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Research Article

Stability analysis of fast food consumption dynamics using a fractional-order framework

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Abstract

In this research, we propose a novel fractional-order model, referred to as $PLSCQ$, which is designed to capture the dynamics of fast food consumption and its impact on both health and social behavior. The model incorporates the Caputo derivative and classifies individuals into five distinct categories: Potential fast food consumers P, moderate consumers L, excessive consumers S, individuals affected by obesity C, and those who have ceased consuming fast food Q. These categories encompass the full range of fast food consumption behaviors, from those who may potentially consume it to those who have experienced its detrimental consequences, such as obesity, cardiovascular diseases, and other social problems associated with unhealthy eating habits.

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The study primarily examines the health risks and behavioral issues associated with fast food consumption. We demonstrate the existence of unique, nonnegative solutions and compute the basic reproduction number R_0. Sensitivity analysis highlights the factors that most significantly influence R_0, while stability analysis reveals that the system is both locally and globally stable at the equilibrium point with no fast food consumption (E_0) when $R_0 \leq 1$, and stable with ongoing consumption (E^*) when $R_0 > 1$. Finally, we present numerical simulations to corroborate the theoretical findings, showing how the order of the partial derivative affects the system's dynamics under varying parameter conditions.

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Keywords: Fast Food; Fractional-order; Mathematical model; Characterization; Pontryagin's maximum principle.

1 Introduction

Obesity has become a significant public health challenge in Morocco, with the number of obese adults increasing sharply from 5.3 million in 2004 to 7 million in 2014 [2]. This trend is not unique to Morocco but is part of a broader global issue, particularly in Pacific Island nations, where a large portion of the population is affected due to a heavy reliance on processed and imported foods. Fast food, which is high in fats, sugars, and salt, plays a crucial role in driving this epidemic. Its consumption has been linked to severe health conditions such as cardiovascular diseases, type 2 diabetes, and other metabolic disorders [5, 3, 8, 18]. In addition to its detrimental physical effects, excessive fast food consumption also disrupts traditional family eating habits, replacing home-cooked meals with convenience-based, fast-food options, thereby contributing to social and cultural imbalances. Given the profound health and societal impacts of this issue, there is an urgent need for policies that promote healthier eating habits and encourage the production of local foods to mitigate this public health crisis [14, 15, 16, 17].

In light of these concerns, this paper proposes a fractional-order $PLSCQ$ model to study the dynamics of fast food consumption and its health impacts. The model includes several categories: P for potential individuals who may consume fast food, L for moderate consumers, S for excessive consumers, C for individuals who have developed obesity as a result of fast food consumption, and Q for those who have quit consuming fast food. This model extends the classic SIRS (Susceptible-Infected-Recovered-Susceptible) framework by incorporating a nonlinear incidence rate, which provides a more accurate representation of fast food consumption dynamics and its effects on public health [6, 13, 7, 19].

By introducing fractional derivatives, the model incorporates memory effects and time-scale influences on eating behaviors, aspects that are typically neglected by traditional local derivatives. This fractional-order approach offers a more comprehensive and precise framework for modeling the dynamics of fast food consumption, accounting for both the long-term and short-term influences on individuals' eating habits and health outcomes.

We show that the model has a unique positive solution, ensuring that the system remains realistic and nonnegative across all categories. Additionally, we conduct a stability analysis to examine the equilibrium states of the system, including the fast-food-free equilibrium and the positive fixed points representing different levels of fast food consumption within the population [1]. By demonstrating the limitations of local derivatives in capturing the time-scale and nonlocal effects of fast food consumption, we argue that the proposed fractional-order model is a more effective and accurate tool for studying the true dynamics of fast food consumption, considering both historical and current behavioral patterns.

The remainder of this paper is organized as follows. Section 2 introduces the preliminary concepts and mathematical foundations necessary for the subsequent analysis. In Section 3, we present the formulation of the fractional-order $PLSCQ$ model, describe its components, and discuss its basic properties. Section 4 provides the stability analysis of the model, covering equilibrium points, local and global stability, and a sensitivity analysis of the basic reproduction number R_0. Section 5 illustrates the numerical simulations and comparative discussions, including two specific scenarios: The healthy-consumption equilibrium and the endemic fast-food equilibrium. Finally, section 6 concludes the paper and highlights future research perspectives.

2 Preliminary

(1) Let $f : \mathbb{R}^+ \to \mathbb{R}$ be a continuous function. The Caputo fractional derivative of order $\alpha > 0$ is defined as

$$D^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-x)^{n-\alpha-1} f^{(n)}(x)\, dx, \tag{1}$$

where $n \in \mathbb{N}$ is chosen such that $n-1 < \alpha < n$, $D = \frac{d}{dt}$, and $\Gamma(\cdot)$ denotes the Gamma function. In particular, for $0 < \alpha < 1$, this expression reduces to

$$D^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f'(x)}{(t-x)^\alpha}\, dx. \tag{2}$$

(2) The Laplace transform of the Caputo derivative is given by

$$\mathcal{L}[D^{\alpha} f(t)] = \lambda^{\alpha} F(\lambda) - \sum_{k=0}^{n-1} f^{(k)}(0)\, \lambda^{\alpha-k-1}, \tag{3}$$

where $F(\lambda)$ represents the standard Laplace transform of $f(t)$.

