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Research Article

Conformable power series method for solving Fisher–Kolmogorov–Petrovskii–Piskunov equation

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Abstract

This work presents the application of the conformable power series method to approximate the nonlinear time-fractional Fisher–Kolmogorov–Petrovskii–Piskunov equation, which models tumor growth and invasion. Using the conformable derivative, the method transforms the fractional equation into a recursive series expansion, yielding accurate semi-analytical approximations. Three numerical examples with different nonlinear powers are solved. Convergence and error analyses of the series solution are established. Results demonstrate excellent agreement with exact and existing numerical solutions, with small absolute errors in all test examples.

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Keywords: Fisher–KPP equation; Power series method; conformable derivative.

1 Introduction

Fractional calculus generalizes classical differentiation and integration to noninteger orders, enabling the mathematical description of systems with memory effects and nonlocal interactions. Since its conceptual origins in a 1695 correspondence between L'Hôpital and Leibniz, fractional calculus has evolved through contributions from Euler, Laplace, Riemann, Liouville, and others, becoming a vital tool in fields such as viscoelasticity, control theory, and transport in porous media [9, 21, 2]. This mathematical framework finds diverse applications in scientific disciplines, particularly for modeling complex systems with memory effects and nonlocal dynamics that cannot be adequately described by classical calculus. Notably, Lacroix became the first to formally mention a fractional derivative in his 1819 publication [34].

Partial differential equations play a central role in modeling dynamical systems, particularly those involving diffusion and reaction processes. Among these, the Fisher–Kolmogorov–Petrovskii–Piskunov (Fisher–KPP) equation, a reaction–diffusion model introduced independently in the 1930s by Fisher and by Kolmogorov, Petrovsky, and Piskunov, has become foundational in mathematical biology [31, 13, 19]. Originally derived from population genetics to model microbial population dynamics, it describes how spatial dispersal affects population densities over time [20]. Contemporary applications extend to analyzing micro-morphogenesis in modern microbiology [32, 37]. This study employs a conformable power series method (CFPSM) to approximate solutions for the nonlinear time-fractional Fisher–KPP equation [37].

Solving nonlinear fractional equations remains challenging, which has prompted the development of various analytical and numerical techniques. Several researchers have investigated the time-fractional Fisher–KPP equation. Angstmann and Henry [3] analyzed both the time-fractional Fitzhugh–Nagumo and Fisher–KPP equations, while Khaled [27] applied the homotopy perturbation method to approximate solutions for the time-fractional nonlinear Fisher–KPP equation. Other studies have examined related aspects: The authors of [10] investigated the time-asymptotic behavior of level sets in Fisher–KPP equations with fractional diffusion in periodic media; Cabré and Roquejoffre [11] studied front propagation in fractional diffusion Fisher–KPP equations; and Cabré and Roquejoffre [12] analyzed the effects of fractional diffusion in these models. Hamel and Roques [24] focused on fast propagation in KPP equations with slowly decaying initial conditions, and Hariharan [25] employed the homotopy analysis method to solve both classical and fractional KPP equations.

Berkovich [8] employed the factorization method to derive analytical invariant solutions of the Fisher–KPP equation. Griffiths and Schiesser [22] analyzed solutions to the Fisher–KPP equation using MATLAB® and Maple™software. Fasano, Herrero, and Rodrigo [18] examined both fast and slow invasion waves in acid-mediated tumor growth models.

Semi-analytical series expansion methods, including the fractional power series (FPS) method, have gained popularity for solving fractional differential equations. These methods provide systematically improvable approximations without spatial discretization. Since nonlinear equations present significant analytical challenges, researchers have developed numerous numerical solution techniques. The FPS method, a well-established semi-analytical technique covered in computational analysis literature, has become fundamental for solving both linear and nonlinear time-fractional differential equations. This approach is particularly valuable in computational mathematics for efficiently deriving functional approximations [4]. Aychluh and Ayalew [6] applied the FPS method to the time-fractional Kuramoto-Sivashinsky equation, while El-Ajou et al. [16] implemented FPS as a generalization of classical power series. Habenom, Suthar, and Mulualem [23] solved the fractional Fokker-Planck equation using the FPS method, while Cui and Hu [14] successfully applied the same approach to fractional differential equations. The current method offers several advantages for solving differential equations, particularly the fractional Fisher–KPP equation. This technique provides semi-analytical approximations through convergent series representations, achieving high accuracy when rapid convergence occurs. The approach is straightforward to implement without requiring complex numerical algorithms or specialized software, making it accessible for various applications. This study implements a conformable fractional power series method (CFPSM) to solve a nonlinear fractional Fisher–KPP model.

Khalil et al. [28] introduced the conformable derivative, a definition that preserves fundamental calculus rules such as the product, quotient, and chain rules. This formulation has attracted substantial attention from researchers, with multiple studies establishing its fundamental characteristics [26, 1]. Silva, Moreira, and Moret [35] investigated the conformable Laplace transform for fractional differential equations. Zhao and Luo [39] examined the physical interpretation of the general conformable operator, while Eroglu, Avci, and Ozdemir [17] applied conformable derivatives to optimal control problems in heat conduction. Özkan and Kurt [33] derived exact solutions for conformable integro-differential equations using the conformable Laplace transform method. The clear properties of conformable derivative simplify the extension of classical series techniques to fractional frameworks, making the CFPSM computationally efficient. While CFPSM has been applied to several fractional models, to our knowledge, it has not yet been employed for the nonlinear time-fractional Fisher–KPP equation.

The proposed noninteger order power series method exhibits a significant limitation in its dependence on initial function selection. The technique's effectiveness hinges on appropriate initial conditions, as poor initial assumptions may lead to slow convergence or inaccurate approximations. Furthermore, the method faces challenges in multi-dimensional applications where managing series convergence becomes increasingly difficult. While the approach provides solutions within a stable convergence radius, highly nonlinear systems often require numerous series terms for adequate approximation, increasing computational demands compared to finite difference, finite element, or finite volume methods. Truncation errors present another limitation, as series termination introduces approximation errors that become particularly significant when computational constraints prevent inclusion of sufficient terms. The fractional nature of the equations introduces additional complexities, especially in computing coefficients for higher-order nonlinear time fractional derivatives. Despite these limitations, the power series method offers distinct advantages over traditional analytical and numerical approaches for fractional differential equations. It maintains applicability across linear and nonlinear problems while achieving high accuracy through series expansion. The method avoids discretization-related numerical instabilities and round-off errors by working with continuous expressions, making it a valuable approach for solving fractional differential equations when appropriate initial conditions are available and computational resources permit sufficient term inclusion.

This research is motivated by the need for a direct and easy method to solve fractional Fisher–KPP equations. The CFPS approach offers an accurate solution due to its simplicity and computational efficiency. To the best of our knowledge, this represents the first application of the CFPSM to solve the time-fractional Fisher–KPP equation. Because the conformable derivative preserves the classical calculus rules (chain rule, product rule, and quotient rule), integer-order power series techniques can be easily extended to fractional domains, offering mathematical simplicity without compromising analytical rigour. CFPSM produces semi-analytical solutions where accuracy can be systematically improved by increasing the series terms.

The paper is organized as follows: Section 2 outlines essential definitions and theorems of conformable calculus and the FPS method. Section 3 details the solution methodology for the fractional Fisher–KPP equation, including numerical examples, graphical and tabular results, and discussion. Section 4 presents concluding remarks.

