\xi_2-\xi_4) + \xi_2\mu_2, \\
\xi_3{}' &= -\frac{\partial H}{\partial A} = \beta_1 \frac{S}{N_h}(1-u(t))\,(\xi_1-\xi_2) + n_1(\xi_3-\xi_5) + \gamma(\xi_3-\xi_4) + \xi_3\mu_3, \\
\xi_4{}' &= -\frac{\partial H}{\partial I} = -1 + \beta_2 \frac{S}{N_h}(1-u(t))\,(\xi_1-\xi_2) + n_2(\xi_4-\xi_5) + \xi_4(\mu_4+\theta), \\
\xi_5{}' &= -\frac{\partial H}{\partial H} = \lambda(\xi_5-\xi_6) + \xi_5\mu_5 \\
\xi_6{}' &= -\frac{\partial H}{\partial R} = \xi_6\mu_6, \\
\xi_7{}' &= -\frac{\partial H}{\partial S_m} = \beta_4 \frac{I_m}{N_m}(\xi_7-\xi_8) + \xi_7\mu_7, \\
\xi_8{}' &= -\frac{\partial H}{\partial I_m} = -1 + \beta_3 \frac{S}{N_h}(1-v(t))\,(\xi_1-\xi_2+\xi_8) + \beta_4 \frac{S_m}{N_m}(\xi_7-\xi_8) + \xi_8\mu_8,
\end{aligned}
$$

with transversal conditions at a final time t_f

$$
\begin{aligned}
&\xi_1(t_f)=0, \quad \xi_2(t_f)=0, \quad \xi_3(t_f)=0, \quad \xi_4(t_f)=1, \\
&\xi_5(t_f)=0, \quad \xi_6(t_f)=0, \quad \xi_7(t_f)=0, \quad \xi_8(t_f)=1.
\end{aligned}
$$

The resolution of the optimal control problem relies on the determination of the control variables u^* and v^*. These optimal control settings are derived through the following process:

$$
\begin{aligned}
u^* &= \min\left(1, \max\left(0, \tfrac{(\xi_2-\xi_1)}{e_1}\left(\beta_1 \tfrac{S(t)A(t)}{N_h} + \beta_2 \tfrac{S(t)I(t)}{N_h}\right)\right)\right), \\
v^* &= \min\left(1, \max\left(0, \tfrac{(\xi_4-\xi_1-\xi_8)}{e_2}\left(\beta_3 \tfrac{S(t)I_m(t)}{N_h}\right)\right)\right).
\end{aligned} \tag{5}
$$

Proof. At any specific moment t, the Hamiltonian, denoted as H, is characterized in the following manner:

$$
\begin{aligned}
H =& A(T) + I(T) + I_m(T) + \frac{e_1}{2}u^2(t) + \frac{e_2}{2}v^2(t) \\
&+ \xi_1\left\{\Pi - \frac{\beta_1 SA}{N_h}(1-u) - \frac{\beta_2 SI}{N_h}(1-u) - \frac{\beta_3 SI_m}{N_h}(1-v) - \mu_1 S\right\} \\
&+ \xi_2\left\{\frac{\beta_1 SA}{N_h}(1-u) + \frac{\beta_2 SI}{N_h}(1-u) + \frac{\beta_3 SI_m}{N_h}(1-v) - (d_1+d_2+\mu_2)E\right\} \\
&+ \xi_3\left\{d_1E - (n_1+\gamma+\mu_3)A\right\} + \xi_4\left\{d_2E + \gamma A - (n_2+\theta+\mu_4)I\right\} \\
&+ \xi_5\left\{n_1A + n_2I - (\lambda+\mu_5)H\right\} + \xi_6\left\{\lambda H - \mu_6 R\right\}
\end{aligned}
$$

$$+\xi_7 \left\{ \Pi_1 - \frac{\beta_4 S_m I_m}{N_m} - \mu_7 S_m \right\} + \xi_8 \left\{ \frac{\beta_4 S_m I_m}{N_m} - \frac{\beta_3 S I_m}{N_h}(1-v) - \mu_8 I_m \right\}.$$

By applying Pontryagin's maximum principle, as referenced in [19], we have successfully formulated both the adjoint equations and the associated transversality conditions. These formulations are presented in the following manner:

$${\xi_1}' = -\frac{\partial H}{\partial Z_i} \quad and \quad \xi_i(t_f) = 0 \qquad for \quad i \in \{1,2,3,5,6,7\},$$

where $(Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7, Z_8) = (S, E, A, I, H, R, S_m, I_m)$