(3) For parameters $\alpha, \beta > 0$, the generalized Mittag–Leffler function $E_{\alpha,\beta}(z)$ is defined by the series:

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}. \tag{4}$$

(4) Its Laplace transform satisfies the relation:

$$\mathcal{L}[t^{\beta-1} E_{\alpha,\beta}(\pm a t^{\alpha})] = \frac{\lambda^{\alpha-\beta}}{\lambda^{\alpha} \pm a}. \tag{5}$$

(5) For all $\alpha, \beta > 0$ and $z \in \mathbb{C}$, the Mittag–Leffler function satisfies the identity:

$$E_{\alpha,\beta}(z) = z\, E_{\alpha,\alpha+\beta}(z) + \frac{1}{\Gamma(\beta)}. \tag{6}$$

(6) Consider now the following fractional-order system, where $0 < \alpha \leq 1$, $n \geq 1$, and $t_0 \in \mathbb{R}$:

$$\begin{cases} D^{\alpha} x(t) = f(t), \\ x(t_0) = x_0. \end{cases} \tag{7}$$

The global existence and uniqueness of the solution of this system are guaranteed by the following lemma.

Lemma 1 (see [9])**.** Let $f : \mathbb{R}^n \to \mathbb{R}^n$ be a function such that **(i)** $f(x)$ and its partial derivative $\frac{\partial f}{\partial x}$ are continuous for all $x \in \mathbb{R}^n$. **(ii)** There exist two positive constants ω, λ such that

$$\|f(x)\| \leq \omega + \lambda \|x\|, \qquad \text{for all } x \in \mathbb{R}^n.$$

Then, system (7) admits a unique solution on the interval $[t_0, +\infty)$.

3 Model formulation

3.1 Model description

In this work, we develop a continuous-time model of fractional order to capture the dynamics of fast food consumption within a population. The structure of the model is inspired by the framework proposed in [10]. The entire population, denoted by $N(t)$, is subdivided into five distinct compartments (Table 1) according to their eating behaviors.

Compartment P (Potential consumers): This class consists of individuals who are not yet consumers of fast food but may adopt this behavior. Their number grows through recruitment at rate b. Transitions out of this class occur either through effective contacts with moderate consumers—reflecting social influence via family, friends, or advertising—at rate β_1, or through natural mortality at rate μ.

Compartment L (Moderate consumers): Individuals in this group consume fast food occasionally or in a socially discreet manner. They are recruited from the potential class at rate β_1. Their number decreases when they shift to the excessive consumers class at rate β_2, or due to natural death at rate μ.

Compartment S (Excessive consumers): This group represents individuals with uncontrolled fast food consumption. Its size increases as moderate consumers become excessive at rate β_2. It decreases when individuals either quit consumption at rate α_2, develop obesity due to persistent habits at rate α_1, die naturally at rate μ, or die prematurely from diet-related diseases at rate δ_1.

Compartment C (Obese individuals): This class contains those who develop obesity as a consequence of overconsumption. It grows through inflow from the excessive consumers class at rate α_1. Losses occur when individuals quit consumption at rate γ or die from obesity-related complications at rate δ_2.

Compartment Q (Quitters): This compartment consists of individuals who successfully abandon fast food consumption. Its population increases at rate γ and decreases solely through natural mortality at rate μ.

The total population at time t, denoted by

$$N(t) = P(t) + L(t) + S(t) + C(t) + Q(t),$$

is assumed to remain constant throughout the study period (see Table 1).

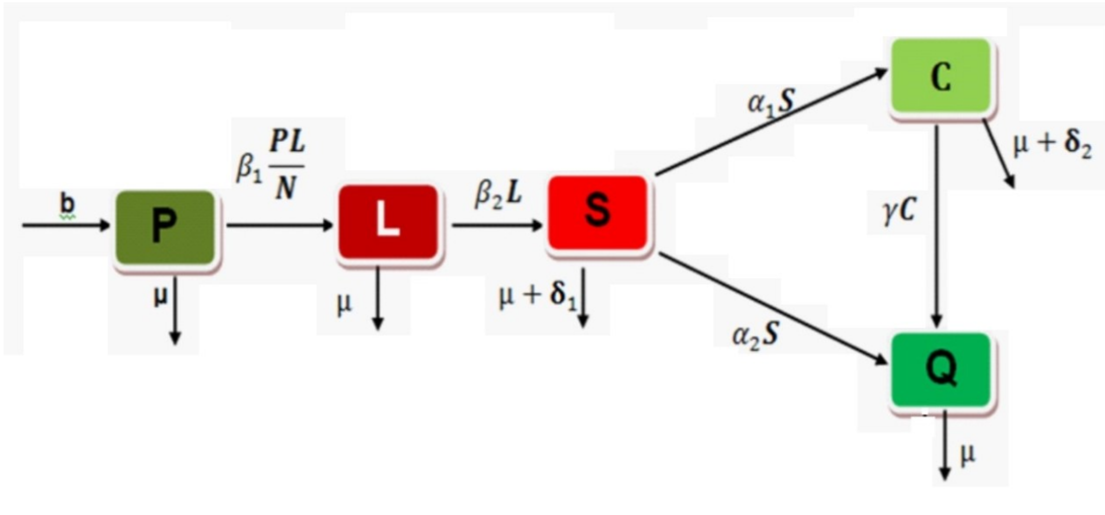


Figure 1: Schematic diagram of the five consumers' fast food classes in the model.

Table 1: State variables and their meanings for fast food consumption.

