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Research Article

Design of a multi-stage hybrid optimizer combining Bayesian and evolutionary algorithms for modeling and optimizing seizure-related information in EEG signals

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Abstract

The prediction of epileptic seizures from EEG signal analysis is modeled as a complex, high-dimensional, multi-objective optimization problem. In this study, we propose and design a novel multi-stage hybrid optimizer that integrates Bayesian optimization with a family of evolutionary and metaheuristic algorithms, including genetic algorithm, evolutionary programming, evolution strategies, ant colony optimization, artificial bee colony, and harmony search. This optimizer is structured to operate across sequential stages of optimization, where each stage is designed to progressively refine the solution space by selecting the optimal subset of EEG channels and time blocks related to the preictal phase. The mathematical model aims to maximize the posterior probability of seizure occurrence while minimizing the temporal distance to the actual seizure, thus formulating a well-defined trade-off in the objective space. Unlike traditional methods, the proposed hybrid framework models and controls the optimization

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trajectory by combining the global exploration capability of evolutionary algorithms with the adaptive search of Bayesian optimization. Experimental results demonstrate that the optimizer successfully converges toward high-informational regions of EEG data that occur early in the preictal phase. A detailed performance comparison and sensitivity analysis further validate the effectiveness and robustness of the proposed approach, offering insights into the design of more generalizable hybrid optimizers for complex modeling tasks.

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1 Introduction

Mathematical modeling has always been a fundamental activity in sciences and engineering. Formulating qualitative questions about observed phenomena into mathematical problems has long been a key driver in the development of mathematics [7]. With the rapid growth of computational tools in recent decades, mathematical modeling has become an essential framework for analyzing and solving complex real-world problems. In many scientific and engineering domains, real-world systems are characterized by high-dimensional, nonlinear, and noisy data. In such situations, reliable interpretation and prediction require mathematical representations that capture the underlying structure of the data rather than merely describing observations. These representations are commonly expressed in terms of functions, stochastic processes, or parametric models [7].

This need is particularly evident in biomedical signal processing, where signals are inherently complex and non-stationary. Among them, electroencephalogram (EEG) signals play a central role in understanding brain dynamics and neurological disorders such as epilepsy [18, 16]. EEG signals provide multichannel time-varying measurements of brain electrical activity, which can be used to detect abnormal patterns associated with neurological events.

Epileptic seizures are caused by sudden and excessive neuronal discharges, which lead to abnormal electrical activity in the brain. These events are reflected in EEG signals as characteristic patterns such as spikes, sharp waves, and rhythmic discharges [11]. One of the most important challenges in this field is the early prediction of seizures during the preictal phase, where subtle changes in EEG signals may indicate an upcoming seizure.

Despite significant progress in signal processing and machine learning methods, seizure prediction remains a challenging problem due to the nonlinear, patient-specific, and highly dynamic nature of EEG signals. Existing approaches often fail to simultaneously achieve high prediction accuracy and early detection capability.

In recent years, optimization-based methods have been widely used for feature selection and pattern discovery in EEG analysis. In particular, evolutionary algorithms and swarm intelligence techniques have shown strong performance in exploring large search spaces and identifying informative signal components [12, 9]. However, most of these methods lack a unified probabilistic framework for modeling uncertainty and incorporating prior knowledge.

Bayesian inference provides a principled framework for probabilistic reasoning and uncertainty modeling. It enables the estimation of posterior probabilities associated with seizure occurrence based on observed EEG data [8]. However, Bayesian methods alone are often insufficient for efficiently exploring large combinational spaces such as EEG channel-time block selection.

To overcome these limitations, this study proposes a novel multi-stage hybrid optimization framework that integrates Bayesian modeling with multiple evolutionary and metaheuristic algorithms, including genetic algorithm (GA), evolutionary programming (EP), evolution strategies (ESs), ant colony optimization (ACO), artificial bee colony (ABC), and harmony search (HS) [14, 20, 12, 10, 9]. The proposed method formulates seizure prediction as a multi-objective optimization problem, aiming to maximize the posterior probability of seizure occurrence while minimizing detection delay.

The main contributions of this work are summarized as follows:

- Development of a Bayesian-based probabilistic framework for modeling EEG channel-time information.
- Definition of a novel cost function that balances seizure probability and temporal proximity.
- Design of a multi-stage hybrid optimization framework integrating Bayesian inference and evolutionary algorithms.
- Comparative evaluation of multiple metaheuristic approaches for EEG feature selection.
- Validation on real EEG datasets demonstrating the effectiveness of the proposed method [6].

The remainder of this paper is organized as follows: Section 2 presents the problem formulation and EEG signal modeling. Section 3 describes the proposed hybrid optimization framework. Section 4 presents experimental results and a discussion. Finally, Section 5 concludes the paper.

2 Materials and methods

2.1 Database

The dataset used in this study consists of EEG recordings from 14 patients collected at the Department of Neurology and Neurophysiology of the University of Siena. The cohort includes 9 male subjects (aged 25–71 years) and 5 female subjects (aged 20–58 years). EEG signals were recorded using the international 10–20 electrode placement system at a sampling frequency of 512 Hz. In addition to EEG recordings, most sessions include one or two electrocardiogram (ECG) channels to support multimodal physiological analysis. All recordings were clinically annotated and validated by a specialist neurologist based on comprehensive evaluation of patients' clinical history and electrophysiological patterns, including seizure onset and type classification [6]. The dataset provides continuous multichannel scalp EEG signals suitable for analyzing preictal and ictal dynamics in seizure prediction tasks.