The transversality conditions stipulate that the adjoint functions should be orthogonal to the reachable set at the terminal state by the final time t_f. This orthogonality is critical to ensure the problem of optimal control is properly defined, thus securing a reliable and consistent solution. To achieve this, we will deploy the optimality conditions derived from these principles to identify the optimal control trajectories for $u^*(t)$ and $v^*(t)$ over the time interval $t \in [0, t_f]$:

$$\frac{\partial H}{\partial u} = 0 \qquad and \qquad \frac{\partial H}{\partial v} = 0$$

Consequently,

$$-\frac{\partial H}{\partial u} = -e_1 u + \beta_1 \frac{S(t)A(t)}{N_h}(\xi_2 - \xi_1) + \beta_2 \frac{S(t)I(t)}{N_h}(\xi_2 - \xi_1) = 0,$$
$$-\frac{\partial H}{\partial v} = -e_2 v + \beta_3 \frac{S(t)I_m(t)}{N_h}(\xi_2 - \xi_1 - \xi_8) = 0.$$

For this reason,

$$u^* = \frac{(\xi_2 - \xi_1)}{C_1} \left(\beta_1 \frac{S(t)A(t)}{N_h} + \beta_2 \frac{S(t)I(t)}{N_h} \right)$$
$$v^* = \frac{(\xi_4 - \xi_1 - \xi_8)}{C_2} \left(\beta_3 \frac{S(t)I_m(t)}{N_h} \right).$$

Given the constraints imposed by the admissible set U_{ad} on the control variables, we are enabled to effectively identify the optimal control functions $u^*(t)$ and $v^*(t)$. These optimal solutions are formalized and detailed in the equations denoted by (5). □

5 Simulation

In this section, we present the findings derived from the numerical solution of our optimization system. To address our control problem, we initially set the conditions for the state variables at the start of the time period ($t = 0$) and the conditions for the adjoint variables at the end of the period ($t = T$). As a result, our optimization framework is structured as a two-point boundary value problem with specific boundary conditions at designated time intervals.

The optimal solution to this system was obtained iteratively. We began by solving the state equations forward in time, followed by solving the adjoint equations backward. This iterative process started with an initial guess for the control variables during the first iteration. Subsequently, before each new iteration, we refined the control variables using a method known as profiling. This iterative cycle continued until convergence was achieved in the results of successive iterations.

The parameter values used in our model were selected based on hypothetical scenarios due to the lack of real-world data. This approach was necessary because actual data to support our choices were unavailable. The relevant parameter data are presented in Table 1.

The control strategy implemented in this study aims to accomplish several objectives. By understanding the dynamics of disease transmission and applying appropriate control measures, we seek to mitigate the spread of infection and safeguard public health.

Discussion

In this section, we delve into the numerical analysis of various optimal control strategies, such as raising awareness through media channels and educational initiatives about the seriousness of the disease, avoiding direct contact with both infected individuals and mosquitoes via safety campaigns, and encouraging vaccination and medical care. Numerical simulations play a crucial role in evaluating how effective these strategies are.

Table 1: Standard parameter values for the system.

Parameter	Description	Value
Π	The rate of new susceptible individual of humans.	350
Π_1	The rate of new susceptible individual of animals.	0.5
β_1	The rate of new infections from contact with asymptomatic individuals.	0.2
β_2	The rate of new infections from contact with symptomatic individuals.	0.2
β_3	The rate of new infections from contact humans with animals.	0.2
β_4	The rate of new infection of animals.	0.2
γ	The count of infected asymptomatic individuals becoming infected and symptomatic.	0.002
$\mathbf{d_1}$	The count of exposed individuals becoming asymptomatic and infectious.	0.3
$\mathbf{d_2}$	The count of exposed individuals becoming symptomatic and infectious.	0.002
μ	Intrinsic mortality rate.	0.0002
$\mathbf{n_1}$	The count of asymptomatic individuals under lockdown.	0.3
$\mathbf{n_2}$	The count of symptomatic individuals under lockdown.	0.13
θ	Mortality rate due to complications.	0.32
λ	The rate of individuals recovering from hospitals.	0.3

By integrating these control measures into our mathematical model, we can observe their influence on the disease's progression. For instance, public awareness campaigns can greatly reduce infection rates by enhancing community knowledge and promoting preventive actions. Similarly, preventing contact with mosquitoes through safety campaigns and advocating for vaccination and medical care can help further reduce disease transmission and enhance public health outcomes.

The numerical calculations were conducted using MATLAB, based on specific parameter values and initial conditions for state variables as outlined in Table 1. This method enables a thorough assessment of various strategies and their efficacy in controlling the disease.

