



A modified flux-wave formula for the solution of one-dimensional Euler equations with gravitational source term

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Abstract

In this paper a novel Godunov-type finite volume technique is presented for the solution of one-dimensional Euler equations. The numerical scheme defined herein is well-balanced and approximates the solution by propagating a set of jump discontinuities from each Riemann cell interface. The corresponding source terms are then treated within the flux-differencing of the finite volume computational cells. First, the capability of the numerical solver under gravitational source term is examined and the results are validated with reference solution and higher-order WENO scheme. Then, the well-balanced property of the scheme for the steady-state is tested and finally the proposed method is employed for the modeling small and large amplitude perturbation imposed to the polytropic atmosphere. It is found out that the defined well-balanced solver provides sensible prediction for all of the given test cases.

Keywords: Wave propagation algorithm, Flux wave formula, Riemann solver, Well-Balanced, Euler equations.

1. Introduction

The Euler equations with gravitational source terms have been extensively used in many scientific aspects such as aerospace, astrophysics, and shock tube problems. Prediction models should be able to capture sharp gradient shocks as well as rarefaction waves that appear within the solution in particular with existence of source terms. Generally, two different class of finite volume methods have been used to model Euler equations in recent

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years mainly reviewed by LeVeque [6, 7] and [16]. The first method is upwind method, which basically uses the Godunov scheme. Despite the complexity of upwind schemes in particular in dealing with associated Jacobian matrix they provide very accurate results for the shock capturing problems [17]. The second approach is central schemes, which applies Lax–Friedrichs or Lax–Wendroff methods. However, they produce rather high diffusion, which eventually affects the methods stability unless a refined mesh is used [7].

Previously, significant attentions have been paid for the solution of the Euler equations with the gravitational source terms mostly based on the finite volume methods. LeVeque and Bale [8] have developed a quasi-state wave propagation algorithm for the solution of the Euler equations. In another work Botta et al. [2] defined a well-balanced finite-volume methods, which maintain certain class of steady states for the nearly hydrostatic flows. A well-balanced approach on the basis of the gas-kinetic scheme has been proposed by Luo et al. [10] for an isolated gravitational hydrodynamic system. Käppeli and Mishra [5] have introduced a second-order well-balanced finite volume scheme for the isentropic hydrostatic equilibrium. Chandrashekar and Klingenberg [3] implemented a well-balanced second-order Godunov-type finite volume method for the Euler equations with gravitation. More recently, Li and Xing designed a high-order well-balanced finite volume WENO (weighted essentially oscillatory) scheme for the Euler equations with gravitational field, which preserves both isothermal equilibrium and the polytropic hydrostatic balance state.

The main purpose of this work is to develop a version of Godunov-type wave propagation algorithm for the solution of one dimensional Euler equations. The proposed method extends the a modified flux-wave solution provided in [11, 12] for the shallow water equations (SWEs) to the one dimensional Euler equations. This approach is well-balanced and treats any source terms within the flux-differencing of the finite-volume computational cells. Additionally, the defined numerical scheme utilizes the advantage of combination both approximate and exact Riemann speeds, which enables the method to avoid non-negative pressure fields for the Euler equations. To the best of author’s knowledge no development of the modified flux-wave approach defined in [11, 12] is used for the solution of the one dimensional Euler equations with gravitational source term. The rest of this paper is organized as follows: In the next section the mathematical equations for the one dimensional Euler equations consisting of gravitational source terms are provided. Then in the second section, the wave propagation algorithm with both first-order and high-resolution accurate terms is expressed. In the third section, the flux-wave formula comprising different choice of wave speeds for the one dimensional Euler equations is described. Fourth section states the validation of the introduced numerical method with the reference solutions and other numerical results available in literature. Finally, the paper ends with the summary of numerical results and conclusions of findings.