2.2 Signal segmentation

Let the discrete EEG signal of channel c be represented as $x_c[n]$, where $n = 0, 1, \ldots, N-1$. The sampling interval is defined as $\Delta t = 1/f_s$, where f_s denotes the sampling frequency, and the total number of samples is given by $N = T \cdot f_s$.

For local temporal analysis and extraction of time-frequency features, each channel signal is segmented into overlapping windows. Each window B_k contains L consecutive samples and is defined as follows:

$$B_k = [x_c[k], x_c[k+1], \ldots, x_c[k+L-1]], \tag{1}$$

where k is the starting index of each window. The window shift is controlled by a step size S, such that

$$k = mS, \qquad m = 0, 1, 2, \ldots. \tag{2}$$

When $S < L$, consecutive windows overlap, which allows capturing fine-grained temporal variations in EEG signals.

This segmentation strategy improves the sensitivity of the model in detecting transient and nonstationary patterns associated with preictal and ictal brain activity, and provides a structured representation of EEG data for subsequent feature extraction and optimization stages. As illustrated in Figure 1, the EEG signal segmentation procedure is performed using overlapping sliding windows as described above.

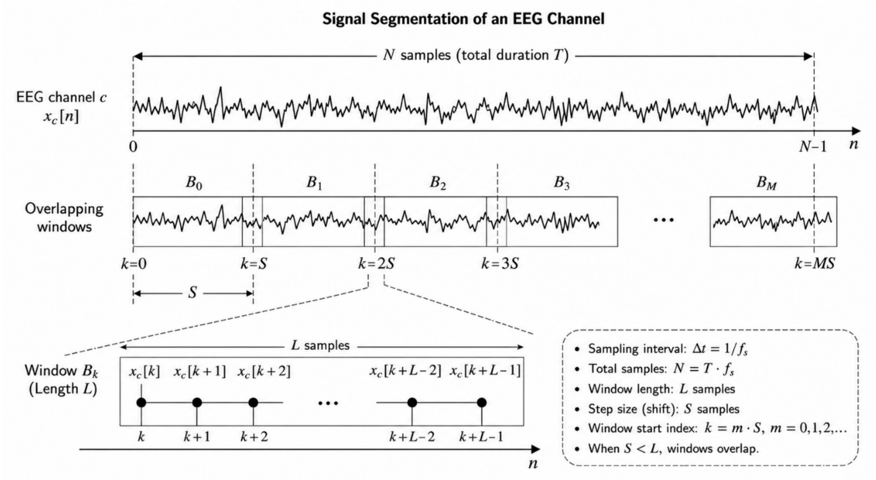

Figure 1: Signal segmentation of an EEG channel using overlapping sliding windows.

2.3 Correlation coefficient

The correlation coefficient is a statistical measure used to quantify the degree of linear dependence between two signals $x(t)$ and $y(t)$. In the context of EEG signal analysis, it is commonly used to evaluate the similarity between different brain signal segments or channels.

The correlation coefficient is defined as

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}, \tag{3}$$

where

- x_i and y_i represent the individual samples of the two signals,
- $\bar{x}$ and $\bar{y}$ denote the mean values of x and y, respectively,
- n is the total number of samples in each segment.

The correlation coefficient takes values in the range $[-1, 1]$, where

- $r = 1$ indicates a perfect positive linear relationship,
- $r = -1$ indicates a perfect negative linear relationship,
- $r = 0$ indicates no linear correlation between the signals [11, 4].

In this study, the correlation coefficient is used to quantify the similarity between EEG segments and seizure-related patterns, which plays a key role in constructing the transition probability in the proposed probabilistic framework.

2.4 Bayes' theorem

Bayes' theorem provides a probabilistic framework for updating prior knowledge about a hypothesis H based on observed data D. It plays a fundamental role in uncertainty modeling and inference in complex systems such as EEG signal analysis.

Mathematically, the posterior probability is defined as

$$P(H|D) = \frac{P(H)P(D|H)}{P(D)}, \tag{4}$$

where

- $P(H)$ is the prior probability representing the initial belief about hypothesis H,
- $P(D|H)$ is the likelihood, representing the probability of observing data D given H,
- $P(H|D)$ is the posterior probability, which updates the belief about H after observing data D,
- $P(D)$ is the marginal likelihood (evidence), acting as a normalization term.

In the context of EEG signal analysis, H represents the occurrence of seizure-related activity within a specific channel or time block, while D corresponds to the observed EEG segment features. Therefore, Bayes' theorem enables the integration of prior temporal knowledge and observed signal characteristics to estimate the probability of seizure occurrence in each EEG segment [18].

2.5 Evolutionary algorithms

Evolutionary algorithms are a class of population-based stochastic optimization methods inspired by the principles of natural evolution, including selection, mutation, and recombination. These methods are widely used for solving complex optimization problems where the search space is large, nonlinear, and not analytically tractable.

In evolutionary optimization, a population of candidate solutions is first initialized randomly within the search space. Each candidate solution $x \in X$ is evaluated using a predefined fitness function $f(x)$, which measures its quality with respect to the optimization objective.

Based on fitness values, individuals with higher performance are selected to generate the next generation. New solutions are created through genetic operators such as mutation and crossover. Mutation is typically defined as

$$\hat{x} = x + \epsilon, \tag{5}$$

where ϵ is a small random perturbation vector that introduces diversity into the population. Crossover combines information from two parent solutions x_1 and x_2 to generate a new offspring solution:

$$x_{\text{new}} = \alpha x_1 + (1 - \alpha)x_2, \qquad 0 < \alpha < 1. \tag{6}$$

This iterative process continues until a stopping criterion is satisfied, such as reaching a maximum number of generations or convergence to a stable solution [7].