State Variable	Meaning
$P(t)$	Potential fast food consumers at time t
$L(t)$	Moderate fast food consumers at time t
$S(t)$	Excessive consumers of fast food at time t
$C(t)$	Obese individuals at time t
$Q(t)$	Recovered and quitters of fast food consumption at time t

We propose a continuous model $PLSCQ$ to describe fast food consumption behavior in the population. The population is divided into five compartments denoted by P, L, S, C, and Q. The graphical representation of the proposed model is shown in Figure 1.The mathematical representation of the model consists of a system of nonlinear differential equations:

$$\begin{cases} \frac{dP^\alpha(t)}{dt^\alpha} = b - \beta_1 \frac{P(t)L(t)}{N} - \mu P(t), \\ \frac{dL^\alpha(t)}{dt^\alpha} = \beta_1 \frac{P(t)L(t)}{N} - (\mu + \beta_2)L(t), \\ \frac{dS^\alpha(t)}{dt^\alpha} = \beta_2 L(t) - (\mu + \delta_1 + \alpha_1 + \alpha_2)S(t), \\ \frac{dC^\alpha(t)}{dt^\alpha} = \alpha_1 S(t) - (\mu + \gamma + \delta_2)C(t), \\ \frac{dQ^\alpha(t)}{dt^\alpha} = \alpha_2 S(t) + \gamma C(t) - \mu Q(t). \end{cases} \tag{8}$$

where $P_0 \geq 0$, $L_0 \geq 0$, $S_0 \geq 0$, $C_0 \geq 0$, and $Q_0 \geq 0$.

3.2 Basic properties of the model

3.2.1 Invariant region

Before analyzing the dynamics of the proposed system, it is essential to verify that its solutions remain biologically meaningful, that is, nonnegative and bounded for all $t > 0$. This is guaranteed by the following lemma.

Lemma 2. Let the feasible region be defined as

$$\Omega = \Big\{(P(t), L(t), S(t), C(t), Q(t)) \in \mathbb{R}^5_+ : \\ 0 \leq P(t) + L(t) + S(t) + C(t) + Q(t) \leq N(0) + \frac{\Lambda}{\mu}\Big\},$$

with nonnegative initial conditions $P(0), L(0), S(0), C(0), Q(0) \geq 0$. Then, the region Ω is positively invariant for system (8).

Proof. Summing all the equations of system (8) gives the fractional derivative of the total population:

$$D^\alpha N(t) = \Lambda - \mu N(t). \tag{9}$$

Hence,

$$D^\alpha N(t) \leq \Lambda - \mu N(t).$$

Applying the Laplace transform, we obtain

$$\mathcal{L}\{N(t)\} - \lambda^{\alpha-1} N(0) \leq \frac{\Lambda}{\lambda} - \mu \mathcal{L}\{N(t)\}. \tag{10}$$

After rearrangement,

$$\mathcal{L}\{N(t)\} \leq \frac{\lambda^{\alpha-1}}{\lambda^\alpha + \mu} N(0) + \frac{\Lambda}{\lambda(\lambda^\alpha + \mu)}. \tag{11}$$

Using the Laplace transform of Mittag–Leffler functions, this yields

$$N(t) \leq E_{\alpha,1}(-\mu t^\alpha) N(0) + \frac{\Lambda}{\mu}\Big(1 - E_{\alpha,1}(-\mu t^\alpha)\Big). \tag{12}$$

Since $0 \leq E_{\alpha,1}(-\mu t^\alpha) \leq 1$ for all $t \geq 0$, it follows that

$$N(t) \leq N(0) + \frac{\Lambda}{\mu}. \tag{13}$$

Thus, all trajectories remain confined within the region Ω. □

3.2.2 Reduced system and existence of solutions

The first three equations of system (8) do not depend explicitly on C and Q. Therefore, it suffices to consider the reduced system:

$$\begin{cases} D^\alpha P(t) = b - \beta_1 \frac{P(t)L(t)}{N} - \mu P(t), \\ D^\alpha L(t) = \beta_1 \frac{P(t)L(t)}{N} - (\mu + \beta_2)L(t), \\ D^\alpha S(t) = \beta_2 L(t) - (\mu + \delta_1 + \alpha_1 + \alpha_2)S(t). \end{cases} \tag{14}$$

Theorem 1. The reduced system (14) admits a unique global solution in the region Ω.

Proof. Let $X(t) = [P(t), L(t), S(t)]^T$, and write $D^\alpha X(t) = F(X(t))$. It can be verified that $F(X)$ and its Jacobian $\partial F/\partial X$ are continuous, satisfying the conditions of Lemma 1. Furthermore, $F(X)$ can be decomposed as

$$F(X(t)) = \Delta + (LM_1 + M_2)X(t),$$

with

$$\Delta = \begin{bmatrix} \Lambda \\ 0 \\ 0 \end{bmatrix}, \quad M_1 = \begin{bmatrix} -\mu & 0 & 0 \\ 0 & -(\mu+\beta_2) & 0 \\ 0 & \beta_2 & -(\mu+\delta_1+\alpha_1+\alpha_2) \end{bmatrix}, \quad M_2 = \begin{bmatrix} -\frac{\beta_1}{N} & 0 & 0 \\ \frac{\beta_1}{N} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Using suitable bounds, one finds

$$\|F(X(t))\| \le \|\Delta\| + (c\|M_1\| + \|M_2\|)\|X(t)\|, \quad c = \frac{N(0) + L}{\mu},$$

which ensures global existence and uniqueness of the solution. □

3.2.3 Positivity of solutions

Theorem 2. For any nonnegative initial conditions $P(0), L(0), S(0), C(0), Q(0) \ge 0$, the solutions of system (8) remain nonnegative for all $t > 0$.