2. Governing equations

The one dimensional Euler equations including source terms can take the conservation law form as

$$\mathbf{U}_t + \mathbf{F}(\mathbf{U})_x = \mathbf{S},$$

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ E \end{bmatrix}, \mathbf{F}(\mathbf{U}) = \begin{bmatrix} \rho u \\ \rho u^2 + P \\ (E + P)u \end{bmatrix}, \mathbf{S} = \begin{bmatrix} 0 \\ -\rho\phi_x \\ -\rho u\phi_x \end{bmatrix},$$

where \mathbf{U} is the vector of unknowns, $\mathbf{F}(\mathbf{U})$ is the flux-term, \mathbf{S} shows corresponding source term, ρ is density, u denotes the particle velocity, P is pressure, ϕ is an time independent gravitational potential, and finally E is the total energy, which can be obtained as

$$E = \frac{P}{\gamma - 1} + \frac{1}{2}\rho u^2,$$

where γ represents the ratio of specific heat. The relevant eigenvalues and eigenvectors for the defined system of equations are expressed as

$$\lambda_1 = u - c, \lambda_2 = u, \lambda_3 = u + c.$$

$$\mathbf{r}_1 = \begin{bmatrix} 1 \\ u - c \\ \zeta - uc \end{bmatrix}, \mathbf{r}_2 = \begin{bmatrix} 1 \\ u \\ u^2/2 \end{bmatrix}, \mathbf{r}_3 = \begin{bmatrix} 1 \\ u + c \\ \zeta + uc \end{bmatrix},$$

where in the above equations c and ζ are called sound speed and the total specific enthalpy, respectively, which can be computed as

$$c = \sqrt{\frac{\gamma P}{\rho}}, \zeta = \frac{E + P}{\rho}.$$

In order to solve the above system of equation the wave propagation algorithm described in the next section is used.

3. Wave propagation algorithm

The one dimensional Godunov-type wave propagation algorithm can be given as [6, 7]

$$\mathbf{U}^{n+1} = \mathbf{U}^n - \frac{\Delta t}{\Delta x} (\mathbf{A}^+ \Delta \mathbf{U}_{i-1/2} + \mathbf{A}^- \Delta \mathbf{U}_{i+1/2}) - \frac{\Delta t}{\Delta x} (\tilde{\mathbf{F}}_{i+1/2}^n - \tilde{\mathbf{F}}_{i-1/2}^n),$$

where \mathbf{U}^{n+1} is the vector of unknowns at the next time step, $\mathbf{A}^+ \Delta \mathbf{U}_{i-1/2}$ and $\mathbf{A}^- \Delta \mathbf{U}_{i-1/2}$ provide the right- and left-going fluctuations, and finally $\tilde{\mathbf{F}}_{i\pm 1}^n$ shows the second-order correction terms required to obtain high-resolution scheme, which can be used with different choice of limiters. If $\tilde{\mathbf{F}} = 0$, the first-order Godunov-type wave propagation algorithm is achieved. The right and left-going fluctuations are then computed, using the following formulations

$$\mathbf{A}^+ \Delta \mathbf{U}_{i-1/2} = \sum_{k:\lambda_{i-1/2} > 0} \boldsymbol{\xi}_{k,i-1/2}, \quad \mathbf{A}^- \Delta \mathbf{U}_{i-1/2} = \sum_{k:\lambda_{i-1/2} < 0} \boldsymbol{\xi}_{k,i-1/2},$$

where $\boldsymbol{\xi}_{k,i-1/2}$ is called the k th flux-wave propagating from cell interface $i - 1/2$ and can be obtained by multiplying particular coefficients into its corresponding eigenvector, say,

$$\boldsymbol{\xi}_{k,i-1/2} = \beta_{k,i-1/2} \mathbf{r}_{k,i-1/2}.$$

4. Flux-wave formula

The flux-wave approach has been first introduced by [1] for the acoustic problem. This approach has been later modified by Mahdizadeh et al. [11,12] for the SWEs with modeling wet/dry front abilities. In this section this modified version of flux-wave approach is extended for the solution of one dimensional Euler equations with gravitational source term. The original flux-wave formula including the treatment of source term can take the form [1]

$$\mathbf{F}(\mathbf{U}_i) - \mathbf{F}(\mathbf{U}_{i-1}) - \mathbf{S}_{i-1/2} \Delta x = \sum_{k=1}^{M_w} \boldsymbol{\xi}_{k,i-1/2},$$

where $\mathbf{F}(\mathbf{U}_i)$ and $\mathbf{F}(\mathbf{U}_{i-1})$ are the fluxes at the left and right side of cell interface $i - 1/2$ and M_w denotes the number of waves, which for the prescribed Euler equations is equal to three and Δx implies finite-volume cell length. To expand the flux-wave approach for the one dimensional Euler equations it is only required that the differences between neighboring fluxes minus the source terms is equalized with the summation of the relevant fluxes. This can be accomplished as