5.1 Strategy 1: Safeguarding vulnerable individuals by ensuring they avoid any contact with those who are infected.

We employ the optimal control function u to safeguard the population from the virus and to diminish the spread of infections among asymptomatic, symptomatic, and exposed individuals. This is aimed at minimizing function

(3) while maintaining other control measures at zero. A range of strategies has been executed, including awareness campaigns and preventive actions. These strategies encompassed educating the public about the severity of the virus via various media channels and promoting preventive measures with strict adherence to medical guidelines. The objective is to foster a comprehensive understanding of the virus and encourage behaviors that mitigate the risk of transmission, ultimately curbing the spread of infection (Figures 2 and 3).

In Figure 4 (illustrating the graph of infected individuals), it is evident that without any control measures, the number of infected individuals continues to rise, peaking at approximately 104. Conversely, with the implementation of control measures, the number of infected individuals begins to decrease, eventually dropping to around 16 individuals.

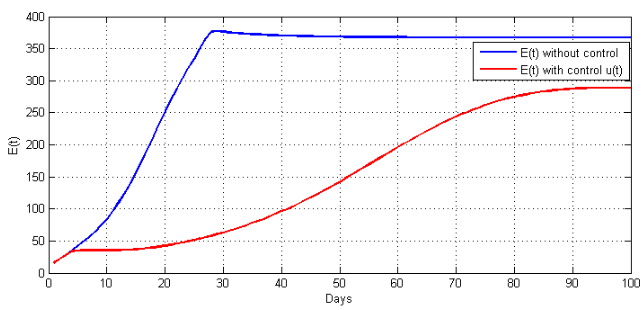

Figure 2: The compartment E without and with control u.

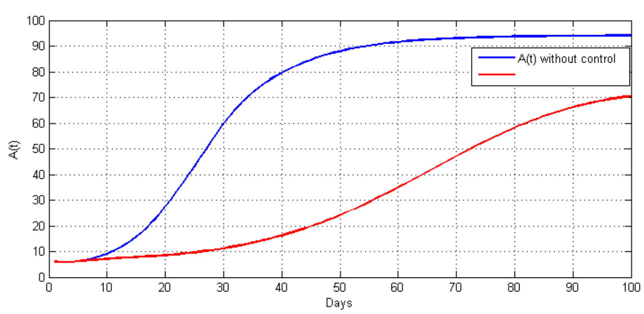

Figure 3: The compartment A without and with control u.

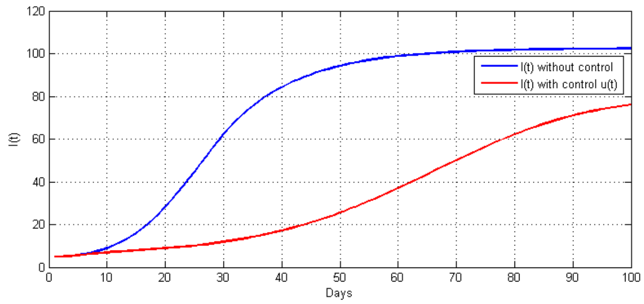

Figure 4: The compartment I without and with control u.

5.2 Strategy 2: Shield vulnerable individuals from exposure to animals (mosquitoes) that are carriers of the virus.

In this approach, we applied control v while maintaining other preventive measures at a minimum. The strategy involved isolating symptomatic individuals and those experiencing severe complications. Quarantine efforts were extended to cover all infected persons, and some individuals received vaccinations against the virus (Figures 5 and 6).

The success of this approach is demonstrated in Figure 7, which shows a marked reduction in both symptomatic and asymptomatic cases following the implementation of quarantine. This method was crucial in managing and containing the spread of the infection within the population.

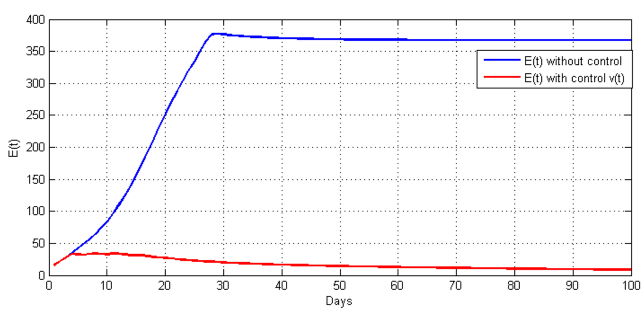

Figure 5: The compartment E without and with control v.

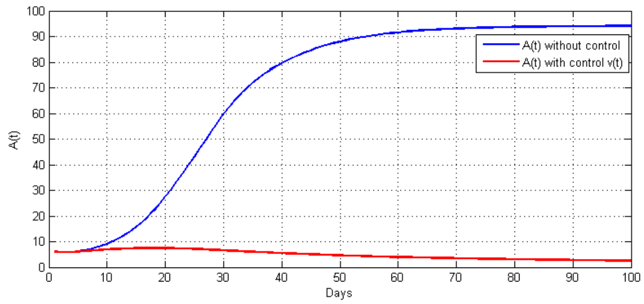

Figure 6: The compartment A without and with control v.