In the context of this study, each candidate solution represents a selection of EEG channel and time block, and the fitness function is defined based on the posterior probability of seizure occurrence. Therefore, evolutionary algorithms are used to efficiently explore the high-dimensional search space of EEG segments and identify the most informative preictal patterns.

2.6 Common types of evolutionary algorithms

Among the most widely used evolutionary and swarm intelligence algorithms in optimization problems are the following methods:

2.6.1 GA

GA is inspired by natural selection and genetic evolution processes. It employs selection, crossover, and mutation operators to evolve a population of candidate solutions over successive generations [17, 1].

The basic procedure of GA can be summarized as follows:

- **Initial population generation:** A set of random candidate solutions (chromosomes) is created.
- **Fitness evaluation:** The objective function is computed for each chromosome to determine its fitness.
- **Selection:** Chromosomes with higher fitness are selected for reproduction.
- **Crossover:** Two parent chromosomes are combined to generate offspring.
- **Mutation:** Random modifications are applied to maintain genetic diversity.
- **Iteration:** The process is repeated until a stopping criterion such as convergence or a maximum number of generations is reached [21].

2.6.2 ES

ESs are optimization algorithms based on selection and mutation, with a strong emphasis on self-adaptation of strategy parameters [10].

The main steps of ES are

- **Initialization:** A population of individuals is randomly generated along with strategy parameters (e.g., mutation variance).
- **Evaluation:** Each individual is evaluated using a fitness function.
- **Selection of parents:** The best individuals are selected as parents.
- **Mutation:** Offspring are generated by adding stochastic perturbations (commonly Gaussian noise) to parents. No crossover is typically used in basic ES.
- **Evaluation of offspring:** The new solutions are evaluated.

- **Replacement:** A new population is formed using either:
  - $(\mu + \lambda)$-ES: selection from both parents and offspring.
  - (μ, λ)-ES: selection only from offspring.
- **Termination:** The process continues until convergence or a maximum number of iterations is reached [10].

2.6.3 EP

EP focuses on optimizing the behavior and structure of solutions, particularly in machine learning and system modeling tasks [11].

Its main procedure includes

- **Initialization:** Random generation of an initial population.
- **Fitness evaluation:** Each individual is evaluated using a fitness function.
- **Mutation:** Only mutation is applied; crossover is not used.
- **Evaluation of offspring:** Newly generated individuals are evaluated.
- **Selection:** The best individuals from parents and offspring are selected, often using tournament selection.
- **Iteration:** The process continues until a stopping criterion is satisfied [11, 6].

2.7 Swarm intelligence based methods

In addition to evolutionary algorithms, swarm intelligence methods are inspired by the collective behavior of biological agents and have been widely used in optimization problems.

2.7.1 ACO

ACO is inspired by the foraging behavior of ants and their use of pheromone trails to find shortest paths [3].

The key steps of ACO include

- **Initialization:** Setting initial pheromone levels.

- **Solution construction:** Each ant constructs a path probabilistically based on pheromone and heuristic information.

  The transition probability is defined as follows:

$$P_{ij}^{(k)} = \frac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l\in allowed}[\tau_{il}]^{\alpha}[\eta_{il}]^{\beta}}. \tag{7}$$

- **Pheromone update:** After all ants complete their paths, pheromone values are updated based on evaporation and deposited pheromone.

- **Termination:** The algorithm stops when a maximum number of iterations is reached or convergence occurs [3, 6].

2.7.2 Particle swarm optimization (PSO)

Particle swarm optimization (PSO) is inspired by the social behavior of birds and fish during food searching [13].

The main steps of PSO include

- **Initialization:** Random initialization of particle positions and velocities.

- **Velocity update:**

$$v_i(t+1) = wv_i(t) + c_1r_1(p_i - x_i(t)) + c_2r_2(g - x_i(t)). \tag{8}$$

- **Position update:**

$$x_i(t+1) = x_i(t) + v_i(t+1). \tag{9}$$

- **Evaluation and update:** Personal and global best solutions are updated.

- **Iteration:** The process continues until convergence or a stopping condition is met [13, 11].

2.7.3 ABC

ABC algorithm is inspired by the foraging behavior of honey bees [10, 12].

Its main phases are

- **Initialization:** Random generation of food sources.
- **Employed bees phase:** Local search around current solutions.
- **Onlooker bees phase:** Probabilistic selection of high-quality solutions.
- **Scout bees phase:** Replacement of abandoned solutions with new random ones.
- **Best solution update:** The best food source is preserved.
- **Iteration:** The process is repeated until stopping criteria are satisfied [10, 12].

2.7.4 HS Algorithm

HS is inspired by the process of musical improvisation [9].

The main steps of HS include

- **Initialization:** Construction of the harmony memory (HM).
- **Parameter setting:** Determination of HMCR and PAR parameters.
- **Improvisation process:**
  - Memory consideration.
  - Pitch adjustment.
  - Random selection.
- **Update of HM:** A new harmony is stored if a better solution is found.
- **Iteration:** The process continues until a stopping condition is reached [9].

These algorithms are widely used in engineering, medical data analysis, artificial intelligence, and optimization problems due to their strong exploration ability, robustness against local minima, and independence from gradient information.