Woodward-Coella

$$\begin{bmatrix} \rho_i \tilde{u}_i \\ \rho_i \tilde{u}_i + P_i \\ (E_i + P_i) \tilde{u}_i \end{bmatrix} - \begin{bmatrix} \rho_{i-1} \tilde{u}_{i-1} \\ \rho_{i-1} \tilde{u}_{i-1}^2 + P_{i-1} \\ (E_{i-1} + P_{i-1}) \tilde{u}_{i-1} \end{bmatrix} - \Delta x \begin{bmatrix} 0 \\ -\rho_i (\phi_{i+1} - \phi_i) / \Delta x \\ -\rho_i u_i (\phi_{i+1} - \phi_i) / \Delta x \end{bmatrix} \quad (1)$$

$$= \beta_1 \begin{bmatrix} 1 \\ \tilde{u}_i - \tilde{c}_i \\ \tilde{\zeta}_i - \tilde{u}_i c_i \end{bmatrix} + \beta_2 \begin{bmatrix} 1 \\ \tilde{u}_i \\ \tilde{u}_i^2/2 \end{bmatrix} + \beta_3 \begin{bmatrix} 1 \\ \tilde{u}_i + \tilde{c}_i \\ \tilde{\zeta}_i + \tilde{u}_i c_i \end{bmatrix},$$

where \tilde{u} and $\tilde{\zeta}$ are the velocity and total specific enthalpy again, which can be obtained through the combination of exact and approximate Riemann wave speeds fully explained in [11], where the approximate Riemann solver utilized herein is based upon the Roe solver [14]. For the Euler equations the approximate velocity and total specific enthalpy can be given as

$$\tilde{u} = \frac{\sqrt{\rho_{i-1}}u_{i-1} + \sqrt{\rho_i}u_i}{\sqrt{\rho_{i-1}} + \sqrt{\rho_i}}, \quad \tilde{\zeta} = \frac{\sqrt{\rho_{i-1}}\zeta_{i-1} + \sqrt{\rho_i}\zeta_i}{\sqrt{\rho_{i-1}} + \sqrt{\rho_i}},$$

and the sound speed \tilde{c}_i can take the form

$$\tilde{c}_i = \sqrt{(\gamma - 1)(\tilde{\zeta}_i - 1/2\tilde{u}_i^2)}.$$

The systems mentioned in equation (1) can be rewritten as

$$\begin{bmatrix} 1 & 1 & 1 \\ \tilde{u}_i - c_i & \tilde{u} & \tilde{u}_i + \tilde{c}_i \\ \tilde{\zeta}_i - \tilde{u}_i c_i & \tilde{u}_i^2/2 & \tilde{\zeta}_i + \tilde{u}_i c_i \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}, \quad (2)$$

where δ_1 , δ_2 , and δ_3 are

$$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} = \begin{bmatrix} \rho_i \tilde{u}_i - \rho_{i-1} \tilde{u}_{i-1} \\ (\rho_i \tilde{u}_i + P_i) - (\rho_{i-1} \tilde{u}_{i-1}^2 + P_{i-1}) + \rho_i (\phi_{i+1} - \phi_i) \\ (E_i + P_i) \tilde{u}_i - (E_{i-1} + P_{i-1}) \tilde{u}_{i-1} + \rho_i u_i (\phi_{i+1} - \phi_i) \end{bmatrix}.$$

By solving the linear system provided in equation (2), the coefficients β_1 , β_2 , and β_3 are computed, which can be later used to calculate the flux-wave $\xi_{k,i-1/2}$ required for obtaining the left- and right-going fluctuations for the first-order Godunov-type wave propagation algorithm. It should be stressed for the solution of linear system given above the LU decomposition algorithm with partial pivoting defined in [13] at each time-step.