2 Fundamental concepts

This section presents the fundamental principles of conformable calculus and the essential concepts of the FPS method formulated within the conformable framework.

Definition 1 ([36]). Let $\upsilon(x,\ t) : \mathbb{R} \times (0, \infty) \to \mathbb{R}$ and let $t > 0$. The conformable time fractional partial derivative of order $\kappa \in (0, 1]$ is defined as follows:

$$T_t^{\kappa} \upsilon(x,\ t) = \lim_{\varepsilon \to 0} \frac{\upsilon(x,\ t + \varepsilon t^{1-\kappa}) - \upsilon(x,\ t)}{\varepsilon}, \qquad t > 0. \tag{1}$$

The above definition has several important properties. For $\kappa \in (0, 1]$, if $\upsilon(x,\ t)$ and $\vartheta(x,\ t)$ are κ-differentiable at a point $t > 0$, then [36]

1. **Linearity:** $T_t^{\kappa}(a\upsilon(x,\ t) \pm b\vartheta(x,\ t)) = aT_t^{\kappa}\upsilon(x,\ t) \pm bT_t^{\kappa}\vartheta(x,\ t)$, for all $a, b \in \mathbb{R}$.
2. **Power rule:** $T_t^{\kappa}(t^{\sigma}) = \sigma t^{\sigma-\kappa}$ for all $\sigma \in \mathbb{R}$.
3. **Constant rule:** $T_t^{\kappa}(\eta) = 0$, for all constant functions $\upsilon(x,\ t) = \eta$.
4. **Quotient rule:** $T_t^{\kappa}(\upsilon/\vartheta) = \dfrac{\vartheta T_t^{\kappa}\upsilon - \upsilon T_t^{\kappa}\vartheta}{\vartheta^2}$.
5. **Product rule:** $T_t^{\kappa}(\upsilon\vartheta) = \upsilon T_t^{\kappa}\vartheta + \vartheta T_t^{\kappa}\upsilon$.

If $\upsilon(x,\ t)$ is differentiable in the classical sense with respect to t, then

$$T_t^{\kappa} \upsilon(x,\ t) = t^{1-\kappa} \frac{\partial \upsilon(x,\ t)}{\partial t}. \tag{2}$$

Definition 2 ([5]). For $0 < \kappa \leq 1$, a power series of the form

$$\upsilon(x,t) = \sum_{i=0}^{\infty} q_i(x) t^{i\kappa} = q_0(x) + q_1(x)t^{\kappa} + q_2(x)t^{2\kappa} + q_3(x)t^{3\kappa} + \cdots, \tag{3}$$

where $x \in I$, $0 \leq t < R^{1/\kappa}$, and $R > 0$, is called a *multiple FPS*, with t as the variable and q_i as the coefficient functions.

All terms in the multiple FPS, except the first, vanish as in classical power series, indicating convergence. The series converges absolutely for

$$0 \leq t < R^{1/\kappa}, \quad (R > 0),$$

where $R^{1/\kappa}$ is the radius of convergence.

Theorem 1 (**Conformable Differentiation of Series**). If $\upsilon(x,t) = \sum_{i=0}^{\infty} q_i(x)t^{i\kappa}$ converges for $0 \leq t < R$, then

$$T_t^{\kappa}\upsilon(x,t) = \kappa \sum_{i=1}^{\infty} i q_i(x) t^{(i-1)\kappa}.$$

Proof. Apply linearity and the power rule of the conformable derivative:

$$\begin{aligned} T_t^{\kappa}\upsilon(x,t) &= T_t^{\kappa}\left(\sum_{i=0}^{\infty} q_i(x)t^{i\kappa}\right) \\ &= T_t^{\kappa}\left(q_0(x) + q_1(x)t^{\kappa} + q_2(x)t^{2\kappa} + q_3(x)t^{3\kappa} + \cdots\right) \\ &= q_0(x)T_t^{\kappa}(1) + q_1(x)T_t^{\kappa}(t^{\kappa}) + q_2(x)T_t^{\kappa}(t^{2\kappa}) + q_3(x)T_t^{\kappa}(t^{3\kappa}) + \cdots \\ &= \sum_{i=1}^{\infty} q_i(x)T_t^{\kappa}\left(t^{i\kappa}\right). \end{aligned}$$

Using the property $T_t^{\kappa}(t^{i\kappa}) = i\kappa t^{(i-1)\kappa}$, we obtain

$$T_t^{\kappa}\upsilon(x,t) = \kappa\sum_{i=1}^{\infty} iq_i(x)t^{(i-1)\kappa}.$$

□

3 Solution for the fractional Fisher–KPP equation

In recent years, the representation of physical problems using fractional operators has yielded significant results. A major challenge in this field involves obtaining solutions for nonlinear equations. Given the practical importance of these solutions, researchers have developed numerous analytical and numerical methods. Notable techniques include the homotopy perturbation method [30], variational iteration method [38], Laplace perturbation method [30], Adomian decomposition method [15], and homotopy decomposition method [7], among others. This study employs the CFPSM to approximate solutions for the nonlinear time-fractional Fisher–KPP equation, which takes the following form:

$$T_t^{\kappa}\upsilon(x,t) = c\frac{\partial^2 \upsilon}{\partial x^2} + a\upsilon + b\upsilon^m, \qquad t > 0,\ \kappa \in (0,1], \tag{4}$$

where a, b, $m \neq 1$, and c are arbitrary constants. For the integer-order case ($\kappa = 1$), an analytical solution is given by [22, p. 173]:

$$\upsilon(x,t) = \left[\beta + \exp\left(\lambda t + \frac{\mu x}{\sqrt{c}}\right)\right]^{\frac{2}{1-m}}, \tag{5}$$

where

$$\lambda = \frac{a(1-m)(m+3)}{2(m+1)}, \quad \mu = \sqrt{\frac{a(1-m)^2}{2(m+1)}}, \quad \beta = \sqrt{\frac{-b}{a}}.$$

In this section, we present three numerical examples ($m = 2, 3, 4$), which demonstrate how the proposed approach can be applied to highly nonlinear problems.

Example 1. Consider the following Fisher–KPP equation:

$$T_t^{\kappa} \upsilon(x,t) = c\frac{\partial^2 \upsilon}{\partial x^2} + a\upsilon + b\upsilon^2, \tag{6}$$

with the initial condition:

$$\upsilon\,(x,\ 0) = \left[\beta + \exp\left(\frac{\mu x}{\sqrt{c}}\right)\right]^{-2}.$$

We assume the solution can be expressed as an FPS:

$$\upsilon(x,\ t) = \sum_{i=0}^{\infty} q_i t^{i\kappa} = q_0 + q_1 t^{\kappa} + q_2 t^{2\kappa} + q_3 t^{3\kappa} + \dots. \tag{7}$$

Applying the conformable derivative to (7) and using the property in (2), we obtain:

$$\begin{aligned} T_t^{\kappa} \upsilon(x,\ t) &= \sum_{i=1}^{\infty} q_i t^{1-\kappa} \frac{d}{dt} t^{i\kappa} = \kappa \sum_{i=1}^{\infty} i q_i t^{(i-1)\kappa} \\ &= \kappa \begin{pmatrix} q_1 + 2q_2 t^{\kappa} + 3q_3 t^{2\kappa} + 4q_4 t^{3\kappa} + 5q_5 t^{4\kappa} \\ + 6q_6 t^{5\kappa} + 7q_7 t^{6\kappa} + 8q_8 t^{7\kappa} + \dots \end{pmatrix}. \end{aligned} \tag{8}$$