3 Proposed method

The proposed framework formulates epileptic seizure prediction as a probabilistic multi-objective optimization problem over segmented EEG event space. The main objective is to identify EEG signal blocks that simultaneously:

- maximize the posterior probability of seizure-related information,
- minimize the temporal distance to seizure onset.

To achieve this, a hybrid optimization architecture is designed by integrating Bayesian inference with evolutionary optimization algorithms. In this framework, Bayesian modeling is responsible for estimating probabilistic relationships within EEG event space, while evolutionary algorithms perform global search over candidate EEG segments.

3.1 Problem formulation and EEG event space modeling

Let the EEG signal be segmented into a set of temporal blocks:

$$B = \{B_1, B_2, \ldots, B_N\}. \tag{10}$$

Each block B_i is associated with

- a channel index,
- a start time,
- a fixed window length.

The goal is to select an optimal block B^* such that it carries maximum seizure-related information while appearing as early as possible in the preictal phase.

The EEG event space is divided into two temporal regimes:

- Normal (preictal) region,
- Seizure (ictal) region.

These regions are defined by two key time instants t_0 and t_1:

$$B_{\text{pre}} = \{B_i \in B \mid t_i \in [0, t_0]\}, \tag{11}$$

$$B_{\text{ictal}} = \{B_i \in B \mid t_i \in [t_0, t_1]\}. \tag{12}$$

Each EEG block is treated as a realization of a stochastic process within this event space.

3.2 Bayesian probabilistic modeling

For each EEG block B_i, three probabilistic quantities are defined:

3.2.1 Prior Probability

The prior probability represents the temporal proximity of an EEG block to seizure onset:

$$p_{\text{prior}}(B_i) = \frac{t_i}{t_s}, \tag{13}$$

where

- t_i denotes the occurrence time of block B_i,
- t_s denotes the seizure onset time.

3.2.2 Transition probability

The transition probability is defined based on the similarity between the EEG block B_i and seizure segments:

$$p(B_i|S) = \max_j \left| \text{corr}(x_i, x_j^{(S)}) \right|, \tag{14}$$

where

- x_i represents the signal corresponding to block B_i,

- $x_j^{(S)}$ represents the j-th seizure segment,
- $\mathrm{corr}(\cdot)$ denotes the correlation coefficient.

The process of computing the transition probability is illustrated in Figure 2.

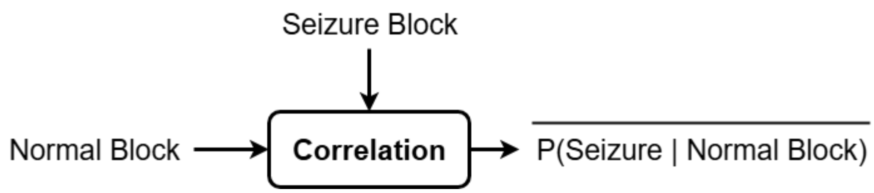

Figure 2: Block diagram for obtaining transition probability.

3.2.3 Posterior probability

Using Bayes' theorem, the posterior probability is defined as follows:

$$p(S|B_i) = \frac{p(S)p(B_i|S)}{p(B_i)}, \tag{15}$$

where

- S denotes the seizure event,
- B_i denotes the EEG block,
- $p(S)$ represents the prior probability of seizure occurrence,
- $p(B_i|S)$ represents the transition probability,
- $p(B_i)$ represents the evidence (normalization term).

To simplify the notation, we define

$$p_{\text{posterior}}(B_i) = p(S|B_i). \tag{16}$$

Since $p(B_i)$ is independent of the class label S, it does not affect the optimization process. Therefore, maximizing the posterior probability reduces to

$$\max_{B_i \in B} p(S|B_i). \tag{17}$$

The posterior maximization can be rewritten as

$$\max_{B_i \in B} p(S)p(B_i|S). \tag{18}$$

In this study, the conditional probability $p(B_i|S)$ is modeled based on the similarity between the EEG block and seizure-related patterns. This similarity is quantified using the correlation coefficient between the candidate block and seizure segments. Hence

$$p(B_i|S) = \max_j \left| \operatorname{corr}(x_i, x_j^{(S)}) \right|. \tag{19}$$

Substituting (19) into (18), the posterior score becomes

$$\max_{B_i \in B} \left\{ p(S) \max_j \left| \operatorname{corr}(x_i, x_j^{(S)}) \right| \right\}. \tag{20}$$

This formulation enables the assignment of a posterior probability to each EEG block, reflecting its relevance to seizure occurrence. The overall process for computing the posterior probability is illustrated in Figure 3.

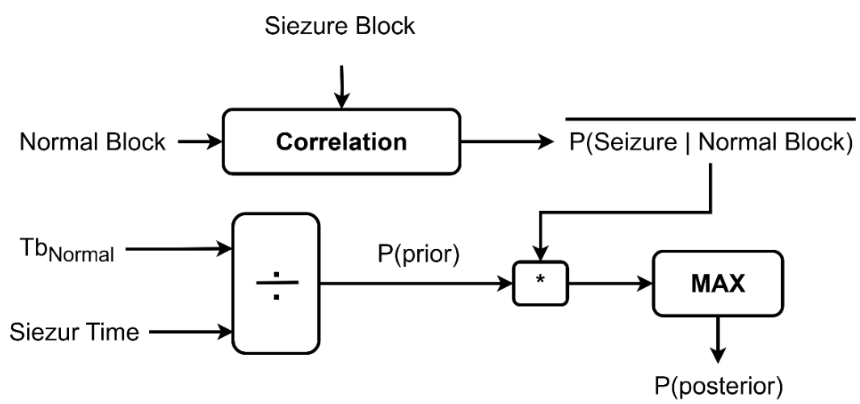

Figure 3: Block diagram for obtaining posterior probability.