4.1 Stability conditions

To ensure the method's stability, the Courant–Friedrichs–Lewy condition (CFL) [4] similar to the SWEs is used. This condition can be given as the following equation for the one dimensional Euler equations:

$$\text{CFL} = \frac{\max(\tilde{\lambda})}{\Delta t} \Delta x,$$

where $\tilde{\lambda} = \max(\tilde{\lambda}_1, \tilde{\lambda}_2, \tilde{\lambda}_3)$ is the maximum amount of wave speeds, where each wave speed is obtained as

$$\tilde{\lambda}_1 = \tilde{u}_i - \tilde{c}_i, \quad \tilde{\lambda}_2 = \tilde{u}_i, \quad \tilde{\lambda}_3 = \tilde{u}_i + \tilde{c}_i.$$

Generally, the wave propagation algorithm uses values of CFL number close to one, which ultimately reduces the computational setup time compared to other Riemann solvers provided in literature.

5. Numerical results

In order to examine the effectiveness of modified flux-wave approach (MFW) for the solution of the one dimensional Euler equations, in this section several numerical test cases are provided. The suitability of the proposed MFW approach in dealing with the gravitational source terms is first investigated, and the obtained numerical results are compared with both reference solutions and available numerical data borrowed from literature. Then, the well-balanced property of the defined method with the existence gravitational field for the isothermal equilibrium is verified, and the calculated numerical results are validated with the nonwell-balanced WENO scheme. Finally, the capability of the proposed well-balanced model in capturing small and large amount of amplitude perturbation is tested. It should be expressed that the MFW model defined herein was solved, using an in-house FORTRAN code on an Intel Core (i7-4790) 3.6 GHz processor with 16GB of RAM. Additionally, the solver employs high-resolution wave propagation algorithm based on the monotonized centered (MC) limiter. The number of computational finite volume cells is defined separately for each test-case.

5.1 Shock tube problem with gravitational field

The purpose of this test case is to assess the performance of the proposed numerical solver for the solution of the one dimensional Euler equations with the existence of gravitational source term. For this problem the Sod test case is considered with the initial condition as

$$(\rho^0, u^0, P^0) = \begin{cases} (1, 0, 1) & \text{if } x \leq 0.5, \\ (0.125, 0, 1) & \text{otherwise,} \end{cases}$$

where ρ^0, u^0 , and P^0 are again the initial data for the defined Riemann problem at time $t = 0$. As the boundary conditions the extrapolation boundary conditions were imposed for both left and right boundaries. In order to consider the effect of gravitational source term a constant exhibit gravitational

field $\phi_x = 1$ is used within the source term. The computation is implemented until time $t = 0.2\text{s}$ with 100 uniform cells and the CFL number equal to 0.9. Figure 2 shows numerical predictions for the density, pressure, energy and velocity obtained based on the MFW approach. The validations are performed with the reference solution achieved using 2000 numerical cell and also with a novel higher-order WENO scheme defined by [9] with the same computational volumes. For the density field a left-going rarefaction wave along with the contact discontinuity and shock wave are created within solution and the obtained results are in qualitative agreement with both reference solution and the WENO approach. Additionally, due to the effect of gravitational source term the density is pulled upward for the left boundary.

Figure 1(b) demonstrates the numerical results for the pressure field. As can be observed, a left-going rarefaction waves as well as right-moving shock are appeared within solution and again the agreement between the obtained numerical results and the reference solution are quite well. Figures 1(c) and 1(d) exhibit the numerical results for the energy and velocity, respectively. In terms of the energy field a rather similar shape to the pressure is seen. For the velocity distributions a negative velocities in some regions are appeared, which is mainly caused by gravitational term.