Proof. Consider the first equation of the reduced system:

$$D^\alpha P(t) \ge -\left(\beta_1 \frac{L(t)}{N} + \mu\right) P(t).$$

Since $L(t)$ is bounded by a constant m, we can write

$$D^{\alpha}P(t) \geq -dP(t), \quad d = \frac{\beta_1 m}{N} + \mu.$$

Applying the Laplace transform gives

$$\mathcal{L}\{P(t)\} \geq \frac{\lambda^{\alpha-1}}{\lambda^{\alpha}+d}P(0).$$

Using the inverse Laplace relation, this implies

$$P(t) \geq E_{\alpha,1}(-dt^{\alpha})P(0).$$

Since $E_{\alpha,1}(-dt^{\alpha}) \geq 0$ for all $t \geq 0$, it follows that $P(t) \geq 0$. The same reasoning applied to the second and third equations ensures that $L(t) \geq 0$ and $S(t) \geq 0$ for all $t \geq 0$. Finally, as the equations for $C(t)$ and $Q(t)$ involve only nonnegative inflows and natural death, they also remain nonnegative. Thus, all model compartments are positive for all $t > 0$. □

4 Stability analysis of the model

4.1 Equilibrium points

In this model, there are two equilibrium points: The disease-free equilibrium point and the disease-present equilibrium point. The equilibrium points are obtained by setting the right-hand side of the system (8) to zero.

The disease-free equilibrium point is given by

$$E^0_{eq} = \left(\frac{b}{\mu}, 0, 0\right),$$

while the disease-present equilibrium point is

$$E^*_{eq} = (P^*, L^*, S^*),$$

is achieved when the disease exists $L \neq 0$ and $S \neq 0$, where

$$P^* = \frac{b}{\mu R_0}, \tag{15}$$

$$L^* = \frac{b(R_0-1)}{\beta_1}, \tag{16}$$

$$S^* = \frac{b\beta_2(R_0 - 1)}{\beta_1(\mu + \delta_1 + \alpha_1 + \alpha_2)}, \tag{17}$$

and

$$R_0 = \frac{\beta_1}{(\mu + \beta_2)}, \tag{18}$$

where R_0 is the basic reproduction number.

4.2 Local stability analysis

We now investigate the local stability of the equilibria E^0 (fast food consumption-free) and E^* (fast food consumption endemic state). The stability conditions are established using the Jacobian matrix and the properties of fractional-order systems.

4.2.1 Stability of the consumption-free equilibrium

Theorem 3. The fast food consumption-free equilibrium E^0 of system (14) is locally asymptotically stable if $R_0 \leq 1$, and unstable if $R_0 > 1$.

Proof. The Jacobian matrix of the system at E^0 is given by

$$J(E^0) = \begin{pmatrix} -\mu & -\beta_1 & 0 \\ 0 & \beta_1 - \mu - \beta_2 & 0 \\ 0 & \beta_2 & -(\mu + \delta_1 + \alpha_1 + \alpha_2) \end{pmatrix}. \tag{19}$$

The characteristic polynomial is obtained from

$$\det(J(E^0) - \lambda I_3) = 0.$$

The eigenvalues are

$$\lambda_1 = -\mu, \quad \lambda_2 = \beta_1 - (\mu + \beta_2), \quad \lambda_3 = -(\mu + \delta_1 + \alpha_1 + \alpha_2).$$

Introducing the basic reproduction number

$$R_0 = \frac{\beta_1}{\mu + \beta_2}, \tag{20}$$

we can rewrite

$$\lambda_2 = (\mu + \beta_2)(R_0 - 1).$$

Thus - If $R_0 < 1$, then $\lambda_2 < 0$, and since $\lambda_1 < 0$ and $\lambda_3 < 0$, all eigenvalues satisfy $|\arg(\lambda_i)| > \frac{\alpha\pi}{2}$ for $i = 1, 2, 3$. Hence, E^0 is locally asymptotically stable. - If $R_0 > 1$, then $\lambda_2 > 0$, so E^0 becomes unstable. □

4.2.2 Stability of the consumption endemic equilibrium

Theorem 4. The fast food consumption equilibrium E^* is locally asymptotically stable whenever $R_0 > 1$, and unstable otherwise.

Proof. The Jacobian matrix evaluated at E^* is

$$J(E^*) = \begin{pmatrix} -\beta_1 \frac{L^*}{N} - \mu & -\beta_1 \frac{P^*}{N} & 0 \\ \beta_1 \frac{L^*}{N} & -\beta_1 \frac{P^*}{N} - (\mu + \beta_2) & 0 \\ 0 & \beta_2 & -(\mu + \delta_1 + \alpha_1 + \alpha_2) \end{pmatrix}. \tag{21}$$

At equilibrium, the components are

$$P^* = \frac{b}{\mu R_0}, \quad L^* = \frac{b(R_0 - 1)}{\beta_1}, \quad S^* = \frac{b\beta_2(R_0 - 1)}{\beta_1(\mu + \delta_1 + \alpha_1 + \alpha_2)}.$$

One eigenvalue is

$$\lambda_1 = -(\mu + \delta_1 + \alpha_1 + \alpha_2) < 0,$$

which clearly satisfies the stability condition. The remaining two eigenvalues satisfy the quadratic polynomial

$$P(\lambda) = \lambda^2 + a_1\lambda + a_2, \tag{22}$$

with coefficients

$$a_1 = \beta_1 \frac{L^*}{N} + \mu > 0, \qquad a_2 = \beta_1^2 \frac{P^*}{N} \frac{L^*}{N} > 0.$$

By the Routh–Hurwitz stability criterion, both roots of $P(\lambda)$ have negative real parts. Therefore, all eigenvalues of $J(E^*)$ satisfy $|\arg(\lambda_i)| > \frac{\alpha\pi}{2}$, and the equilibrium E^* is locally asymptotically stable. □

4.3 Global stability

In this section, we establish the global stability of both the consumption-free equilibrium E^0 and the endemic equilibrium E^* of system (8). The analysis is carried out using the Lyapunov direct method combined with LaSalle's invariance principle.

4.3.1 Global stability of the consumption-free equilibrium

Theorem 5. The fast food consumption-free equilibrium E^0 is globally asymptotically stable whenever $R_0 < 1$, and unstable otherwise.