3.3 Cost function formulation

The objective of this study is to identify EEG signal blocks that maximize the posterior probability of seizure occurrence while minimizing the time of occurrence. To achieve this goal, a cost function is defined as follows:

$$J(B_i) = \frac{p(B_i)}{p(S|B_i)}, \tag{21}$$

where $p(B_i)$ represents the prior probability associated with block B_i, modeled as the normalized time index of the block. Accordingly, earlier blocks correspond to smaller prior values, which is desirable for early prediction.

From the posterior probability formulation derived in the previous section:

$$p(S|B_i) = p(S) \max_j \left| \operatorname{corr}(x_i, x_j^{(S)}) \right|, \tag{22}$$

where x_i denotes the EEG signal segment corresponding to block B_i, and $x_j^{(S)}$ represents the j-th seizure segment. The operator $\operatorname{corr}(\cdot, \cdot)$ measures the similarity between the candidate block and seizure patterns.

This formulation implies that the posterior probability of each block is determined by its strongest similarity with seizure segments. Therefore, blocks with higher correlation values are considered more informative with respect to seizure occurrence.

Substituting (22) into (21), the cost function becomes

$$J(B_i) = \frac{p(B_i)}{p(S) \max_j \left| \operatorname{corr}(x_i, x_j^{(S)}) \right|}. \tag{23}$$

Since $p(S)$ is constant for all blocks, it does not influence the optimization process and can be removed. Therefore, the final optimization objective is simplified as

$$J(B_i) = \frac{p(B_i)}{\max_j \left| \operatorname{corr}(x_i, x_j^{(S)}) \right|}. \tag{24}$$

The final optimization problem is formulated as

$$\min_{B_i \in B} J(B_i). \tag{25}$$

The proposed cost function ensures a balanced trade-off between:

- **Temporal optimality:** selecting earlier EEG blocks through $p(B_i)$,
- **Seizure relevance:** maximizing similarity with seizure segments through correlation.

As a result, the optimization framework identifies EEG blocks that contain strong seizure-related information while appearing in the early preictal phase, which is critical for effective seizure prediction systems.

3.4 Evolutionary optimization framework

The primary objective of this stage is to identify EEG signal blocks that contain maximum seizure-related information while occurring as early as possible in time. Therefore, the optimization problem is formulated as a trade-off between maximizing seizure-related probability and minimizing the temporal distance to seizure onset.

In practical seizure prediction systems, early detection is more critical than late-stage identification. Although seizure probability generally increases as the onset approaches, informative patterns often appear as early local increases before reaching the global maximum. Hence, the goal is to detect the earliest block in which a significant rise in seizure likelihood is observed.

More formally, the target is to identify EEG blocks where

- the seizure probability exhibits an increasing trend,
- the probability is significantly higher compared to preceding blocks, even if it has not reached its maximum value.

This concept is directly incorporated into the cost function, ensuring that selected blocks simultaneously:

- contain high seizure-related information,
- occur at earlier time instances.

To solve this optimization problem, evolutionary algorithms are employed. These algorithms operate on a population of candidate solutions, where each individual represents an EEG block and is evaluated using a fitness function derived from the proposed cost formulation.

In this framework, each EEG block is encoded as an individual in the population and described by

- EEG channel index,
- normalized time position within the range $[0, 1]$.

The fitness of each individual is computed using the defined cost function, where lower values indicate more optimal solutions. The evolutionary process iteratively improves the population through selection and variation operators, guiding the search toward blocks that minimize the cost function.

In the following subsections, different evolutionary algorithms are applied to optimize the proposed objective function and determine the optimal prediction time.

3.5 Evolutionary algorithms for EEG optimization

3.5.1 GA

In this section, a GA is employed to identify the optimal EEG signal block that maximizes seizure-related information while ensuring early temporal occurrence. The objective is to select a block in which the probability of seizure-related activity is relatively high compared to other blocks, while still belonging to the pre-seizure phase, enabling early prediction.

The GA performs a global search by iteratively evolving a population of candidate solutions toward minimizing the defined cost function. Each candidate solution represents an EEG block characterized by both spatial (channel) and temporal (time) information.

3.5.2 Index sets definition

To ensure consistency across all optimization algorithms, the following index sets are defined.

Let the solution space be defined over

Channel set:

$$C = \{1, 2, \ldots, 29\}, \tag{26}$$

Time block set:

$$B = \{0, 1, \ldots, B - 1\}. \tag{27}$$

Each candidate solution is defined as

$$x = (c, b), \qquad c \in C,\ b \in B, \tag{28}$$

where c denotes the EEG channel index and b represents the time block index.

3.5.3 Chromosome encoding

Each individual (chromosome) in the population is encoded as a 16-bit binary string representing the properties of an EEG block:

- The first 6 bits represent the EEG channel index (1 to 29).
- The next 10 bits represent the normalized time of the block within the interval $[0.70, 0.80]$.

This encoding allows simultaneous optimization over both spatial (channel selection) and temporal (block position) dimensions.

3.5.4 Algorithm configuration

The GA is configured as follows:

- Population size: 50.
- Number of generations: 100.
- Crossover rate: 0.8.
- Mutation rate: 0.05.