The spatial derivative term is:

$$\begin{aligned} \frac{\partial^2 \upsilon}{\partial x^2} &= \frac{\partial^2}{\partial x^2}\left[\sum_{i=0}^{\infty} q_i t^{i\kappa}\right] \\ &= \frac{\partial^2}{\partial x^2} q_0 + \frac{\partial^2}{\partial x^2} q_1 t^{\kappa} + \frac{\partial^2}{\partial x^2} q_2 t^{2\kappa} + \frac{\partial^2}{\partial x^2} q_3 t^{3\kappa} + \dots. \end{aligned} \tag{9}$$

The nonlinear term $\upsilon^2(x,t)$ expands to

$$\begin{aligned} \upsilon^2(x,t) = q_0^2 &+ 2q_0 q_1 t^{\kappa} + \left(q_1^2 + 2q_0 q_2\right) t^{2\kappa} + 2\left(q_0 q_3 + q_1 q_2\right) t^{3\kappa} \\ &+ \left(q_2^2 + 2q_0 q_4 + 2q_1 q_3\right) t^{4\kappa} + 2\left(q_0 q_5 + q_1 q_4 + q_2 q_3\right) t^{5\kappa} \\ &+ \left(q_3^2 + 2q_0 q_6 + 2q_1 q_5 + 2q_2 q_4\right) t^{6\kappa} \\ &+ 2\left(q_0 q_7 + q_1 q_6 + q_2 q_5 + q_3 q_4\right) t^{7\kappa} + \cdots. \end{aligned} \tag{10}$$

Substituting (7)–(10) into (6) and matching coefficients of like powers of t^{κ} yields the recurrence relations:

$$\begin{cases} q_0 = \upsilon_0, \\ q_1 = \dfrac{c\dfrac{\partial^2 q_0}{\partial x^2} + aq_0 + bq_0^2}{\kappa}, \\ q_2 = \dfrac{c\dfrac{\partial^2 q_1}{\partial x^2} + aq_1 + 2bq_0q_1}{2\kappa}, \\ q_3 = \dfrac{c\dfrac{\partial^2 q_2}{\partial x^2} + aq_2 + b\left[2q_0q_2 + q_1^2\right]}{3\kappa}, \\ q_4 = \dfrac{c\dfrac{\partial^2 q_3}{\partial x^2} + aq_3 + 2b\left[q_0q_3 + q_1q_2\right]}{4\kappa}, \\ q_5 = \dfrac{c\dfrac{\partial^2 q_4}{\partial x^2} + aq_4 + b\left[2q_0q_4 + 2q_1q_3 + q_2^2\right]}{5\kappa}, \\ q_6 = \dfrac{c\dfrac{\partial^2}{\partial x^2}q_5 + aq_5 + 2b\left[q_0q_5 + q_1q_4 + q_2q_3\right]}{6\kappa}, \\ q_7 = \dfrac{c\dfrac{\partial^2}{\partial x^2}q_6 + aq_6 + b\left[2q_0q_6 + 2q_1q_5 + 2q_2q_4 + q_3^2\right]}{7\kappa}, \\ q_8 = \dfrac{c\dfrac{\partial^2}{\partial x^2}q_7 + aq_7 + 2b\left[q_0q_7 + q_1q_6 + q_2q_5 + q_3q_4\right]}{8\kappa}. \\ \vdots \end{cases} \tag{11}$$

Higher-order terms follow similarly. Thus, the series solution is

$$\begin{aligned} \upsilon(x,t) = q_0 + q_1t^{\kappa} + q_2t^{2\kappa} + q_3t^{3\kappa} + q_4t^{4\kappa} \\ + q_5t^{5\kappa} + q_6t^{6\kappa} + q_7t^{7\kappa} + q_8t^{8\kappa} + \dots . \end{aligned}$$

Convergence Analysis

Theorem 2 (Convergence of the FPS Solution)**.** Let $\upsilon(x,t)$ be defined in a Banach space $\mathfrak{B}$ with norm $\|\cdot\|$. Consider the series solution of the fractional Fisher–KPP equation given by

$$\upsilon(x,t) = \sum_{n=0}^{\infty} \upsilon_n(x,t),$$

where $\upsilon_0(x,t) = q_0(x)$ is the initial approximation and subsequent terms $\upsilon_n(x,t)$ are determined recursively via the CFPSM procedure. If there exists a constant $\delta \in (0,1)$ such that

$$\|\upsilon_{n+1}(x,t)\| \leq \delta\|\upsilon_n(x,t)\| \quad \text{for all } n \in \mathbb{N}, \tag{12}$$

then the series solution converges uniformly in $\mathfrak{B}$.

Proof. Let $w_n(x,t) = \sum_{i=0}^{n} \upsilon_i(x,t)$ denote the partial sum of the series. We prove that $\{w_n\}_{n=0}^{\infty}$ is a Cauchy sequence in $\mathfrak{B}$. First, from condition (12), we have

$$\|\upsilon_n(x,t)\| \leq \delta^n\|\upsilon_0(x,t)\| \quad \text{for all } n \in \mathbb{N}.$$

Now, for any $n > m \geq 0$, consider

$$\begin{aligned}
\|w_n - w_m\| &= \left\|\sum_{i=0}^{n} \upsilon_i\left(x,\ t\right) - \sum_{i=0}^{m} \upsilon_i\left(x,\ t\right)\right\| \\
&= \left\|\sum_{i=m+1}^{n} \upsilon_i\left(x,\ t\right)\right\| \\
&\leq \sum_{i=m+1}^{n} \|\upsilon_i\left(x,\ t\right)\| \quad \text{(by the triangle inequality)} \\
&\leq \sum_{i=m+1}^{n} \delta^i\|\upsilon_0\left(x,\ t\right)\|.
\end{aligned}$$

Since $0 < \delta < 1$, the geometric series converges

$$\sum_{i=m+1}^{\infty} \delta^i = \frac{\delta^{m+1}}{1-\delta}.$$

Thus, for all $n > m$:

$$\|w_n - w_m\| \leq \frac{\delta^{m+1}}{1-\delta}\|\upsilon_0\left(x,\ t\right)\|.$$

Given $\|\upsilon_0\| < \infty$ and $\delta \in (0,1)$, we have

$$\lim_{m\to\infty} \frac{\delta^{m+1}}{1-\delta}\|\upsilon_0\left(x,\ t\right)\| = 0.$$

Therefore, for any $\epsilon > 0$, there exists $N_0 \in \mathbb{N}$ such that for all $n, m \geq N_0$,

$$\|w_n - w_m\| < \epsilon.$$

Since $\mathfrak{B}$ is a Banach space, $\{w_n\}$ converges to some $v^* \in \mathfrak{B}$. Each w_n satisfies the CFPSM recurrence derived from the original equation. Therefore, the series converges to the exact solution. □