5.2 One dimensional gas-falling into fixed external potential

This test case was originally introduced in [15, 18] and utilized to verify the well-balanced property of the proposed MFW approach for the isothermal equilibrium within gravitational field. The initial conditions of the problem can be given as

$$\rho = \rho_0 \exp\left(-\frac{\phi}{RT}\right), \quad u = 0, \quad \text{and} \quad p = RT \left(-\frac{\phi}{RT}\right), \quad (3)$$

where gravitational field is expressed as

$$\phi(x) = -\phi_0 \frac{L}{2\pi} \sin\left(\frac{2\pi x}{L}\right),$$

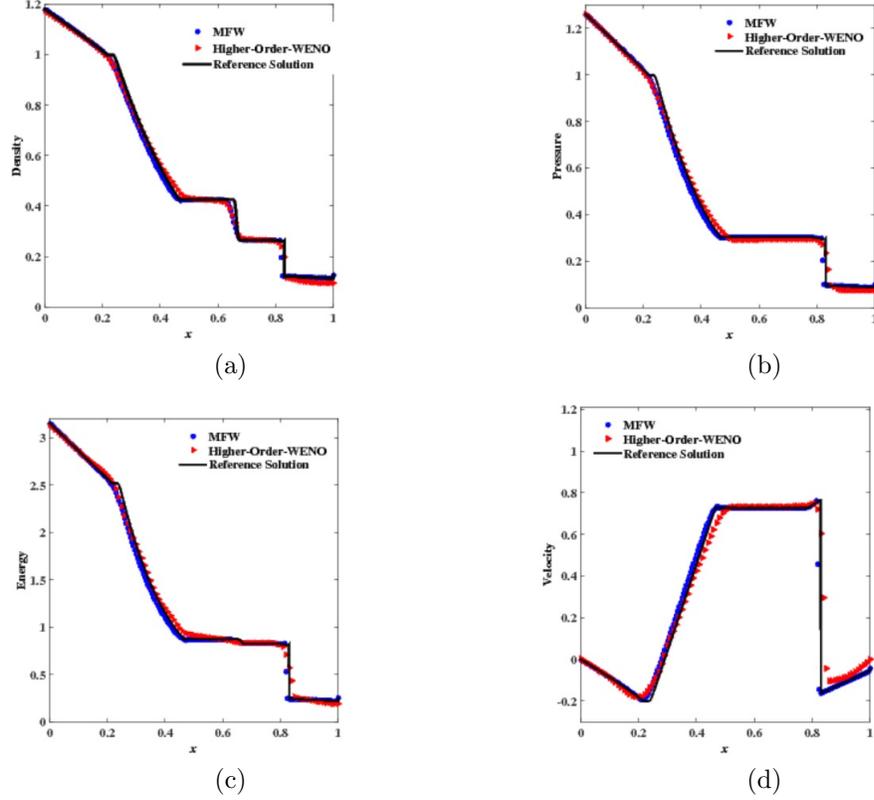


Figure 1: Numerical solution of the shock tube problem with gravitational source term performed at $t = 0.2s$. (a) Density, (b) pressure, (c) energy, (d) velocity

where L is the computational domain set equal to 64 and other parameters are: $\rho_0 = 1$, $\gamma = 5/3$, $T = 0.6866$ and $\phi_0 = .02$. The solution was then performed in double precision using 200 and 400 computational cells until time $t=50s$, where the steady-state condition is reached. Table 1 demonstrates the Euclidean norm achieved for the conserved variables ρ , ρu and E at the steady-state condition. As can be seen for all variables relatively small amount of error is obtained, which clearly state that the initial conditions have been preserved during the simulations.

In order to compare the performance of the defined MFW approach with the unbalanced scheme a small perturbation was imposed into the steady-state solution provided in (3). Therefore the initial condition related to pressure becomes

$$p = RT \left(-\frac{\phi}{RT} \right) + 0.001 \exp(-10(x - 32)^2).$$

Table 1: Euclidean error norm computed based on the MFW approach for the steady-state

Number of Cells	ρ	ρu	E
200	2.6493E-04	9.7563E-05	4.3708E-04
400	6.8740E-05	2.2979E-05	8.9109E-05

The simulation was implemented up to time $t = 179980.53s$, which is much larger than the reference sound crossing time approximated as $\tau = 120s$. Figure 2 illustrates the numerical results obtained based upon the second-order MFW approach in comparison with non-well-balanced WENO scheme borrowed from [9]. As can be observed the initial condition for the velocity profile was maintained during computation in contrast to the non-well-balanced scheme, which was not able to properly neutralize the effect of gravitational source terms with the flux gradient for the isothermal equilibrium state.