3.5.5 Genetic operators

The evolutionary process is guided by the following operators:

- **Selection:** Roulette wheel selection is used to probabilistically favor individuals with lower cost values (higher fitness).
- **Crossover:** Single-point crossover is applied to generate offspring by combining parent chromosomes.
- **Mutation:** A single random bit mutation is used to introduce diversity and prevent premature convergence.

3.5.6 Optimization objective

The GA minimizes the cost function defined in Section 3.3, thereby selecting EEG blocks that

- maximize the posterior probability of seizure occurrence,
- correspond to earlier time instances in the signal.

This approach enables the identification of informative pre-seizure blocks, which are critical for early warning systems.

3.5.7 Optimization results for different block sizes

This section presents the results obtained from applying the GA to EEG signal blocks with different window sizes, specifically 32, 64, and 128 samples. The reported values correspond to the optimal solution identified by the algorithm, which maximizes the seizure-related information while minimizing the cost function.

The results are summarized in Table 1.

Table 1: GA optimization results for various block sizes in seizure prediction

Block Size (Samples)	Maximum Seizure Probability	Selected Channel	Normalized Block Time
32	0.764	14	0.712
64	0.755	7	0.769
128	0.751	11	0.793

The results indicate that the GA successfully identifies EEG blocks that contain early indicators of seizure onset. Specifically, the selected blocks satisfy two important conditions:

- They capture the initial statistical patterns associated with seizure activity.
- They correspond to earlier time instances where seizure probability begins to increase but has not yet reached its global maximum.

Although the maximum seizure probability varies slightly across different block sizes, the algorithm consistently identifies blocks located in the pre-seizure phase, which are critical for early warning systems.

These findings demonstrate that the proposed GA-based optimization framework is effective in extracting informative temporal segments from EEG signals for seizure prediction.

3.5.8 ESs

To achieve optimal selection of EEG channels and corresponding temporal segments associated with seizure occurrence, an ES algorithm is employed. ES is a population-based stochastic optimization method inspired by biological evolution, particularly mutation and selection mechanisms, and is well-suited for continuous and discrete search spaces.

3.5.9 Representation and encoding

In this framework, each candidate solution (individual) is encoded as a 16-bit binary chromosome that simultaneously represents both spatial (channel) and temporal (time block) information.

Specifically,

- **First 6 bits:** represent the EEG channel index, corresponding to integer values in the range $[0, 63]$.

  Although the encoding allows up to 64 values, only 29 channels are physically available in the dataset; therefore, values outside the valid channel range are discarded during evaluation.

- **Next 10 bits:** represent the time block index.

The EEG signal is partitioned into a predefined number of temporal segments (e.g., 1000 blocks), allowing indices in the range $[0, 999]$.

By decoding the binary representation into decimal values, each chromosome uniquely defines:

- the selected EEG channel,
- the corresponding time block.

3.5.10 Role in optimization

This encoding enables the ES algorithm to perform a joint optimization over spatial and temporal dimensions, allowing simultaneous selection of

- the most informative EEG channels,
- the most relevant time intervals for seizure prediction.

The fitness of each individual is evaluated using the previously defined cost function, and the ES algorithm iteratively improves the population through mutation-based exploration and selection of high-quality candidates.

3.5.11 ES algorithm settings

The parameter configuration of the ES algorithm is presented in Table 2.

Table 2: ES algorithm parameters for EEG channel and time block selection

Parameter	Value
Initial Population Size	50
Number of Generations	50
Mutation Rate	0.2
Recombination Rate	0.6
Mutation Type	Single-point Binary
Stopping Criterion	Fixed number of generations or no improvement

3.5.12 Preliminary results

Table 3 presents a sample of the top-performing chromosomes obtained from the ES algorithm. These chromosomes correspond to solutions with higher fitness values, indicating a greater amount of seizure-related information within the selected EEG blocks.

Table 3: Sample chromosomes evaluated by ES algorithm

No.	Chromosome	Channel Address	Time Block Address	Fitness Value
1	0100111110100110	19	598	0.842
2	0001100111011101	6	477	0.835
3	0011000010100100	12	324	0.828
4	0100001011010011	16	723	0.822
5	0001011101110010	5	466	0.818

The fitness value represents the amount of seizure-related information contained in the EEG block selected by each chromosome. Higher fitness values indicate stronger relevance to seizure onset patterns and improved predictive capability.

These preliminary results demonstrate that the ES algorithm is capable of effectively exploring the search space and identifying informative EEG channel-time block combinations.

3.5.13 Initial Conclusion

As can be seen from the table, the ES algorithm has managed to find combinations of channels and time blocks that contain more information about the occurrence of seizures compared to other signal points. These preliminary results could serve as a useful guide for subsequent phases of analysis or the training of reinforcement learning models.

3.5.14 EP

In this section, the EP algorithm is employed to identify optimal EEG channel-time block combinations containing significant seizure-related information. EP is a population-based optimization method inspired by mutation-driven evolution. Unlike other evolutionary algorithms, EP primarily relies on mutation and selection, without using crossover operators, making it well-suited for problems requiring fine local search refinement.

3.5.15 Chromosome representation in EP

Each individual (chromosome) is encoded as a 16-bit binary string representing both spatial and temporal information:

- **First 6 bits:** represent the EEG channel index in the range $[0, 63]$. Only valid channels in the set $C = \{1, \ldots, 29\}$ are considered.
- **Next 10 bits:** represent the time block index.