Theorem 3. Let $\upsilon(x,t)=\sum_{n=0}^{\infty}\upsilon_n(x,t)$ be the exact series solution of the fractional Fisher–KPP equation obtained by CFPSM, and let $w_n(x,t)=\sum_{i=0}^{n}\upsilon_n(x,t)$ be the n-term partial sum approximation. If there exists $\delta\in(0,1)$ such that

$$\|\upsilon_{n+1}(x,t)\|\leq\delta\|\upsilon_n(x,t)\|\quad\text{for all } n\geq 0,$$

then the truncation error $e_n(x,t)=|\upsilon(x,t)-w_n(x,t)|$ satisfies

$$e_n(x,t)\leq\frac{\delta^{n+1}}{1-\delta}\|\upsilon_0(x,t)\|. \tag{13}$$

Proof. The exact solution can be expressed as follows:

$$\upsilon(x,t)=w_n(x,t)+\sum_{i=n+1}^{\infty}\upsilon_i(x,t).$$

Therefore,

$$e_n(x,t)=\left|\sum_{i=n+1}^{\infty}\upsilon_i(x,t)\right|\leq\sum_{i=n+1}^{\infty}|\upsilon_i(x,t)|\leq\sum_{i=n+1}^{\infty}\delta^i\|\upsilon_0(x,t)\|.$$

The last inequality follows from the contraction condition $\|\upsilon_n\|\leq\delta^n\|\upsilon_0\|$. Evaluate the geometric series:

$$\sum_{i=n+1}^{\infty}\delta^i=\delta^{n+1}(1+\delta+\delta^2+\cdots)=\frac{\delta^{n+1}}{1-\delta}.$$

Thus,

$$e_n(x,t)\leq\frac{\delta^{n+1}}{1-\delta}\|\upsilon_0(x,t)\|.$$

$\square$

For numerical Example 1, setting $a=1$, $b=-1$, $c=0.1$, and $m=2$, we compute the first few coefficients:

$$q_0(x)=\frac{1}{\left(\exp\left(\frac{\sqrt{15}x}{3}\right)+1\right)^2},$$

$$q_1(x)=\frac{5\exp\left(\frac{\sqrt{15}x}{3}\right)}{3\kappa\left(\exp\left(\frac{\sqrt{15}x}{3}\right)+1\right)^3},$$

$$q_2(x) = \frac{25\exp\left(\frac{\sqrt{15}x}{3}\right)\left[2\exp\left(\frac{\sqrt{15}x}{3}\right)-1\right]}{36\kappa^2\left(\exp\left(\frac{\sqrt{15}x}{3}\right)+1\right)^4},$$

$$q_3(x) = \frac{125\exp\left(\frac{\sqrt{15}x}{3}\right)\left[4\exp\left(\frac{2\sqrt{15}x}{3}\right)-7\exp\left(\frac{\sqrt{15}x}{3}\right)+1\right]}{648\kappa^3\left(\exp\left(\frac{\sqrt{15}x}{3}\right)+1\right)^5},$$

$$q_4(x) = \frac{625\exp\left(\frac{\sqrt{15}x}{3}\right)\left[\begin{array}{r}8\exp\left(\sqrt{15}x\right)+18\exp\left(\frac{\sqrt{15}x}{3}\right)\\ -33\exp\left(\frac{2\sqrt{15}x}{3}\right)-1\end{array}\right]}{15552\kappa^4\left(\exp\left(\frac{\sqrt{15}x}{3}\right)+1\right)^6},$$

$$q_5\left(x\right) = \frac{625\exp\left(\frac{\sqrt{15}x}{3}\right)\left(\begin{array}{l}171\exp\left(\frac{\sqrt{2}15x}{3}\right)-41\exp\left(\frac{\sqrt{15}x}{3}\right)\\ -131\exp\left(\sqrt{15}x\right)+16\exp\left(\frac{\sqrt{4}15x}{3}\right)+1\end{array}\right)}{93312\kappa^5(\exp\left(\frac{\sqrt{15}x}{3}\right)+1)^7},$$

$$q_6\left(x\right) = \frac{3125\exp\left(\frac{\sqrt{15}x}{3}\right)\left(\begin{array}{l}1208\exp\left(\sqrt{15}x\right)+88\exp\left(\frac{\sqrt{15}x}{3}\right)\\ -718\exp\left(\frac{\sqrt{2}15x}{3}\right)-473\exp\left(\frac{\sqrt{4}15x}{3}\right)\\ +32\exp\left(\frac{\sqrt{5}15x}{3}\right)-1\end{array}\right)}{3359232\kappa^6(\exp\left(\frac{\sqrt{15}x}{3}\right)+1)^8},$$

$$q_7(x) = \frac{15625\exp\left(\frac{\sqrt{15}x}{3}\right)\begin{pmatrix}1+64\exp\left(\sqrt{215}x\right)-8422\exp\left(\sqrt{15}x\right)\\ -183\exp\left(\frac{\sqrt{15}x}{3}\right)+2682\exp\left(\frac{\sqrt{215}x}{3}\right)\\ +7197\exp\left(\frac{\sqrt{415}x}{3}\right)-1611\exp\left(\frac{\sqrt{515}x}{3}\right)\end{pmatrix}}{141087744\kappa^7\left(\exp\left(\frac{\sqrt{15}x}{3}\right)+1\right)^9},$$

$$q_8(x) = \frac{78125\exp\left(\frac{\sqrt{15}x}{3}\right)\begin{pmatrix}49780\exp\left(\sqrt{15}x\right)-5281\exp\left(\sqrt{215}x\right)\\ +374\exp\left(\frac{\sqrt{15}x}{3}\right)-9327\exp\left(\frac{\sqrt{215}x}{3}\right)\\ -78095\exp\left(\frac{\sqrt{415}x}{3}\right)+38454\exp\left(\frac{\sqrt{515}x}{3}\right)\\ +128\exp\left(\frac{\sqrt{715}x}{3}\right)-1\end{pmatrix}}{6772211712\kappa^8\left(\exp\left(\frac{\sqrt{15}x}{3}\right)+1\right)^{10}}.$$

This section employs the CFPSM to obtain numerical solutions for the time-fractional Fisher–KPP equation. The results presented in the reference tables show that the fractional order κ significantly influences the solution behavior. Both tabular data and graphical representations confirm that the proposed method accurately captures the key features of the Fisher–KPP model across different nonlinearities and fractional orders. The tables provide a quantitative assessment of the CFPSM's performance by comparing the approximate solutions with exact analytical values and, where applicable, with results from existing methods. The close agreement observed validates the accuracy and reliability of the proposed approach.

Figure 1(a) displays the approximate solution $\upsilon(x,t)$ obtained via the CFPSM for the Fisher–KPP equation under the initial condition specified in Equation (6), while Figure 1(b) illustrates the corresponding exact solution. To evaluate the accuracy of the proposed method, error analysis is presented in Table 1 for fixed x and varying t, confirming the effectiveness of the approach in solving the Fisher–KPP equation with the initial condition from Example 1. Figure 2 shows a surface plot of the absolute error for Example 1, revealing a maximum absolute error of 2.5670×10^{-8}.