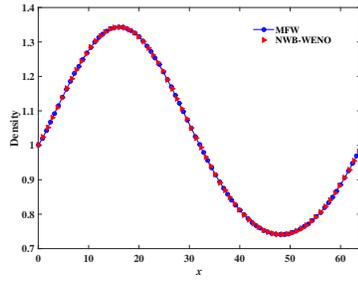
5.3 Small and large amplitude wave propagation

The final test case employed herein was introduced in [5] and investigates the performance of the well-balanced MFW approach in dealing with small and large disturbances for the polytropic atmosphere in the gravitational field $\phi(x) = gx$. In doing so, the polytropic steady state solution is defined as

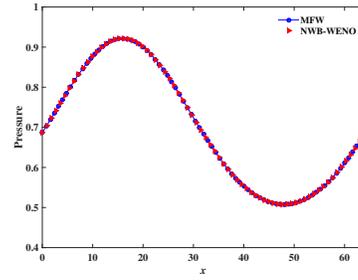
$$\rho(x) = \left(\rho_0^{\gamma-1} - \frac{1}{K_0} \frac{\gamma-1}{\gamma} gx \right)^{\frac{1}{\gamma-1}}, \quad u(x) = 0, \quad p(x) = K_0 \rho(x)^\gamma.$$

with the constants: $g = 1$, $\gamma = 5/3$, $\rho_0 = 1$, $p_0 = 1$, and $K_0 = p_0/(\rho_0^\gamma)$. The computational domain of the polytropic atmosphere was set equal to $[0, 2]$ and the following periodic velocity perturbation was imposed at the bottom boundary

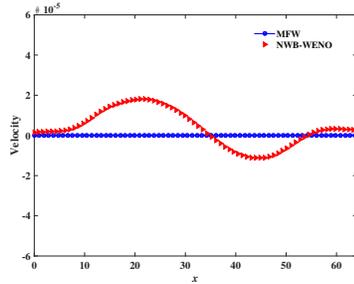
$$u(0, t) = A \sin(4\pi t), \quad (4)$$



(a)



(b)



(c)

Figure 2: The numerical results achieved with the MFW method along with the non well-balanced WENO scheme for the perturbation problem given in section 5.2 at $t = 179.980.53s$. (a) Density, (b) pressure, (c) velocity

In the first numerical study the amount of A in above equation was chosen to 10^{-6} which provides a small perturbation into the solution and the simulation was run until time $t = 1.5s$. In Figure 2 the pressure perturbation and velocity profiles calculated based upon the second-order MFW approach with 400 numerical cells for the small disturbance were shown. These results have been also compared with the reference solution with 8000 numerical cells together with the higher-order WENO scheme given in [9]. It is clear that the results are in good agreement with both reference solution and the higher-order WENO scheme, which confirms that even for a small-perturbation the effect of gravitational source term has been accurately treated within the flux-differences of the neighboring cells for the finite volume method.

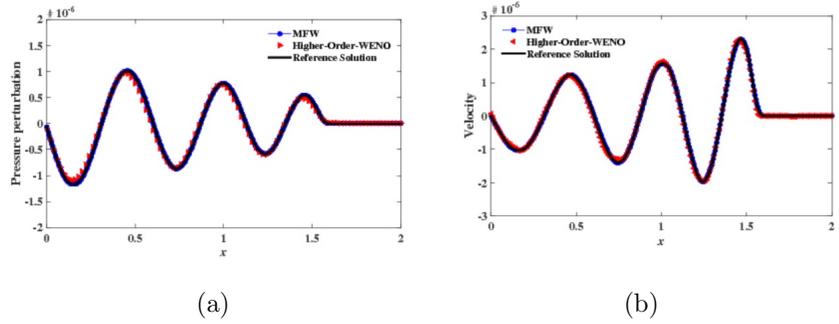


Figure 3: Numerical results obtained based on the MFW approach for the small perturbation imposed for the polytropic steady-state solution in comparison with the higher-order WENO scheme and reference. solution (a) Pressure perturbation,(b) velocity

Eventually the amount of A in the equation (4) was set to 0.1, which creates rather large amplitude disturbance into the solution. The simulation based on the second-order MFW approach with 200 computational cells and $CFL=0.9$ was then implemented until $t=1.5s$. Figure 4 shows plots for the pressure perturbations and velocity along with the results calculated using reference solution with 2000 cells and higher-order WENO scheme. As can be observed the large amplitude perturbation was also captured by the defined MFW approach and the results are in qualitative agreement with the higher-order WENO method.