The EEG signal is divided into a fixed number of temporal segments (e.g., 1000 blocks), resulting in indices in the range $[0, 999]$.

Each chromosome is decoded into a pair:

$$(channel\ index,\ time\ block\ index), \tag{29}$$

which uniquely defines an EEG segment.

3.5.16 EP algorithm settings

The parameter configuration of the EP algorithm is presented in Table 4.

Table 4: EP algorithm parameters for EEG channel and time block selection

Parameter	Value
Initial Population Size	50
Number of Generations	60
Mutation Rate	0.25
Selection Method	Tournament Selection
Mutation Type	Gaussian Perturbation (on decoded integer values)
Stopping Criterion	No improvement in best fitness for 10 generations or maximum generations

As shown in Table 4, a population size of 50 and a mutation rate of 0.25 were employed to effectively explore the search space of EEG channel and temporal segment combinations.

3.5.17 Preliminary results

The preliminary results obtained from the EP algorithm are presented in Table 5. The table lists the top-ranked chromosomes together with their corresponding EEG channel addresses, time block indices, and fitness values. Higher fitness values indicate a greater amount of seizure-related information contained in the selected EEG segments.

Table 5: Sample chromosomes evaluated by EP algorithm

No.	Chromosome	Channel Address	Time Block Address	Fitness Value
1	0010011101100110	9	614	0.849
2	0001110011010101	7	437	0.841
3	0010101001011010	10	698	0.834
4	0001101100111111	6	831	0.829
5	0011010010001000	13	528	0.823

The fitness value reflects the amount of seizure-relevant information in the selected time block and channel combination.

3.5.18 Initial conclusion

The results demonstrate that the EP algorithm is capable of effectively identifying EEG channel-time block combinations that contain significant pre-seizure information. Compared to random selection, the algorithm consistently converges toward solutions with higher information content.

These findings confirm the suitability of EP for this optimization problem and suggest that the extracted blocks can serve as informative candidates for early seizure prediction.

3.5.19 ACO

In this section, the ACO algorithm is employed to identify optimal combinations of EEG channels and time blocks that contain the highest amount of seizure-related information. ACO is a population-based metaheuristic inspired by the foraging behavior of ants in nature, especially their ability to find the shortest paths to food sources using pheromone trails.

In the context of EEG signal analysis, this algorithm can effectively discover optimal paths (channel-time combinations) associated with early seizure indicators.

3.5.20 Problem formulation

In this problem, each path traveled by an ant consists of selecting an EEG channel and a time block within the signal. The objective is to find combinations with the highest seizure likelihood or preictal information.

The ACO search space is defined as follows:

- **Nodes:**
  - Channel nodes: 29 EEG channels.
  - Time nodes: EEG signal divided into 1000 temporal blocks.
- **Ant path:** Selection of one channel node and one time block node.
- **Heuristic value:** Reflects the seizure-related information content of a given channel–time combination.

The transition probability for selecting a candidate node is defined as

$$P_{ij}^{(k)} = \frac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l\in allowed}[\tau_{il}]^{\alpha}[\eta_{il}]^{\beta}}, \tag{30}$$

where τ_{ij} represents the pheromone level, η_{ij} represents the heuristic information, and α and β control the relative importance of pheromone and heuristic information, respectively.

3.5.21 ACO algorithm parameters

To effectively guide the search process in the ACO algorithm, the parameter configuration listed in Table 6 was used. These settings were selected to ensure a proper trade-off between convergence speed and solution diversity.

Table 6: ACO algorithm parameters for EEG channel and time block selection

Parameter	Value
Number of Ants	30
Number of Iterations	50
Pheromone Evaporation Rate (ρ)	0.1
Pheromone Importance (α)	1
Heuristic Importance (β)	2
Exploration Rate (q_0)	0.8
Stopping Criterion	Maximum iterations or no improvement in best path

3.5.22 Preliminary results

As shown in Table 7, the ACO algorithm successfully identified several optimal EEG channel-time block combinations with high fitness values, indicating strong seizure-related information content in the selected segments.

Table 7: Optimal paths identified by ACO

No.	Selected Channel	Selected Time Block	Fitness Value
1	14	721	0.861
2	9	658	0.853
3	5	493	0.847
4	17	780	0.839
5	12	542	0.834

3.5.23 Initial conclusion

The results demonstrate that the ACO algorithm successfully identifies combinations of EEG channels and time blocks containing high levels of pre-seizure information. These early indicators are valuable for timely seizure prediction and can also serve as informative inputs for training reinforcement learning models or other downstream decision-making modules.

3.5.24 ABC Algorithm

In this section, the ABC algorithm is applied to identify optimal combinations of EEG channels and time blocks that maximize seizure-related information content. ABC is a population-based metaheuristic inspired by the foraging behavior of honey bees and is characterized by a strong balance between exploration and exploitation, making it suitable for high-dimensional EEG optimization problems.

3.5.25 Problem representation in ABC

Each candidate solution represents an EEG channel-time block pair defined as

$$x = (c, b), \tag{31}$$

where c denotes the EEG channel index and b represents the time block index.

The quality of each solution is evaluated using the cost function defined in Section 3.3.

3.5.26 ABC algorithm parameters

To ensure effective exploration of the search space, the ABC algorithm was configured using the parameters listed in Table 8. These settings were selected to balance convergence speed and solution quality in EEG channel and time block optimization.