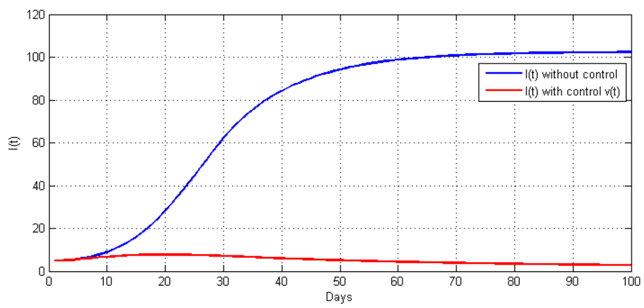

Figure 7: The compartment I without and with control v.

5.3 The third approach: Safeguarding at-risk individuals by ensuring they avoid contact with both infected people and mosquitoes, and guiding them to quarantine facilities.

We employed the optimal control functions u and v to shield the population from dengue fever. The core focus of this critical strategy was on administering comprehensive tests to all individuals suspected of virus infection, with a particular emphasis on testing family members, relatives, and neighbors of confirmed cases. Additionally, preventive measures were implemented across public transportation networks, such as subways, trains, and airports. These measures included monitoring efforts and ensuring that all identified patients were directed to hospitals and quarantine centers. The primary objective of this approach was to curb the spread of the virus into areas that had not yet been affected, thereby minimizing its overall reach and impact, as illustrated in Figures 8, 9, and 10.

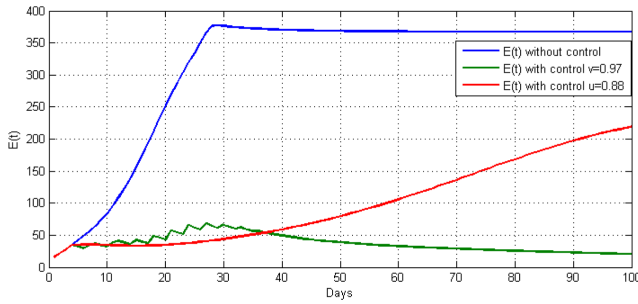

Figure 8: The compartment E without and with control u and v.

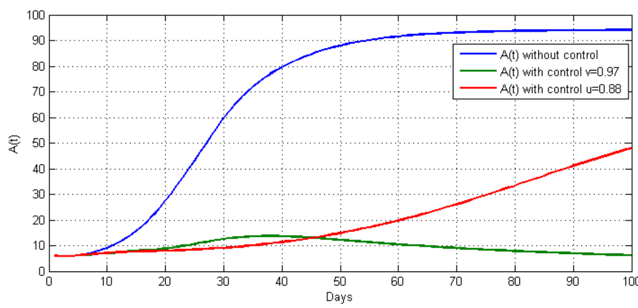

Figure 9: The compartment A without and with control u and v.

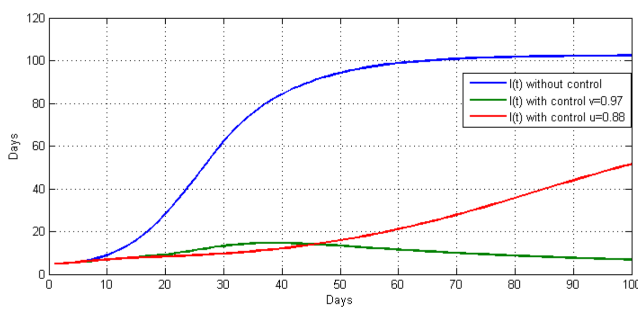

Figure 10: The compartment I without and with control u and v.

6 Conclusion

To summarize, this study delves into the complexities of dengue transmission by developing a comprehensive mathematical model. The main aim was to assess the efficacy of various optimal control strategies aimed at curbing the spread of dengue. These strategies included avoiding contact with both infected individuals and mosquitoes. Additionally, the model incorporated critical interventions such as the treatment of dengue patients and their timely referral to hospitals and quarantine facilities.

The mathematical framework, which includes both symptomatic and asymptomatic cases, is grounded in robust control theory. By applying Pontryagin's maximum principle in discrete time, we were able to derive optimal control strategies, which were subsequently resolved through an iterative numerical method. Moreover, the study also explored the cost-effectiveness of these controls, offering key insights into their economic viability.

Looking ahead, our future research will focus on incorporating fractional derivatives into a spatiotemporal model. This sophisticated approach seeks to improve our understanding of dengue transmission by capturing more complex spatial and temporal patterns, thus enhancing the precision of disease forecasts and control measures. This ongoing work is expected to make a significant contribution to public health strategies and reduce the global impact of dengue.

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