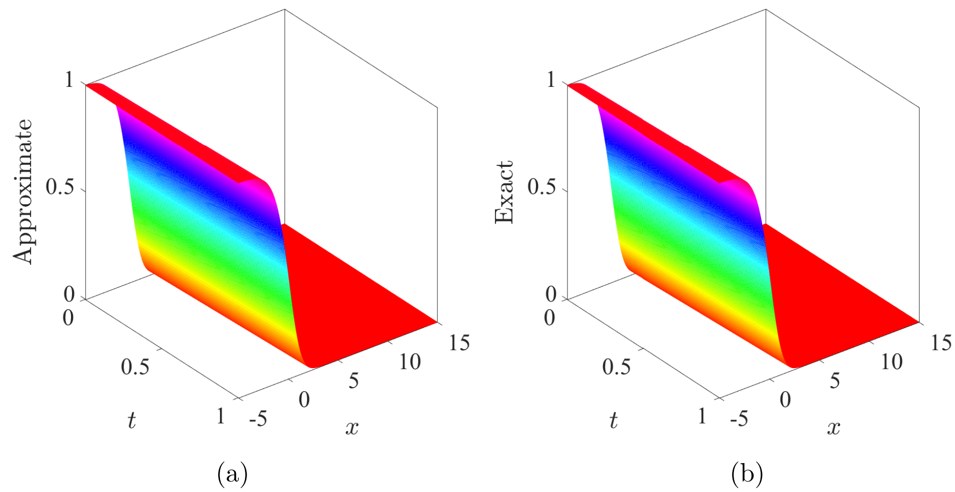


Figure 1: Comparison between exact solution and approximate results for Example 1 with $a = 1,\ b = -1,\ c = 0.1, \kappa = 1$

Table 1: Comparisons of the present results against analytical values for Example 1 at $\kappa = 1$

x	t	Exact	CFPSM	Abs. Error
0.2	0.2	0.22765581	0.22765581	3.3928×10^{-12}
	0.4	0.26912727	0.26912727	1.5652×10^{-09}
	0.6	0.31377638	0.31377643	5.3170×10^{-08}
	0.8	0.36086503	0.36086564	6.1156×10^{-07}
	1.0	0.40953226	0.40953609	3.8266×10^{-06}

Figure 3(a) illustrates the solution profile of $v(x,t)$ for Example 1 at $\kappa = 0.85$ and different time levels. Figure 3(b) shows the absolute difference between the exact and CFPSM results for $\kappa = 1$ and $t = 0.1$. Figures 4(a)–4(b) display the solution behavior of $v(x,t)$ for Example 1 at $\kappa = 0.25$ and $\kappa = 0.5$, respectively.

Table 2: Absolute errors for Example 1 at different values of κ

$(x,\ t)$	κ	Absolute error
(2.0, 0.1)	0.99	2.9952×10^{-05}
	0.97	9.3705×10^{-05}
	0.95	1.6304×10^{-04}
	0.93	2.3856×10^{-04}
	0.91	3.2094×10^{-04}

Tables 2, 4, and 8 present the absolute errors for different fractional orders κ of the Fisher–KPP equation in Examples 1–3. These results demonstrate that as the fractional order κ in-

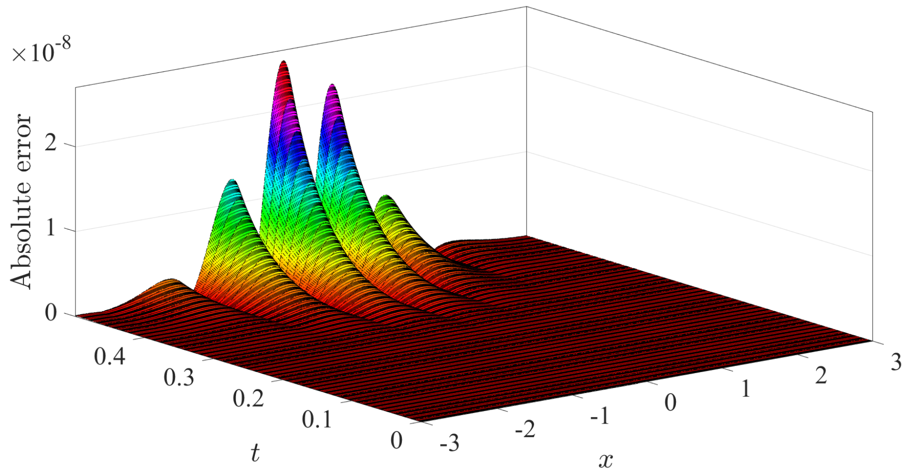


Figure 2: Absolute Error max $= 2.5670 \times 10^{-08}$, for Example 1 with $a = 1,\ b = -1,\ c = 0.1, \kappa = 1$

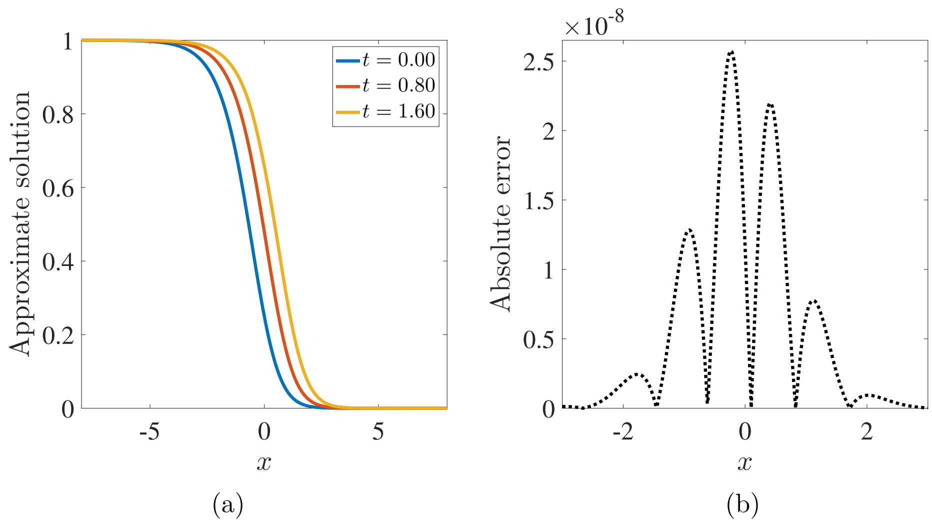


Figure 3: a. Approximate solution for $\kappa = 0.85$; b. Absolute error plot for Example 1 at $\kappa = 1,\ t = 0.1$

creases, the numerical solutions obtained using the proposed technique converge toward the exact solution.

Example 2. Consider the time fractional Fisher–KPP equation:

$$T_t^{\kappa} \upsilon(x,t) = c\frac{\partial^2 \upsilon}{\partial x^2} + a\upsilon + b\upsilon^3,\ t \geq 0,\ 0 < \kappa \leq 1, \tag{14}$$

with the initial condition

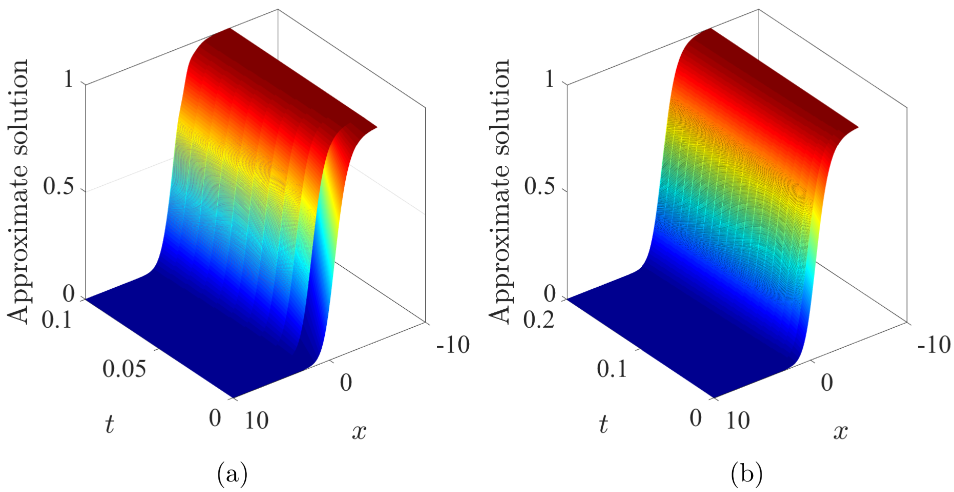


Figure 4: (a) Surf plot of Example 1 for $\kappa = 0.25$. (b) Surf plot of Example 1 at $\kappa = 0.5$

$$\upsilon\left(x,\ 0\right) = \left[\beta + \exp\left(\frac{\mu x}{\sqrt{c}}\right)\right]^{-1}. \tag{15}$$