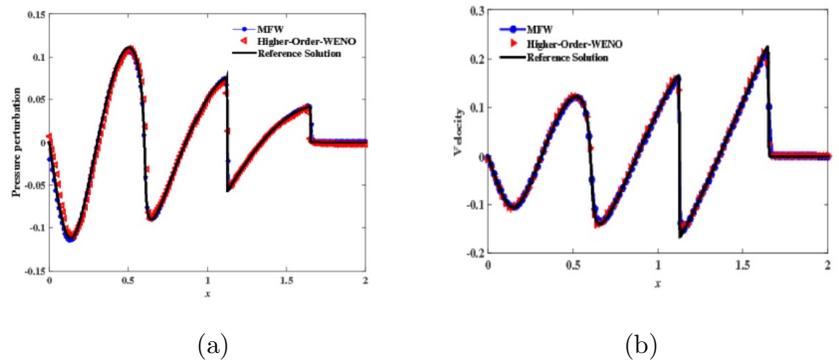


Figure 4: Numerical results obtained based on the MFW approach for the large perturbation imposed for the polytropic steady-state solution in comparison with the higher-order WENO scheme and reference. solution (a) Pressure perturbation, (b) velocity

Conclusions

In this paper a modified flux-wave formula is presented for the solution one dimensional Euler equations including gravitational source term. The numerical solver defined herein is well-balanced and deals with gravitational source terms inside the differences between fluxes for the finite volume computational cells. In order to cope with difficulties with nonphysical results, a combination of exact and approximate Riemann solution was utilized. A modification for the Sod problem was first considered with the gravitational source term and the validations was made with the higher-order WENO scheme and rather identical results was achieved. Then, the well-balanced property of the scheme was tested and the ability of the method in modeling the perturbation applied into the steady-state equilibrium in comparison with unbalanced scheme was demonstrated. For the problem with small and large amplitude of perturbations, the MFW approach gave the same results equal to higher-order WENO scheme and accurately captured disturbances imposed to the polytropic atmosphere.

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روش شار موج تغییر یافته برای حل معادله اولر یک بعدی با ترمهای منبع ثقلی

حسین مهدیزاده

دانشگاه بیرجند، گروه عمران

دریافت مقاله ۲۳ مهر ۱۳۹۵، دریافت مقاله اصلاح شده ۱۱ بهمن ۱۳۹۶، پذیرش مقاله ۹ اسفند ۱۳۹۶

چکیده: در این مقاله یک روش نوین حجم محدود نوع گودونوف برای حل معادلات اولر یک بعدی ارائه گردیده است. روش عددی معرفی شده در اینجا خوش توازن بوده و قادر به تقریب روش حل با بخش مجموعه ای از نا پیوستگی ها از هر سطح مشترک سلول ریمان می باشد. ترمهای منبع مرتبط نیز در داخل تفاضل شارهای دو سلول محاسباتی روش حجم محدود رفتار می گردند. ابتدا توانایی حل کننده عددی تحت تاثیر ترمهای منبع ثقلی بررسی گردیده و نتایج با حل های مرجع و روش ونو با دقت مرتبه بالا مقایسه می گردد. سپس ویژگی خوش توازن بودن روش مورد نظر برای حل مسائل با حالت ماندگار امتحان گردیده و نهایتاً روش پیشنهادی برای مدل سازی اغتشاشات با دامنه کوچک و بزرگ بکار گرفته می شود. نتایج عددی نشان می دهد که روش خوش توازن معرفی شده پیش بینی های دقیقی برای مسائل داده شده ارائه می نماید.

کلمات کلیدی: روش پخش موج؛ روش شار موج؛ حل کننده ریمان؛ خوش توازن؛ معادلات اولر.