Proof. We define the Lyapunov function candidate

$$V(P, L, S, C, Q) = \tfrac{1}{2}\Big[(P - P_0)^2 + L^2\Big] + \frac{(2\mu + \beta_1^2)}{\beta_1} L,$$

where $P_0 = \frac{b}{\mu}$ represents the steady-state of potential consumers in the absence of fast food consumption.

The time derivative of V along solutions of system(16) yields

$$\frac{dV}{dt} \leq \big[(P - P_0) + L\big]\left(\frac{dP}{dt} + \frac{dL}{dt}\right) + \frac{(2\mu + \beta_2)}{\beta_1}\frac{dL}{dt}.$$

Substituting the system equations and simplifying, we obtain

$$\frac{dV}{dt} \leq -\mu(P - P_0)^2 - (\mu + \beta_2)L^2 - \frac{(\mu + \beta_2)(2\mu + \beta_1^2)N}{\beta_1}(1 - R_0)L.$$

Hence,

$$\frac{dV}{dt} \leq 0 \quad \text{for all} \quad R_0 \leq 1.$$

Moreover, $\frac{dV}{dt} = 0$ holds only if $P = P_0$ and $L = 0$. By LaSalle's invariance principle [11], the equilibrium E^0 is therefore globally asymptotically stable when $R_0 < 1$. □

4.3.2 Global stability of the endemic equilibrium

Theorem 6. The fast food consumption endemic equilibrium E^* is globally asymptotically stable whenever $R_0 > 1$.

Proof. Consider the following Lyapunov function defined on the positive orthant

$$V(P,L) = \left(P - P^*\right) \ln\left(\frac{P}{P^*}\right) + \left(L - L^*\right) \ln\left(\frac{L}{L^*}\right),$$

where (P^*, L^*) denote the endemic equilibrium values.

The derivative of V along the trajectories of system (16) is

$$\frac{dV}{dt} \leq \left(1 - \frac{P}{P^*}\right) \frac{dP}{dt} + \left(1 - \frac{L}{L^*}\right) \frac{dL}{dt}.$$

Using the model equations for $\frac{dP}{dt}$ and $\frac{dL}{dt}$, we obtain after simplification

$$\frac{dV}{dt} \leq -b \left(P - P^*\right)^2 \frac{P}{P^*} \leq 0.$$

Clearly, $\frac{dV}{dt} = 0$ only when $P = P^*$. Thus, the largest invariant set in which $\frac{dV}{dt} = 0$ corresponds exactly to the endemic equilibrium E^*.

Applying LaSalle's invariance principle [11], we conclude that E^* is globally asymptotically stable when $R_0 > 1$. □

4.4 Sensitivity analysis of R_0

Sensitivity analysis is a useful tool for assessing the robustness of a model with respect to parameter variations. It allows us to determine which parameters (Table 2) have the greatest impact on the basic reproduction number R_0. Following the method described in Chitnis, Hyman, and Cushing [4], we calculate the normalized forward sensitivity indices of R_0.

Let $\Upsilon_{R_0}^{m}$ denote the sensitivity index of R_0 with respect to the parameter m, defined as

$$\Upsilon_{R_0}^{m} = \frac{m}{R_0} \frac{\partial R_0}{\partial m}.$$

Applying this formula, we obtain

$$\begin{cases} \Upsilon_{R_0}^{\beta_1} = 1, \\ \Upsilon_{R_0}^{\beta_2} = -\dfrac{\beta_2}{\mu + \beta_2}, \\ \Upsilon_{R_0}^{\mu} = -\dfrac{\mu}{\mu + \beta_2}. \end{cases} \tag{23}$$

These results reveal that the parameter β_1 (effective contact rate) has the strongest influence on R_0. Specifically, an increase in β_1 leads to a proportional increase in R_0, while a decrease reduces it accordingly. On the other hand, both the natural death rate μ and the transition rate β_2 exert a negative influence on R_0, but their effects are proportionally smaller in magnitude.

From a public health perspective, increasing mortality (μ) or artificially accelerating progression to excessive consumption (β_2) are neither ethical nor practical strategies. Therefore, the most realistic and effective approach is to target β_1, that is, reducing the effective contact rate through awareness campaigns, limiting exposure to aggressive marketing, and promoting healthier social norms. In this context, prevention efforts appear to be more efficient than treatment-oriented interventions.

Table 2: Sensitivity indices of the parameters influencing R_0.

Parameter	**Description**	**Sensitivity Index**
β_1	Effective contact rate	$+1$
μ	Natural death rate	-0.35
β_2	Transition rate from L to S	-0.65

In summary, the sensitivity analysis highlights that preventive measures aimed at reducing β_1 (the effective contact rate) are the most promising strategy for controlling and mitigating the spread of fast food consumption behavior in the population.

5 Numerical simulations and discussion

To validate and illustrate the theoretical results obtained in the previous sections, we now present numerical simulations of the fractional-order fast food consumption model. The numerical experiments are carried out using the *generalized Euler method* (GEM), which extends the classical Euler scheme to fractional-order systems (See [12]). This method is widely used in the literature due to its ease of implementation and its ability to provide accurate approximations of fractional dynamics.