The exact solution for the nonfractional case at $\kappa = 1$ is

$$\upsilon(x,t) = \left[\beta + \exp\left(\lambda t + \frac{\mu x}{\sqrt{c}}\right)\right]^{-1}. \tag{16}$$

The nonlinear term $\upsilon^3\left(x,\ t\right)$ can be computed as below:

$$\begin{aligned} &\upsilon^3(x) \\ &= \left(\begin{array}{l} q_0^3 + 3q_1q_0t^\kappa + 3\left(q_0q_1^2 + q_0^2q_2\right)t^{2\kappa} + \left(q_1^3 + 3q_0^2q_3 + 6q_0q_1q_2\right)t^{3\kappa} \\ + \left(3q_0q_2^2 + 3q_1^2q_2 + 3q_0^2q_4 + 6q_0q_1q_3\right)t^{4\kappa} \\ + \left(3q_1q_2^2 + 3q_1^2q_3 + 3q_0^2q_5 + 6q_0q_1q_4 + 6q_0q_2q_3\right)t^{5\kappa} \\ + \left(q_2^3 + 3\left(q_0q_3^2 + q_1^2q_4 + q_0^2q_6 + 2q_0q_1q_5 + 2q_0q_2q_4 + 2q_1q_2q_3\right)\right)t^{6\kappa} \\ + 3\left(\begin{array}{l} q_1q_3^2 + q_2^2q_3 + q_1^2q_5 + q_0^2q_7 \\ + 2\left(q_0q_1q_6 + q_0q_2q_5 + q_0q_3q_4 + q_1q_2q_4\right) \end{array}\right)t^{7\kappa} + \dots \end{array}\right). \end{aligned} \tag{17}$$

Substituting (7)–(9) and (17) into (14) and looking at the coefficients of t^κ, hence, we obtain

$$\begin{cases} q_1 = \dfrac{c\dfrac{\partial^2 q_0}{\partial x^2} + aq_0 + bq_0^3}{\kappa}, \quad q_2 = \dfrac{c\dfrac{\partial^2 q_1}{\partial x^2} + aq_1 + 3bq_0^2 q_1}{2\kappa}, \\ q_3 = \dfrac{c\dfrac{\partial^2 q_2}{\partial x^2} + aq_2 + 3b\left(q_0 q_1^2 + q_0^2 q_2\right)}{3\kappa}, \quad q_4 = \dfrac{c\dfrac{\partial^2 q_3}{\partial x^2} + aq_3 + b\left(q_1^3 + 3q_0^2 q_3 + 6q_0 q_1 q_2\right)}{4\kappa} \\ q_5 = \dfrac{c\dfrac{\partial^2 q_4}{\partial x^2} + aq_4 + 3b\left(q_0 q_2^2 + q_1^2 q_2 + q_0^2 q_4 + 2q_0 q_1 q_3\right)}{5\kappa}, \\ q_6 = \dfrac{c\dfrac{\partial^2 q_5}{\partial x^2} + aq_5 + 3b\left(q_1 q_2^2 + q_1^2 q_3 + q_0^2 q_5 + 2q_0 q_1 q_4 + 2q_0 q_2 q_3\right)}{6\kappa}, \\ q_7 = \dfrac{c\dfrac{\partial^2 q_6}{\partial x^2} + aq_6 + b\left[q_2^3 + 3\begin{pmatrix} q_0\left(q_3^2 + q_0 q_6 + 2q_1 q_5 + 2q_2 q_4\right) \\ + q_1^2 q_4 + 2q_1 q_2 q_3 \end{pmatrix}\right]}{7\kappa}, \\ q_8 = \dfrac{c\dfrac{\partial^2 q_7}{\partial x^2} + aq_7 + 3b\begin{bmatrix} q_1 q_3^2 + q_2^2 q_3 + q_1^2 q_5 + q_0^2 q_7 \\ + 2\left(q_0\left(q_1 q_6 + q_2 q_5 + q_3 q_4\right) + q_1 q_2 q_4\right) \end{bmatrix}}{8\kappa}. \end{cases} \tag{18}$$

Following the same procedure, the remaining iterative terms can be obtained. The coefficients

Table 3: Comparisons of the present results against analytical values for Example 2 at $\kappa = 1$

x	t	Exact	CFPSM	Absolute Error
2.0	0.2	0.015185146	0.015185146	2.4356×10^{-10}
	0.4	0.020389481	0.020389463	1.7384×10^{-08}
	0.6	0.027327977	0.027327771	2.0605×10^{-07}
	0.8	0.036539559	0.036538489	1.0698×10^{-06}
	1.0	0.048700671	0.048697711	2.9600×10^{-06}

are as follows:

$$q_1 = \frac{3\exp\left(\sqrt{5}x\right)}{2\kappa(\exp\left(\sqrt{5}x\right)+1)^2},$$

$$q_2 = \frac{9\exp\left(\sqrt{5}x\right)\left(\exp\left(\sqrt{5}x\right)-1\right)}{8\kappa^2(\exp\left(\sqrt{5}x\right)+1)^3},$$

$$q_3(x) = \frac{9\exp\left(\sqrt{5}x\right)\left(\exp\left(\sqrt{25}x\right)-4\exp\left(\sqrt{5}x\right)+1\right)}{16\kappa^3(\exp\left(\sqrt{5}x\right)+1)^4},$$

$$q_4(x) = \frac{27\exp\left(\sqrt{5}x\right)\begin{bmatrix} 11\exp\left(\sqrt{5}x\right) - 11\exp\left(\sqrt{25}x\right) \\ + \exp\left(\sqrt{35}x\right) - 1 \end{bmatrix}}{128\kappa^4(\exp\left(\sqrt{5}x\right)+1)^5},$$

$$q_5(x) = \frac{81\exp\left(\sqrt{5}x\right)\begin{bmatrix}66\exp\left(\sqrt{25}x\right) - 26\exp\left(\sqrt{5}x\right)\\ -26\exp\left(\sqrt{35}x\right) + \exp\left(\sqrt{45}x\right) + 1\end{bmatrix}}{1280\kappa^5(\exp\left(\sqrt{5}x\right) + 1)^6},$$