Table 8: ABC algorithm parameters for EEG channel and time block selection

Parameter	Value
Number of Employed Bees	25
Number of Onlooker Bees	25
Number of Iterations	50
Mutation Rate	0.25
Abandonment Limit for Scout Phase	5
Stopping Criterion	Maximum iterations or no improvement

3.5.27 ABC algorithm phases

The ABC algorithm consists of the following main steps:

1. Initialization of candidate solutions.
2. Employed bee phase for local search around existing solutions.
3. Onlooker bee phase based on fitness probability.

4. Scout bee phase for abandoning poor solutions and generating new candidates.

5. Memorization of the best solution.

3.5.28 Preliminary results

As shown in Table 9, the ABC algorithm successfully identified several high-quality solutions corresponding to EEG channel-time block combinations with high fitness values, indicating strong seizure-related information content in the selected segments.

Table 9: ABC algorithm results for EEG channel and time block selection

No.	Selected Channel	Selected Time Block	Fitness Value
1	13	645	0.865
2	8	527	0.858
3	16	734	0.850
4	4	498	0.846
5	21	801	0.839

3.5.29 Initial Conclusion

The results demonstrate that the ABC algorithm effectively identifies EEG channel-time block combinations containing high seizure-related information. These blocks correspond to informative preictal patterns and can be utilized in downstream predictive models, such as reinforcement learning-based seizure prediction systems.

3.5.30 HS algorithm

To optimally select EEG channels and time blocks related to seizure prediction, the HS algorithm was employed. Inspired by the improvisation process in musical harmony, HS is a metaheuristic algorithm suitable for both discrete and continuous optimization problems.

In this application, each harmony vector was encoded as a 16-bit chromosome. The first 6 bits represented the EEG channel address, and the remaining 10 bits encoded the time block index.

3.5.31 HS algorithm parameters

The parameter settings used for the HS algorithm are summarized in Table 10.

Table 10: HS algorithm parameters for EEG channel and time block selection

Parameter	Value
Harmony Memory Size (HMS)	20
Harmony Memory Considering Rate (HMCR)	0.9
Pitch Adjusting Rate (PAR)	0.3
Number of Iterations	100

In each iteration, the algorithm generates new harmonies by considering memory-based components and random adjustments. The generated solutions are evaluated using the defined cost function, and the combinations with lower cost values are retained in the HM.

3.5.32 Initial results

The preliminary results obtained from the HS algorithm are presented in Table 11. Each row corresponds to a candidate solution (chromosome) representing a combination of EEG channel and time block selected during the optimization process.

Table 11: HS algorithm results for EEG channel and time block selection

Iteration	Selected Chromosome	Channel Address	Time Block (sec)	Cost Function Value
1	0101010001010110	21	86	0.034
2	1000110010100011	35	163	0.028
3	0011101100011010	14	106	0.031
4	1110000100110101	56	213	0.027
5	0110111001011100	27	93	0.029
6	0010101011001111	10	187	0.033
7	1001010110001010	37	120	0.030
8	0101110001110101	23	143	0.032
9	0001101010110001	12	98	0.031
10	1010100100101110	42	175	0.026

The table presents preliminary results of the HS algorithm for the optimal selection of EEG channels and time blocks. Each row corresponds to a candidate solution representing a combination of a specific channel and a particular time segment in the EEG signal.

The cost function value serves as the quality metric for these combinations, where lower values indicate better selections with higher seizure-related information content. The results demonstrate that the HS algorithm successfully found solutions with low-cost values ranging from 0.026 to 0.034, reflecting its effectiveness in exploring the solution space efficiently.

Moreover, the selected channels are distributed across a wide range of channel addresses, indicating comprehensive search behavior in the state space. Similarly, the diversity in chosen time blocks reflects the algorithm's capability to identify different pre-seizure intervals within the EEG signals.

Overall, these preliminary results provide a foundation for further detailed analysis, additional optimization, and training more accurate seizure prediction models.

4 Conclusion

In this study, six metaheuristic optimization algorithms (GA, EP, ES, ACO, ABC, and HS) were evaluated for EEG channel and time-block selection in the context of early seizure prediction. The results demonstrated that all methods are capable of identifying EEG segments with high seizure-related information content, although differences in optimization performance were observed. Among the evaluated methods, the ABC and ACO algorithms achieved the highest fitness values (0.865 and 0.861, respectively), indicating superior exploration capability and solution quality. The HS algorithm achieved the lowest cost value (0.026), reflecting efficient convergence in the defined search space. The EP and ES algorithms demonstrated stable performance with consistent mid-range fitness values, suggesting robustness against local optima and noise. Across all methods, EEG channels 13, 14, 9, and 12 were most frequently selected in optimal solutions, indicating their strong association with pre-seizure activity patterns. The comparative performance of all algorithms is summarized in Table 12.

Table 12: Final comparative table of algorithm performance

Algorithm	Best Fitness / Cost	Best Channel	Best Time Block	Most Frequent Channels
GA	0.764 (Fitness)	14	0.712	14, 7, 11
ES	0.842 (Fitness)	19	598	12, 6, 5
EP	0.849 (Fitness)	9	614	9, 10, 7
ACO	0.861 (Fitness)	14	721	14, 9, 12
ABC	0.865 (Fitness)	13	645	13, 8, 16
HS	0.026 (Cost $\downarrow$)	42	175	21, 35, 14

To further enhance seizure prediction performance, future work should focus on integrating these metaheuristic optimization methods with deep learning models and developing real-time adaptive systems for clinical applications.

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Conflicts of Interest

The authors declare no conflicts of interest.

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