Let the integration interval be $[0, T]$, which is divided into k subintervals of equal length $h = \frac{T}{k}$. The discrete nodes are denoted by $t_j = jh$, with $j = 0, 1, \dots, k-1$. For a system of order $0 < \alpha \leq 1$, the GEM update rule is given by

$$P_{j+1} = P_j + \frac{h^\alpha}{\Gamma(\alpha+1)} f_1(P_j, L_j, S_j, C_j, Q_j),$$

$$L_{j+1} = L_j + \frac{h^\alpha}{\Gamma(\alpha+1)} f_2(P_j, L_j, S_j, C_j, Q_j),$$

$$S_{j+1} = S_j + \frac{h^\alpha}{\Gamma(\alpha+1)} f_3(P_j, L_j, S_j, C_j, Q_j),$$

$$C_{j+1} = C_j + \frac{h^\alpha}{\Gamma(\alpha+1)} f_4(P_j, L_j, S_j, C_j, Q_j),$$

$$Q_{j+1} = Q_j + \frac{h^\alpha}{\Gamma(\alpha+1)} f_5(P_j, L_j, S_j, C_j, Q_j),$$

where f_i corresponds to the right-hand side of the fractional system

$$\frac{d^\alpha P(t)}{dt^\alpha} = f_1(P, L, S, C, Q, t), \quad \frac{d^\alpha L(t)}{dt^\alpha} = f_2(P, L, S, C, Q, t),$$

$$\frac{d^\alpha S(t)}{dt^\alpha} = f_3(P, L, S, C, Q, t), \quad \frac{d^\alpha C(t)}{dt^\alpha} = f_4(P, L, S, C, Q, t),$$

$$\frac{d^\alpha Q(t)}{dt^\alpha} = f_5(P, L, S, C, Q, t).$$

In this study, the values presented in Table 3 were used based on hypothetical scenarios due to the lack of real data, in order to construct and analyze the model. This approach allows exploration of different model behaviors and identification of potential limitations, to be validated later once real data become available.

Table 3: Parameters for the fast food consumption model (1).

Parameter	**Description**	**Value in d^{-1}**	**Source**
b	Recruitment rate of susceptibles	159.5	Assumed
β_1	Contact rate with moderate consumers	0.4	Assumed
μ	Natural death rate	0.1595	Assumed
β_2	Proportion joining addicted class	0.3	Assumed
α_1	Rate of violent consumers	0.03	Assumed
α_2	Rate of accident-prone consumers	0.02	Assumed
α_3	Rate of recovery	0.3	Assumed
δ_1	Death rate from heavy consumption	0.035	Assumed
δ_2	Death rate due to traffic accidents	0.002	Assumed
δ_3	Death rate from violence	0.001	Assumed
γ_1	Recovery rate of violent consumers	0.001	Assumed
γ_2	Recovery rate of accident-prone consumers	0.001	Assumed

Case 1: Healthy-consumption equilibrium ($R_0 < 1$)

Figures 2 and 4 illustrate the temporal dynamics of the five compartments when $R_0 = 0.87 < 1$. As predicted by the local and global stability analyses, all trajectories converge toward the consumption-free equilibrium E^0. Specifically, the compartments of moderate consumers $L(t)$, excessive consumers $S(t)$, obese individuals $C(t)$, and quitters $Q(t)$ asymptotically decay to zero. In contrast, the class of potential consumers $P(t)$ approaches a positive constant value.

This confirms that if $R_0 < 1$, the system evolves toward a healthy equilibrium where harmful consumption habits vanish from the population, independently of initial conditions. The fractional order α plays a role in the rate of convergence:
- Higher values of α accelerate stabilization,
- Smaller values of α prolong transient oscillations due to the memory effect intrinsic to fractional-order models.

Hence, the order of the fractional derivative acts as a regulator of the convergence speed, without altering the qualitative long-term dynamics.

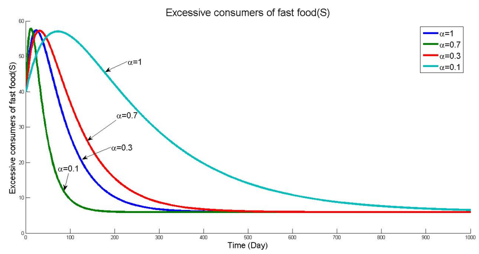
Case 2: Endemic fast-food equilibrium ($R_0 > 1$)

When the basic reproduction number exceeds unity ($R_0 = 1.30 > 1$), the trajectories converge towards the endemic equilibrium E^*, as illustrated in Figures 3 and 5. In this scenario, excessive consumers $S(t)$, moderate consumers $L(t)$, obese individuals $C(t)$, and quitters $Q(t)$ all stabilize at positive values, reflecting the persistence of unhealthy eating patterns in the population. Meanwhile, potential consumers $P(t)$ approach an intermediate plateau that ensures a continuous inflow into the excessive-consumer class. Once again, the fractional order α controls only the rate of convergence: Larger α accelerates stabilization, while smaller values prolong the transient oscillations.

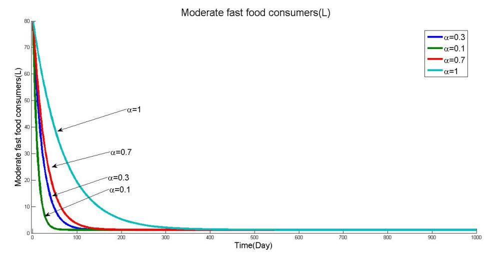
Comparative discussion

The numerical simulations confirm the decisive role of the basic reproduction number R_0 as a threshold parameter governing the system dynamics. When $R_0 \leq 1$, the healthy-consumption equilibrium E^0 is globally stable, leading to the eventual elimination of excessive fast-food consumption. In contrast, when $R_0 > 1$, the endemic equilibrium E^* becomes globally attractive, ensuring the persistence of unhealthy dietary habits in the population.

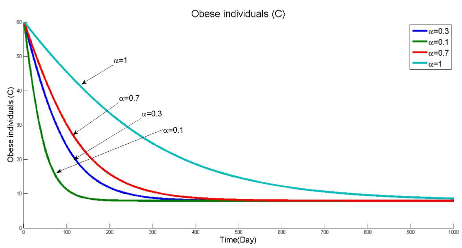
In both regimes, the fractional order α does not affect the nature of the long-term equilibrium but influences the rate of convergence toward it. Higher values of α accelerate the stabilization process, whereas lower values prolong transient oscillations due to the memory effect characteristic of fractional-order systems.