$$q_6(x) = \frac{81\exp\left(\sqrt{5}x\right)\begin{bmatrix}168\exp\left(\sqrt{5}x\right) - 897\exp\left(\sqrt{25}x\right)\\ +752\exp\left(\sqrt{35}x\right) + 83\exp\left(\sqrt{45}x\right)\\ -56\exp\left(\sqrt{55}x\right) + \exp\left(\sqrt{65}x\right) - 3\end{bmatrix}}{5120\kappa^6(\exp\left(\sqrt{5}x\right) + 1)^8},$$

$$q_7(x) = \frac{243\exp\left(\sqrt{5}x\right)\begin{bmatrix}11662\exp\left(\sqrt{35}x\right) - 1214\exp\left(\sqrt{25}x\right)\\ -106\exp\left(\sqrt{5}x\right) - 18476\exp\left(\sqrt{45}x\right)\\ +6194\exp\left(\sqrt{55}x\right) + 358\exp\left(\sqrt{65}x\right)\\ -118\exp\left(\sqrt{75}x\right) + \exp\left(\sqrt{85}x\right) + 3\end{bmatrix}}{71680\kappa^7(\exp\left(\sqrt{5}x\right) + 1)^{10}},$$

$$q_8(x) = \frac{243\exp\left(\sqrt{5}x\right)\begin{bmatrix}852\exp\left(\sqrt{5}x\right) - 31025\exp\left(\sqrt{25}x\right)\\ +302352\exp\left(\sqrt{35}x\right) - 1037658\exp\left(\sqrt{45}x\right)\\ +1500040\exp\left(\sqrt{55}x\right) - 844434\exp\left(\sqrt{65}x\right)\\ +132336\exp\left(\sqrt{75}x\right) + 4131\exp\left(\sqrt{85}x\right)\\ -732\exp\left(\sqrt{95}x\right) + 3\exp\left(\sqrt{105}x\right) - 9\end{bmatrix}}{1146880\kappa^8(\exp\left(\sqrt{5}x\right) + 1)^{12}}.$$

Figure 5(a) shows the absolute error for the Fisher–KPP solution in Example 2, while Figure 5(b) displays the solution computed using the CFPSM at $\kappa = 0.75$. Figure 6 presents a mesh plot of the absolute error for Example 2 at $\kappa = 1$. Table 3 compares the exact and CFPSM results for Example 2 at $\kappa = 1$, demonstrating near-perfect agreement.

Table 4: Absolute errors for Example 2 at different values of κ

$(x,\ t)$	κ	Absolute error for $\upsilon(x,\ t)$
$(2.0,\ 0.1)$	0.99	6.5365×10^{-05}
	0.98	1.3348×10^{-04}
	0.97	2.0448×10^{-04}
	0.96	2.7853×10^{-04}
	0.95	3.5577×10^{-04}

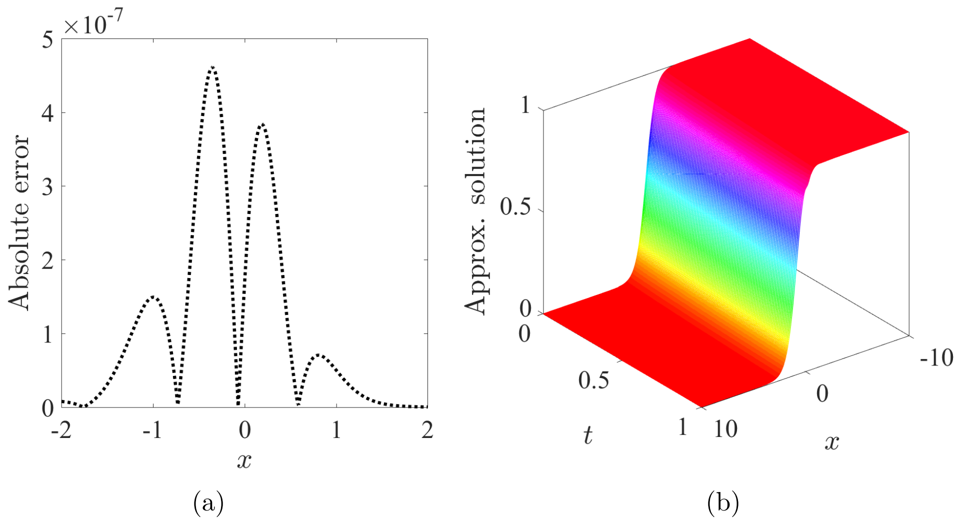


Figure 5: (a) Absolute error plot for Example 2 at $\kappa = 1,\ t = 0.2$. (b) Approximate solution of Example 2 for $\kappa = 0.75$

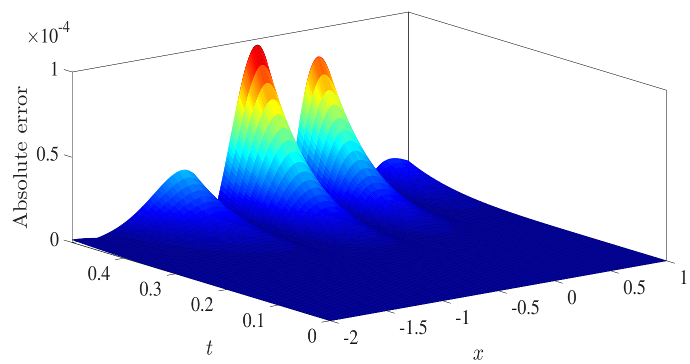


Figure 6: Absolute error for Example 2 with $\kappa = 1$

Table 5: Method comparison for Example 1 with $a = -1,\ b = 0,\ c = 1$ and $q_0(x) = x + \exp(-x)$

x	t	$\kappa = 0.98$	$\kappa = 0.98$
		Method in [29]	Present method
0.5	0.1	1.05647673	1.05586096
	0.2	1.01266710	1.01151583
	0.3	0.97347374	0.97194245
	0.4	0.93809566	0.93646575
	0.5	0.90587041	0.90458680

Tables 5 and 6 compare the CFPSM with the method presented in [29]. These results confirm that the solutions obtained by the proposed method for the Fisher–KPP equation are in excellent agreement with existing numerical results.

Table 6: Method comparison for Example 1 with $a=-1,\ b=0,\ c=1$ and $q_0(x)=x+\exp(-x)$

x	t	$\kappa=1.0$	$\kappa=1.0$
		Method in [29]	Present method
0.5	0.1	1.0589494100	1.058949370
	0.3	0.9769494100	0.976939770
	0.5	0.9099160766	0.909795992
	0.7	0.8554494100	0.854823364
	0.9	0.7940306600	0.809815979

Example 3. Consider the time fractional Fisher–KPP equation:

$$T_t^{\kappa}\upsilon(x,t)=c\frac{\partial^2\upsilon}{\partial x^2}+a\upsilon+b\upsilon^4, \tag{19}$$

with the initial condition

$$\upsilon(x,\ 0)=\left[\beta+\exp\left(\frac{\mu x}{\sqrt{c}}\right)\right]^{-2/3}. \tag{20}$$

The exact solution for the nonfractional case at $\kappa=1$ is

$$\upsilon(x,t)=\left[\beta+\exp\left(\lambda t+\frac{\mu x}{\sqrt{c}}\right)\right]^{-2/3}. \tag{21}$$