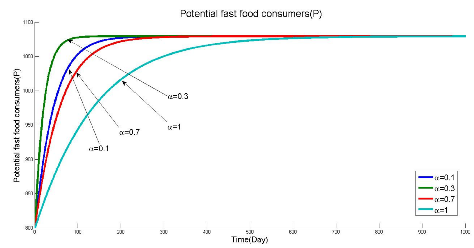
(a) Excessive consumers of fast food $S(t)$



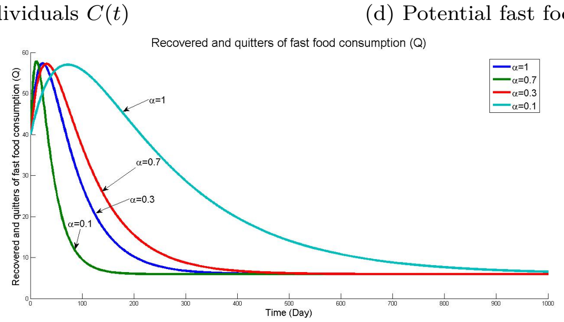
(b) Moderate fast food consumers $L(t)$



(c) Obese individuals $C(t)$



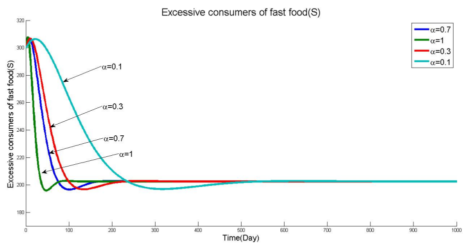
(d) Potential fast food consumers $P(t)$



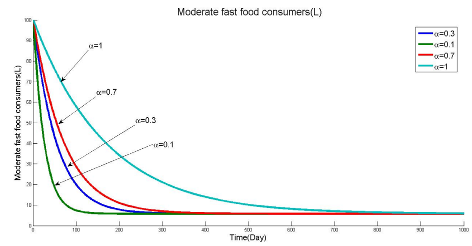
(e) Recovered and quitters $Q(t)$

Figure 2: Dynamics of the fast-food consumption model under the condition $R_0 = 0.87 < 1$. All trajectories converge towards the healthy-consumption equilibrium E^0. The excessive consumers $S(t)$, moderate consumers $L(t)$, and obese individuals $C(t)$ gradually vanish, while the potential consumers $P(t)$ stabilize at a steady state and the recovered group $Q(t)$ plays a transient role. The order of the fractional derivative α strongly influences the rate of convergence: Larger α leads to faster stabilization, whereas smaller α prolongs the persistence of unhealthy habits due to memory effects.

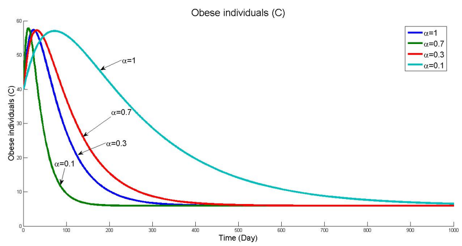
These findings underscore the joint influence of epidemiological thresholds (R_0) and fractional-order effects (α) in shaping long-term outcomes. From a practical standpoint, such insights provide valuable guidance for the design of effective intervention strategies aimed at mitigating the spread of fast-food consumption and reducing its associated health risks.



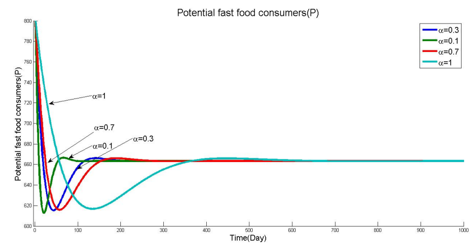
(a) Excessive consumers of fast food $S(t)$



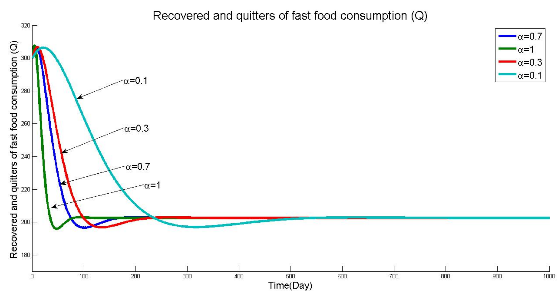
(b) Moderate fast food consumers $L(t)$



(c) Obese individuals $C(t)$



(d) Potential fast food consumers $P(t)$

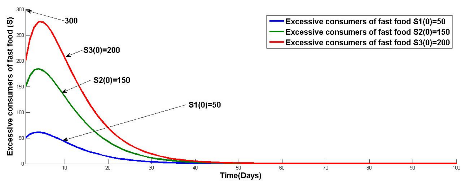


(e) Recovered and quitters $Q(t)$

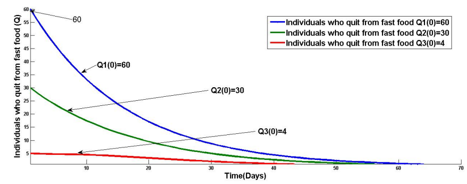
Figure 3: Dynamics of the fast-food consumption model under the condition $R_0 = 1.30 > 1$. All trajectories converge towards the endemic equilibrium E^*, where excessive consumption persists. The populations of excessive consumers $S(t)$ and obese individuals $C(t)$ stabilize at positive values, reflecting the persistence of unhealthy eating habits. Moderate consumers $L(t)$ and recovered individuals $Q(t)$ decrease but remain nonzero, while potential consumers $P(t)$ converge to an intermediate plateau. The fractional order α influences only the speed of convergence: Larger α leads to faster stabilization, whereas smaller α prolongs the transient dynamics due to memory effects.