The nonlinear term $\upsilon^4(x,\ t)$ can be computed as follows:

$$\begin{aligned}&\upsilon^4(x,t)\\ &=\begin{pmatrix}4q_0^3q_1t^{\kappa}+q_0^2\left(4q_0q_2+6q_1^2\right)t^{2\kappa}+4q_0\left(q_0^2q_3+3q_0q_1q_2+q_1^3\right)t^{3\kappa}\\ +q_0^4+\left(q_1^4+4q_0^3q_4+6q_0^2q_2^2+12q_0q_1^2q_2+12q_0^2q_1q_3\right)t^{4\kappa}\\ +4\left(q_1^3q_2+q_0^3q_5+3q_0q_1q_2^2+3q_0q_1^2q_3+3q_0^2q_1q_4+3q_0^2q_2q_3\right)t^{5\kappa}\\ +4\begin{pmatrix}q_0q_2^3+q_1^3q_3+q_0^3q_6+3q_0q_1^2q_4+3q_0^2q_1q_5+3q_0^2q_2q_4\\ +1.5q_0^2q_3^2+1.5q_1^2q_2^2+6q_0q_1q_2q_3\end{pmatrix}t^{6\kappa}+\dots\end{pmatrix}.\end{aligned}$$

The coefficients are as follow:

$$q_0=\frac{1}{(\exp(3x)+1)^{2/3}},$$

$$q_1=\frac{7\exp(3x)}{5\kappa(\exp(3x)+1)^{5/3}},$$

$$q_2 = \frac{49\exp(3x)(2\exp(3x)-3)}{100\kappa^2(\exp(3x)+1)^{8/3}},$$

$$q_3 = \frac{343\exp(3x)(4\exp(6x)-27\exp(3x)+9)}{3000\kappa^3(\exp(3x)+1)^{11/3}},$$

$$q_4 = \frac{2401\exp(3x)(234\exp(3x)-171\exp(6x)+8\exp(9x)-27)}{120000\kappa^4(\exp(3x)+1)^{14/3}},$$

$$\vdots$$

Table 7: Comparisons of the present results against analytical values for Example 3 at $\kappa = 1$

x	t	Exact	CFPSM	Abs. Error
0.2	0.02	0.50981094	0.50981094	1.2606×10^{-10}
	0.04	0.51889992	0.51889991	4.0763×10^{-09}
	0.06	0.583909237	0.583903284	3.1268×10^{-08}
	0.08	0.53712165	0.53712152	1.3305×10^{-07}
	0.1	0.54624177	0.54624136	4.0988×10^{-07}

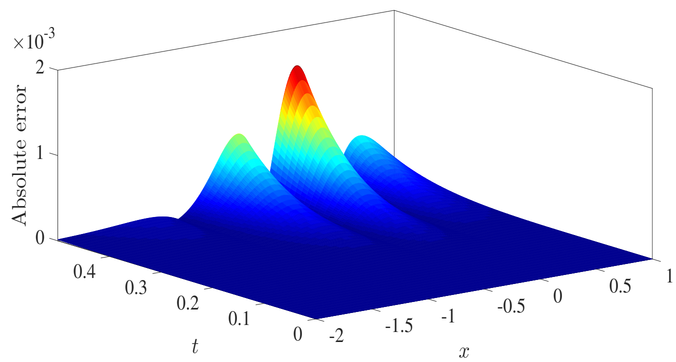


Figure 7: Absolute error for Example 3 with $\kappa = 1$

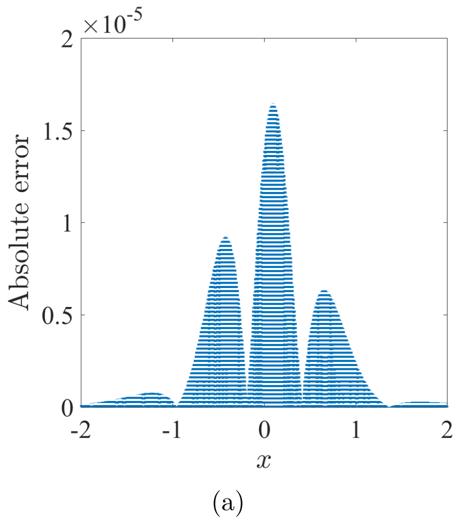
For Example 3, Table 7 compares the exact and CFPSM results at $\kappa = 1$, showing excellent agreement. Figure 7 displays the error function for Example 3, further illustrating the method's accuracy and applicability.

The consistency of these results underscores the reliability of the proposed method for solving highly nonlinear equations. Both the excellent agreement with exact solutions and the behavior of the error plots confirm the accuracy and robustness of the CFPSM.

Figure 8(a) displays the absolute error, while Figure 8(b) shows the approximate solution for Example 3 at $\kappa = 0.85$. In summary, the accuracy of the CFPSM has been validated

Table 8: Absolute errors for Example 3 at different values of κ

$(x,\ t)$	κ	Absolute error for $\upsilon\,(x,\ t)$
$(0.2,\ 0.08)$	0.99	$2.05900637 \times 10^{-3}$
	0.98	$4.21096997 \times 10^{-3}$
	0.97	$6.43844304 \times 10^{-3}$
	0.96	$8.74387351 \times 10^{-3}$
	0.95	$1.11297450 \times 10^{-2}$



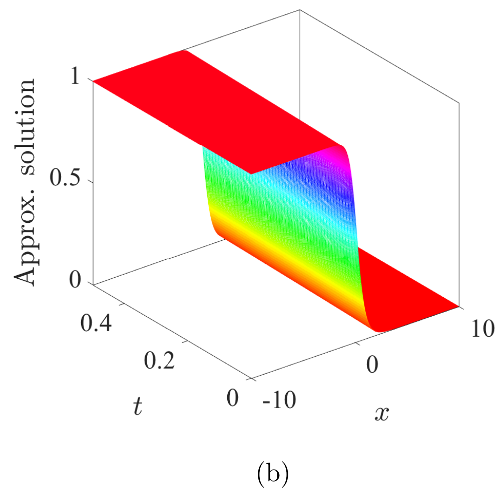


Figure 8: (a) Absolute error plot for Example 3 at $\kappa = 1,\ t = 0.2$. (b) Approximate solution of Example 3 for $\kappa = 0.85$

by comparing the computed results with exact solutions and existing numerical results from the literature. The near-perfect agreement demonstrates the applicability and reliability of the proposed technique for approximating nonlinear fractional differential equations.

4 Conclusion

This work has successfully applied the CFPSM to solve the nonlinear time-fractional Fisher–KPP equation, a widely used model for biological invasion and tumor growth. By leveraging the mathematical simplicity and rule-preserving properties of the conformable derivative, we derived accurate semi-analytical approximations without the need for spatial discretization or complex numerical algorithms. The method was implemented for three representative cases with varying nonlinear powers ($m = 2, 3, 4$) and fractional orders $\kappa \in (0, 1]$. In all examples, the CFPSM solutions show excellent agreement with the corresponding exact solutions. In addition, convergence and error analyses of the series solution were established. The graphical and tabular results consistently illustrated that as $\kappa \to 1$, the fractional solutions converge smoothly to the classical integer-order solutions. All computations, including the derivation of series coefficients, and graphical visualizations, were performed using MATLAB R2016a. In conclusion, the CF-PSM shows excellent approximation behavior for solving nonlinear time-fractional Fisher–KPP equations. Future work may extend the method to multi-dimensional systems, variable-order fractional operators, or coupled nonlinear models.

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Conflicts of Interest

The authors declare no conflicts of interest.

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