6 Conclusion

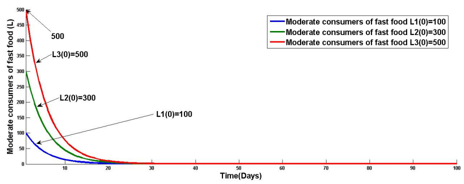
In this work, we proposed and analyzed a fractional-order mathematical model describing the dynamics of fast-food consumption in a population. The basic reproduction number R_0 was explicitly derived, and a sensitivity analysis was conducted to determine the parameters exerting the greatest influence on its value.



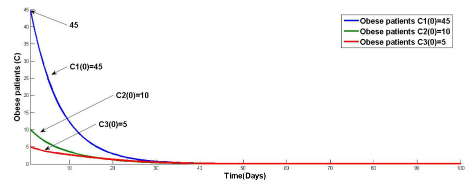
(a) Excessive consumers of fast food $S(t)$



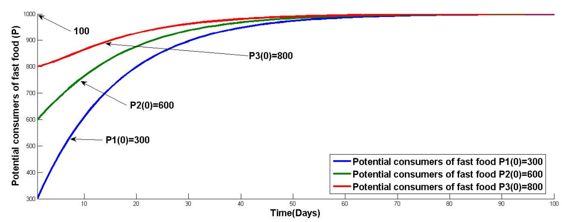
(b) Individuals who quit fast food $Q(t)$



(c) Moderate consumers of fast food $L(t)$



(d) Obese patients $C(t)$



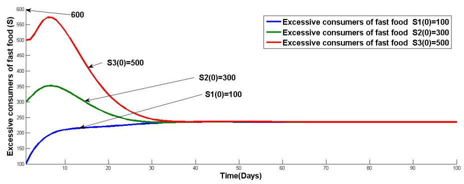
(e) Potential consumers of fast food $P(t)$

Figure 4: Dynamics of the fast-food consumption model with $R_0 = 0.87 < 1$ under different initial conditions. In all scenarios, the trajectories converge towards the healthy-consumption equilibrium E^0. Excessive consumers $S(t)$, moderate consumers $L(t)$, obese individuals $C(t)$, and quitters $Q(t)$ decay asymptotically to zero, confirming the disappearance of unhealthy eating habits. Conversely, the population of potential consumers $P(t)$ increases and stabilizes at a positive steady state, indicating that while individuals remain at risk, the transition to excessive consumption is not sustained. These results validate the global stability of E^0 when $R_0 < 1$, regardless of the initial conditions.

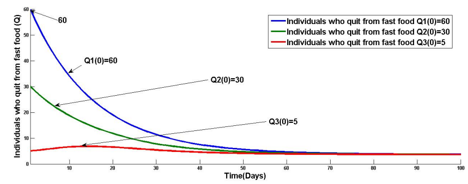
The stability analysis revealed that the consumption-free equilibrium E^0 is locally and globally asymptotically stable when $R_0 \leq 1$, whereas the endemic equilibrium E^* becomes locally and globally asymptotically stable when $R_0 > 1$. The global stability of both equilibria was established through suitable Lyapunov functions combined with LaSalle's invariance principle.

Numerical simulations were then performed using the Generalized Euler Method to validate these theoretical findings. The results confirmed that while the fractional order α does not modify the equilibrium states themselves, it significantly influences the convergence speed: Larger α accelerates stabilization, while smaller values amplify transient dynamics due to memory effects.

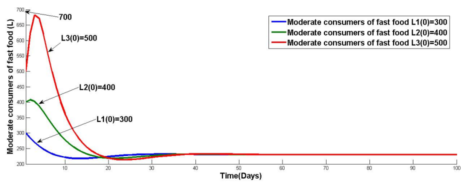
As directions for future research, the current model could be extended by incorporating demographic heterogeneity (e.g., age or gender structures) or socioeconomic factors influencing dietary habits. Particular attention could be given to adolescents and young adults, who are



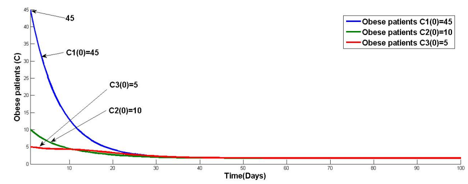
(a) Excessive consumers of fast food $S(t)$



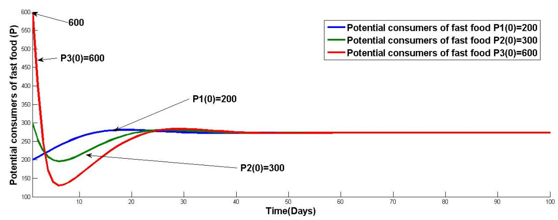
(b) Individuals who quit fast food $Q(t)$



(c) Moderate consumers of fast food $L(t)$



(d) Obese patients $C(t)$



(e) Potential consumers of fast food $P(t)$

Figure 5: Dynamics of the fast-food consumption model with $R_0 = 1.30 > 1$ under different initial conditions. All trajectories converge towards the endemic equilibrium E^*. Excessive consumers $S(t)$, moderate consumers $L(t)$, obese individuals $C(t)$, and quitters $Q(t)$ stabilize at positive values, confirming the persistence of unhealthy eating habits in the population. The potential consumers $P(t)$ decrease initially before reaching a steady state, indicating a continuous flow towards excessive consumption. These results highlight the stability of E^* when $R_0 > 1$, independent of initial conditions.

generally more vulnerable to aggressive fast-food marketing strategies and at higher risk of developing obesity-related health complications.

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Conflicts of Interest

The author declares no conflicts of interest.

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